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A NOTE ON THE CYLINDRICAL WAVES WITH TRANSVERSE DISTORTION IN A PLASMA WITH VORTEX ELECTRON DISTRIBUTION

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Abstract. In the present work, employing the conventional reductive perturbation method and the nonlinear field equations of a plasma consisting of a cold electron fluid, hot electrons obeying a non-isothermal (trapped/vortex -like) distribution and stationary ions with transverse distortion, we studied the propagation of nonlinear waves in such a plasma medium and obtained the modified CKP equation. Seeking a progressive wave solution to this evolution equation we obtained the exact analytical solution. It is observed that the speed of the solitary wave is directional dependent and the wave front is not circularly cylindrical surface any more

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1. Introduction

In examining the properties of electron- acoustic solitary wave structures, Dubouloz et al.[1] assumed a one-dimensional, collisionless plasma model composed of cold electrons, hot electrons with Maxwellian distribution and the stationary ions. But, in applications, due to the formation of phase space holes caused by the trapping of hot electrons in a wave potential, the hot electrons may not obey the Maxwellian distribution. Therefore, in most space plasma the hot electrons follow the trapped/vortex-like distribution (Schamel[2]; Abbasi et. al.[3]). For that reason, in the present work, we shall consider a plasma model consisting of a cold electron fluid, hot electrons obeying a non-isothermal (trapped/vortex-like) distribution, and stationary ions.

For the case of one dimensional plasma models, the propagation of small-but-finite amplitude planar waves had been studied by several researchers (see, for instance, Washimi and Taniuti [4], Mamun and Shukla [5]) by employing the standard reductive perturbation method[6]. Due to nonlinearity and dispersive nature of the governing equations, in the long-wave limit, the KdV or modified KdV equations are obtained, for the lowest order term in the perturbation expansion. But the unbounded planar geometry may not be a realistic model in laboratory devices, and the space and geometric distortion on waves

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may always exist. Franz et al. [7] have shown that a purely one dimensional model cannot account for the observed features in the auroral region, especially at the higher polar altitudes. Recent theoretical studies [8-11] indicate that the properties of solitary waves in bounden non-planar cylindrical geometry are very different from those of in unbounded planar geometry. But all those studies are confined to the radial symmetry case. It is well known that the transverse distortion will always exist in higher dimensional systems, and the wave structure of the medium will be changed.

In the present work, employing the conventional reductive perturbation method and the nonlinear field equations of a plasma consisting of a cold electron fluid, hot electrons obeying a non-isothermal (trapped/vortex -like) distribution and stationary ions with transverse distortion, we studied the propagation of nonlinear waves in such a plasma medium and obtained the modified CKP equation. Seeking a progressive wave solution to this evolution equation we obtained the exact analytical solution. It is observed that the speed of the solitary wave is directional dependent and the wave front is not circularly cylindrical surface any more. It is shown that a solitary wave can be generated in a bounded cylindrical region under transverse disturbances.

2. Basic Field Equations

The dynamics of electron-acoustic waves is governed by the following equations:

$$\frac{\partial n_c}{\partial t} + \nabla.(n_c \mathbf{v}) = 0, \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} - \alpha \nabla \phi = 0, \tag{2}$$

$$\nabla^2 \phi = \frac{1}{\alpha} n_c + n_h - (1 + \frac{1}{\alpha}), \tag{3}$$

where n_c is the normalized cold electron number density, n_h is the normalized hot electron number density, $\mathbf{v}$ is the cold electron fluid velocity vector, ϕ is the electrostatic potential and the coefficient α is defined by $\alpha = n_{h0}/n_{c0}$, where n_{c0} and n_{h0} are the equilibrium values of the cold and hot electron number densities, respectively. The hot electron number density n_h (for $\beta < 0$) can be expressed by Schamel[2],

$$n_h = [1 - erf(\phi)]\exp(\phi) + \frac{2}{\sqrt{-\pi\beta}} \exp(-\phi^2) \int_0^{\phi} \exp(y^2) dy, \tag{4}$$

where $erf(\phi)$ is the error function. For $\phi << 1$ the equation (4) gives

$$n_h = 1 + \phi - \frac{4}{3\sqrt{\pi}}(1-\beta)\phi^{3/2} + \frac{\phi^2}{2} - \frac{8}{15\sqrt{\pi}}(1-\beta^2)\phi^{5/2} + \frac{\phi^3}{6} + \dots . \tag{5}$$

