

DISTANCE SPECTRA OF SOME GRAPH OPERATIONS AND SOME NEW DISTANCE EQUIENERGETIC GRAPHS OF DIAMETER 3

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ABSTRACT. Two graphs of same order are said to be distance equienergetic if their distance energies are same. In this paper, we first give a partial insight on the distance spectrum of Mycielskian graphs and then we focus on constructing distance equienergetic graphs by introducing three new graph operations. As an application of our results, we construct some new class of distance equienergetic graphs of diameter 3 on $18+2n$ vertices for all $n \geq 1$.

Keywords: Distance matrix, distance equienergetic graphs, Mycielskian graphs.

AMS Subject Classification: 05C50.

1. INTRODUCTION

Graphs considered in this paper are simple, i.e., graphs without multiple edges and loops. In what follows, for a graph G with vertex labels v_1, v_2, \dots, v_n , the adjacency matrix of G , denoted by $A(G)$, is the $n \times n$ matrix $[a_{ij}]$, where $a_{ij} = 1$ if v_i and v_j are adjacent and 0 otherwise, and the spectrum of G is the collection of the eigenvalues of $A(G)$. Since $A(G)$ is a real symmetric matrix, all its eigenvalues are real and so they can be ordered. We denote the i th largest eigenvalue of $A(G)$ (or G) by $\lambda_i(G)$. There is an extensive literature available on the adjacency matrix and we refer the reader to a classical book by Cvetković, Doob and Sachs [11] for more knowledge about this topic. The distance between two vertices v_i and v_j in G , denoted by $d(v_i, v_j)$, is the length of a shortest path between them and the maximum distance between any pair of vertices in G is known as the diameter of the graph G . The matrix $D(G)=[d_{ij}]_{n \times n}$, where $d_{ij} = d(v_i, v_j)$, is the distance matrix of a graph G and the corresponding spectrum is known as distance spectrum of G . The distance matrix and its eigenvalues are used to study the problems in data communication systems [14]. Recently, the study of distance spectra of graph operations have attracted many researchers, for example, see [1, 18] and also refer [5].

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§ Manuscript received: July 29, 2017; accepted: May 24, 2018.

TWMS Journal of Applied and Engineering Mathematics, Vol.9, No.3, © Işık University, Department of Mathematics, 2019; all rights reserved.

In the year 1978, Gutman [15] introduced and defined the energy of a graph G as the sum of the absolute values of the eigenvalues of $A(G)$. In a similar fashion, Indulal et al. introduced the distance energy [17] of a graph, which is defined as the sum of the absolute values of the eigenvalues of the distance matrix $D(G)$. In recent years, the concept of graph energy has been extensively studied by many researchers. Studies on graph energy can be found in a book [20] by Li et al. and therein cited references. The chemical connection of graph energy can be traced back to the 1930s (Hückel molecular orbital (HMO) theory) see [20] and that of distance energy can be found in [10]. Two graphs of same order are said to be distance equienergetic if their distance energies are same. In [22], Ramane et al. presented a pair of distance equienergetic graphs of diameter 2 on $9 + n$ vertices for all $n \geq 1$. Various constructions of distance equienergetic graphs and some other types of equienergetic graphs can be found in [2, 3, 4, 16, 6, 9, 13, 19, 23, 24, 25, 26] and therein cited references.

Let $G = (V, E)$ be a graph with vertex set $V = V(G)$ and edge set $E = E(G)$. The Mycielskian $\mu(G)$ of a graph $G = (V, E)$ is the graph with vertex set $V(\mu(G)) = V \cup V' \cup \{u\}$, where $V' = \{x' : x \in V\}$ and edge set $E(\mu(G)) = E \cup \{yx' : xy \in E\} \cup \{x'u : x' \in V'\}$. The Mycielskian graph operation was introduced in the year 1955 by Mycielski [21] in order to construct triangle-free graphs with arbitrarily large chromatic number. It may be noted that the diameter of the Mycielskian graph $\mu(G)$ is at most 4, see [8]. Recently, Balakrishnan et al. [7] computed the spectrum of the Mycielskian of a regular graph. Motivated by the works on distance spectrum and distance equienergetic graphs, in this paper, we first give a partial insight on the distance spectrum of Mycielskian graphs and then we focus on constructing distance equienergetic graphs by introducing three new graph operations. As an application of our results, we construct some new class of distance equienergetic graphs of diameter 3 on $18 + 2n$ vertices for all $n \geq 1$.

2. MAIN RESULTS

In this section, we compute the distance spectrum of Mycielskian and three new graph operations. Let G and H be two graphs with vertex sets $V(G) = \{u_1, u_2, \dots, u_n\}$ and $V(H) = \{v_1, v_2, \dots, v_m\}$, respectively. Let G_i ($i = 1, 2$) and H_i denote the i^{th} copy of G and H , respectively. Using G_i and H_i , we define three new graph operations denoted by $\Theta_1(G, H)$, $\Theta_2(G, H)$ and $\Theta_3(G, H)$ as follows:

Definition 2.1. The graph $\Theta_1(G, H)$ is obtained from G_i and H_i ($i = 1, 2$) by joining each

- a. vertex of G_i to every vertex of H_i (for $i = 1, 2$)
- b. vertex of G_1 to every vertex of G_2
- c. vertex of H_1 to the corresponding vertex in H_2 .

Definition 2.2. $\Theta_2(G, H)$ is defined as the graph obtained from $\Theta_1(G, H)$ by removing all the edges between H_1 and H_2 , and then adding an edge between the vertex v_i of H_1 and v_j of H_2 ($i \neq j$ and $i, j = 1, 2, \dots, m$), whenever v_i and v_j are not adjacent in H .

Definition 2.3. The graph $\Theta_3(G, H)$ is obtained from $\Theta_2(G, H)$ by removing all edges $u_i u_j$ ($i \neq j$ and $i, j = 1, 2, \dots, n$) between G_1 and G_2 .

Remark 2.1. Let v_{ik} ($i = 1, 2$) denote the vertex v_k of H in the i^{th} copy of H , i.e., H_i . From the definition of $\Theta_1(G, H)$, it follows that the diameter of $\Theta_1(G, H)$ is either 2 or 3. If H has a pair of non adjacent vertices v_i and v_j , then the distance between v_{1i} and v_{2j} in $\Theta_1(G, H)$ is 3. If H is a complete graph, then it is clear that the diameter of $\Theta_1(G, H)$ is

2. Hence the diameter of $\Theta_1(G, H)$ is 2 if and only if H is a complete graph. Now, from the definition of $\Theta_i(G, H)$ ($i = 2, 3$), we see that the diameter of $\Theta_i(G, H)$ is at most 3. Since the distance between the vertices v_{1j} and v_{2j} ($j = 1, 2, \dots, m$) in $\Theta_i(G, H)$ is 3, it follows that $\Theta_i(G, H)$ ($i = 2, 3$) is of diameter 3.

Example 2.1. Fig.1 describes the three graphs $\Theta_1(K_3, K_2)$, $\Theta_2(K_1, C_4)$ and $\Theta_3(2K_1, C_4)$. The dotted edges are those in graphs G, H and the solid edges are from the operation.

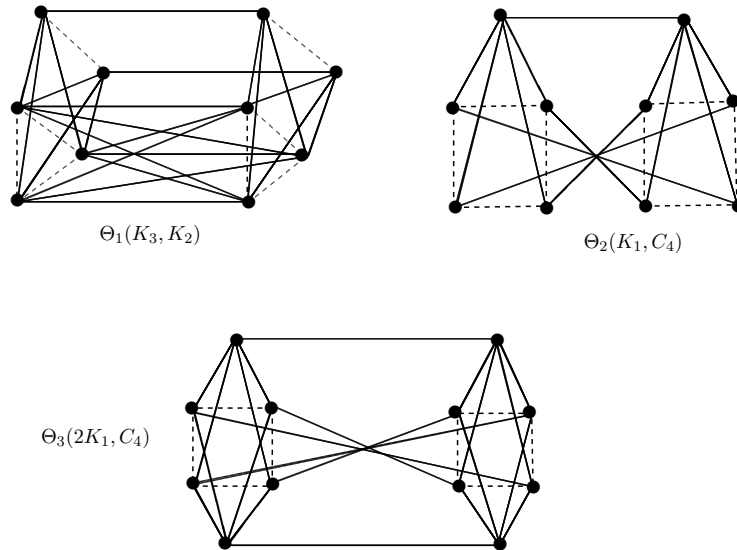


FIGURE 1. Graphs $\Theta_1(K_3, K_2)$, $\Theta_2(K_1, C_4)$ and $\Theta_3(2K_1, C_4)$.

The following theorem gives the distance spectrum of $\mu(G)$ in some cases.

Theorem 2.1. Let G be an r -regular graph with n vertices and let the diameter of G be at most 2. Then the distance spectrum of the Mycielskian graph $\mu(G)$ of G consists of $\frac{\lambda_i(G)(-1 \pm \sqrt{5}) - 4}{2}$ for $i = 2, 3, \dots, n$ and the three roots of the polynomial $t^3 + (-4n + r + 4)t^2 + (2nr - r^2 - 13n + 2r + 4)t + 2n^2 + 3nr - 10n$.