In this work, we shall assume that the field quantities are functions of (r, θ) in the cylindrical coordinates as well as the time variable t. For this case, the field equations (1)-(5) take the following form

$$\frac{\partial n_c}{\partial t} + \frac{\partial}{\partial r}(n_c u) + \frac{1}{r}(n_c u) + \frac{1}{r}\frac{\partial}{\partial \theta}(n_c v) = 0, \tag{6}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} - \frac{v^2}{r} - \alpha \frac{\partial \phi}{\partial r} = 0, \tag{7}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + \frac{uv}{r} - \alpha \frac{1}{r}\frac{\partial \phi}{\partial \theta} = 0, \tag{8}$$

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = -\frac{1}{\alpha} + \frac{n_c}{\alpha} + \phi - \frac{4}{3\sqrt{\pi}}(1-\beta)\phi^{3/2} + \dots \ , \tag{9}$$

where u and v are the cold electron fluid velocity components in r and θ directions, respectively.

For the asymptotic analysis of the field equations, we shall utilize the standard reductive perturbation method and introduce the following stretched coordinates:

$$\epsilon^{1/2}(r-t)=\xi,\quad \eta=\epsilon^{-1/2}\theta,\quad \tau=\epsilon^{3/2}r, \tag{10}$$

where ϵ is a small parameter characterizing the order of nonlinearity and dispersion. The dependent variables may be expanded as:

$$\begin{aligned} u &= \epsilon^2(u^{(1)}+\epsilon u^{(2)}+\epsilon^2 u^{(3)}+...),\\ v &= \epsilon^{5/2}(v^{(1)}+\epsilon v^{(2)}+\epsilon^2 v^{(3)}+...),\\ n_c &= 1+\epsilon^2(n_c^{(1)}+\epsilon n_c^{(2)}+\epsilon^2 n_c^{(3)}+...),\\ \phi &= \epsilon^2(\phi^{(1)}+\epsilon\phi^{(2)}+\epsilon^2\phi^{(3)}+...). \end{aligned} \tag{11}$$

Introducing the expansions (10) and (11) into the field equations (6)-(9) and setting the coefficients of like powers of ϵ equal to zero the following sets of differential equations are obtained:

O(ϵ^2) equations:

$$-\frac{\partial n_c^{(1)}}{\partial\xi}+\frac{\partial u^{(1)}}{\partial\xi}=0,\quad \frac{\partial u^{(1)}}{\partial\xi}+\alpha\frac{\partial\phi^{(1)}}{\partial\xi}=0,\quad n_c^{(1)}=-\alpha\phi^{(1)}. \tag{12}$$

O(ϵ^3) equations:

$$\begin{aligned} &-\frac{\partial n_c^{(2)}}{\partial\xi}+\frac{\partial u^{(2)}}{\partial\xi}+\frac{u^{(1)}}{\tau}+\frac{\partial u^{(1)}}{\partial\tau}+\frac{1}{\tau}\frac{\partial v^{(1)}}{\partial\eta}=0,\\ &\frac{\partial u^{(2)}}{\partial\xi}+\alpha\frac{\partial\phi^{(2)}}{\partial\xi}+\alpha\frac{\partial\phi^{(1)}}{\partial\tau}=0,\\ &\frac{\partial v^{(1)}}{\partial\xi}+\frac{\alpha}{\tau}\frac{\partial\phi^{(1)}}{\partial\eta}=0,\\ &n_c^{(2)}=\alpha\frac{\partial^2\phi^{(1)}}{\partial\xi^2}-\alpha\phi^{(2)}+\frac{4\alpha}{3\sqrt{\pi}}(1-\beta)(\phi^{(1)})^{3/2}. \end{aligned} \tag{13}$$

2.1. **Solution of the field equations.** From the solution of the set (12) we obtain

$$u^{(1)}=n_c^{(1)}=-\alpha\varphi,\quad \phi^{(1)}=\varphi(\xi,\eta,\tau), \tag{14}$$

where $\varphi(\xi,\eta,\tau)$ is an unknown function whose governing equation will be obtained later.