Proof. Using the fact that G is of diameter at most 2 and by proper labeling of the vertices of $\mu(G)$ the distance matrix of $\mu(G)$ can be written as follows:

$$D(\mu(G)) = \begin{bmatrix} 0 & \mathbf{1}_n^T & 2\mathbf{1}_n^T \\ \mathbf{1}_n & 2(J_n - I_n) & 2J_n - A(G) \\ 2\mathbf{1}_n & 2J_n - A(G) & 2(J_n - I_n) - A(G) \end{bmatrix},$$

where $\mathbf{1}_n = (1, 1, \dots, 1)^T$ of size n and J_n is the $n \times n$ matrix having all its entries as 1.

Let $X_1 = (1/\sqrt{n})(1, 1, \dots, 1)^T$ and $\{X_i\}_{i=1}^n$ be orthonormal eigenvectors of $A(G)$ and let δ_{ik} ($\neq 0$) be any scalar. Then for $i = 2, 3, \dots, n$, we have

$$D(\mu(G)) \begin{bmatrix} 0 \\ \delta_{ik}X_i \\ X_i \end{bmatrix} = - \begin{bmatrix} 0 \\ (2\delta_{ik} + \lambda_i(G))X_i \\ ((\delta_{ik} + 1)\lambda_i(G) + 2)X_i \end{bmatrix}.$$

The above matrix equation implies that $\begin{bmatrix} 0 \\ \delta_{ik}X_i \\ X_i \end{bmatrix}$ is an eigenvector of $D(\mu(G))$ corresponding to the eigenvalue $-\left(2 + \frac{\lambda_i(G)}{\delta_{ik}}\right)$ if and only if $2 + \frac{\lambda_i(G)}{\delta_{ik}} = \lambda_i(G)(\delta_{ik} + 1) + 2$, i.e., if and only if $\delta_{ik} = \frac{1}{2}(-1 \pm \sqrt{5})$. Therefore, setting $\delta_{ik} = \frac{1}{2}(-1 + (-1)^k\sqrt{5})$ ($k = 1, 2$), it follows that $\frac{\lambda_i(G)((-1)^k\sqrt{5} - 1) - 4}{2}$ is an eigenvalue of $D(\mu(G))$ corresponding to the eigenvector $\begin{bmatrix} 0 \\ \delta_{ik}X_i \\ X_i \end{bmatrix}$ ($i = 2, 3, \dots, n$ and $k = 1, 2$) and thus we have obtained $2(n - 1)$ of $2n + 1$ linearly independent eigenvectors of $D(\mu(G))$.

Now, we find the remaining three linearly independent eigenvectors of $D(\mu(G))$. Let $Y_1 = (1, 0, \dots, 0)^T$, $Y_2 = (0, \mathbf{1}_n^T, 0, \dots, 0)^T$ and $Y_3 = (0, \dots, 0, \mathbf{1}_n^T)^T$ be column vectors of size $2n + 1$. Then one can easily see that the vectors $\begin{bmatrix} 0 \\ \delta_{ik}X_i \\ X_i \end{bmatrix}$ ($i = 2, 3, \dots, n$ and $k = 1, 2$), Y_1 , Y_2 and Y_3 form a linearly independent set of order $2n+1$. Thus the remaining three linearly independent eigenvectors of $D(\mu(G))$ must be of the form $Z := \alpha Y_1 + \beta Y_2 + \gamma Y_3$ for some scalars (not all zero) α , β and γ . Hence

$$D(\mu(G))Z = tZ$$

for some scalar t .
i.e.,

$$\begin{aligned} n\beta + 2n\gamma - t\alpha &= 0 \\ \alpha + (2(n - 1) - t)\beta + (2n - r)\gamma &= 0 \\ 2\alpha + (2n - r)\beta + (2(n - 1) - r - t)\gamma &= 0. \end{aligned}$$

Upon eliminating α and β from the above equations, one can notice that the equation $D(\mu(G))Z = tZ$ holds if and only if t is a root of the polynomial $t^3 + (-4n + r + 4)t^2 + (2nr - r^2 - 13n + 2r + 4)t + 2n^2 + 3nr - 10n$. Thus we have listed all the eigenvalues of $D(\mu(G))$. This completes the proof. \square

Now, we compute the distance spectrum of $\Theta_i(G, H)$ ($i = 1, 2, 3$) using the following lemma.