To obtain the solution for $O(\epsilon^3)$ equations, we introduce (14) into (13) to have

$$\begin{aligned} &-\frac{\partial n_c^{(2)}}{\partial\xi}+\frac{\partial u^{(2)}}{\partial\xi}-\alpha\frac{\partial\varphi}{\partial\tau}-\alpha\frac{\varphi}{\tau}+\frac{1}{\tau}\frac{\partial v_1}{\partial\eta}=0,\\ &\frac{\partial u^{(2)}}{\partial\xi}+\alpha\frac{\partial\phi^{(2)}}{\partial\xi}+\alpha\frac{\partial\varphi}{\partial\tau}=0,\\ &\frac{\partial v_1}{\partial\xi}=-\frac{\alpha}{\tau}\frac{\partial\varphi}{\partial\eta},\\ &n_c^{(2)}=\alpha\frac{\partial^2\varphi}{\partial\xi^2}-\alpha\phi^{(2)}+\frac{4\alpha}{3\sqrt{\pi}}(1-\beta)\varphi^{3/2}. \end{aligned} \tag{15}$$

Eliminating v_1, $n_c^{(2)}$, $u^{(2)}$ and $\phi^{(2)}$ between the equations (15) we obtain the following evolution equation

$$\frac{\partial}{\partial\xi}[\frac{\partial\varphi}{\partial\tau}+\frac{1}{2\tau}\varphi+\frac{(1-\beta)}{\sqrt{\pi}}\varphi^{1/2}\frac{\partial\varphi}{\partial\xi}+\frac{1}{2}\frac{\partial^3\varphi}{\partial\xi^3}]+\frac{1}{2\tau^2}\frac{\partial^2\varphi}{\partial\eta^2}=0. \quad (16)$$

This evolution equation is known as the cylindrical Kadomtsev-Petviashvili (CKP) equation that describes the acoustic waves in such a plasma medium.

2.2. **Progressive wave solution.** In this sub-section we shall seek a progressive wave solution to the CKP equation given in (16). For that purpose we set

$$\varphi=\psi(\zeta),\quad \zeta=\xi-(\frac{1}{2}\eta^2+\kappa\eta+c)\tau, \quad (17)$$

where κ is a constant and c corresponds to the wave speed along the line $\theta(\eta)=0$. Introducing (17) into (16) we obtain

$$[-(\frac{1}{2}\eta^2+\kappa\eta+c)\psi'+\frac{1}{2\tau}\psi+\frac{(1-\beta)}{\sqrt{\pi}}\psi^{1/2}\psi'+\frac{1}{2}\psi''']'$$
$$+\frac{1}{2\tau^2}[-\tau\psi+(\eta+\kappa)^2\tau^2\psi']'=0, \quad (18)$$

where the prime denotes the differentiation of the corresponding quantity with respect to ζ. Integrating the equation (18) twice with respect to ζ we obtain

$$(\frac{\kappa^2}{2}-c)\psi+\frac{2(1-\beta)}{3\sqrt{\pi}}\psi^{3/2}+\frac{1}{2}\psi''=A\zeta+B, \quad (19)$$

where A and B are two integration constants.

In the present work we shall be dealing with localized solutions, i. e., ψ and its derivatives vanish as $\zeta\to\pm\infty$. In this case the equation (19) reduces to

$$(\frac{\kappa^2}{2}-c)\psi+\frac{2(1-\beta)}{3\sqrt{\pi}}\psi^{3/2}+\frac{1}{2}\psi''=0. \quad (20)$$

This nonlinear ordinary differential equation assumes the solitary wave solution of the form

$$\psi=a\ \mathrm{sech}^4\mu\zeta, \quad (21)$$

where a is the constant wave amplitude and the other quantities are defined by

$$\mu^2=\frac{(1-\beta)}{15\sqrt{\pi}}a^{1/2},\quad \kappa^2=2c-\frac{16(1-\beta)}{15\sqrt{\pi}}a^{1/2}>0. \quad (22)$$

The velocity v_p of the solitary wave may be defined by

$$v_p=(\frac{1}{2}\eta^2+\kappa\eta+c). \quad (23)$$

As is seen from the equation (23) the velocity of propagation is a function of the direction angle $\eta(\epsilon^{-1/2}\theta)$.

We further note that the wave front ($\zeta=constant\ \ surface$) is not a circularly cylindrical surface, it is rather a distorted surface in the polar coordinates.

In conclusion, a modified cylindrical KP equation describing the acoustic waves with transverse distortion in an electron-acoustic plasma with vortex electron distribution is derived by the standard reductive perturbation method. An exact solitary wave solution of the modified CKP equation is obtained. It is shown that a solitary wave can be generated in a bounded cylindrical region under transverse disturbances. We further note that the speed of propagation is directional dependent and the wave front ($\zeta = constant\ surface$)is not a circularly cylindrical surface anymore, it is rather distorted.

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