Lemma 2.1 (see [11]). *If A, B, C, D are matrices with A being a non-singular matrix, then*

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |A||D - CA^{-1}B|.$$

Theorem 2.2. *Let G be an r_1 -regular graph on n vertices and H be an r_2 -regular graph on m vertices. Then the distance spectra of $\Theta_1(G, H)$ consists of $-(\lambda_i(G) + 2)$ repeating twice for $i = 2, 3, \dots, n$, $-2(\lambda_i(H) + 2)$ ($i = 2, 3, \dots, m$), 0 repeating $m - 1$ times and the four roots of the polynomial*

$$\begin{aligned} &[x^2 + (r_1 + m - n + 2)x + m(r_1 - 2n + 2)] \times \\ &[x^2 + (r_1 + 2r_2 - 5m - 3n + 6)x + (2r_2 - 5m + 4)r_1 + (-6n + 4)r_2 + (6n - 10)m - 12n + 8]. \end{aligned}$$

Proof. One can suitably label the vertices of $\Theta_1(G, H)$ so that its distance matrix is

$$\begin{bmatrix} 2(J_n - I_n) - A(G) & J_n & J_{n \times m} & 2J_{n \times m} \\ J_n & 2(J_n - I_n) - A(G) & 2J_{n \times m} & J_{n \times m} \\ J_{m \times n} & 2J_{m \times n} & 2(J_m - I_m) - A(H) & 3J_m - 2I_m - A(H) \\ 2J_{m \times n} & J_{m \times n} & 3J_m - 2I_m - A(H) & 2(J_m - I_m) - A(H) \end{bmatrix},$$

where J is a matrix whose all entries are 1.

Using the fact that G, H are r_1, r_2 regular graphs and $A(G), A(H)$ are orthogonally diagonalizable it can be easily seen that the above matrix is similar to $B :=$

$$\begin{bmatrix} 2(nJ'_n - I_n) - DG & nJ'_n & \sqrt{nm}J'_{n \times m} & 2\sqrt{nm}J'_{n \times m} \\ nJ'_n & 2(nJ'_n - I_n) - DG & 2\sqrt{nm}J'_{n \times m} & \sqrt{nm}J'_{n \times m} \\ \sqrt{nm}J'_{m \times n} & 2\sqrt{nm}J'_{m \times n} & 2(mJ'_m - I_m) - DH & 3mJ'_m - 2I_m - DH \\ 2\sqrt{mn}J'_{m \times n} & \sqrt{mn}J'_{m \times n} & 3mJ'_m - 2I_m - DH & 2(mJ'_m - I_m) - DH \end{bmatrix}.$$

Here J' is a matrix obtained from J by replacing all its entries by 0 except the first diagonal entry and $DG = \text{diag}(\lambda_1(G), \lambda_2(G), \dots, \lambda_n(G))$.

Employing Laplace's method [12] along i^{th} ($i = 2, 3, \dots, n, n+2, \dots, 2n$) column of $|xI - B|$, we have

$$|xI - B| = \prod_{i=2}^n (x + \lambda_i(G_1) + 2)^2 \times |xI - C|,$$

where

$$C := \begin{bmatrix} 2(n-1) - r_1 & n & \sqrt{nm}e_{1m}^T & 2\sqrt{nm}e_{1m}^T \\ n & 2(n-1) - r_1 & 2\sqrt{nm}e_{1m}^T & \sqrt{nm}e_{1m}^T \\ \sqrt{nm}e_{1m} & 2\sqrt{nm}e_{1m} & 2(mJ'_m - I_m) - DH & 3mJ'_m - 2I_m - DH \\ 2\sqrt{mn}e_{1m} & \sqrt{mn}e_{1m} & 3mJ'_m - 2I_m - DH & 2(mJ'_m - I_m) - DH \end{bmatrix}$$

and $e_{1m} = (1, 0, \dots, 0)^T$ of size m .

Rest of the proof follows by applying Lemma 2.1 to $|xI - C|$. □

Proofs of the following theorems are in similar lines to that of Theorem 2.2. Hence we simply state them. We denote the neighborhood set of a vertex x in a graph G by $N_G(x)$, i.e., $N_G(x) := \{y \in V(G) : x \text{ and } y \text{ are adjacent in } G\}$.

Theorem 2.3. *Let G be an r_1 -regular graph on n vertices and H be an r_2 -regular graph on m vertices. Suppose $N_H(x) - N_H(y) \neq \{y\}$ for all adjacent vertices x and y in H , then the distance spectrum of $\Theta_2(G, H)$ consists of $-(\lambda_i(G) + 2)$ repeating twice for $i = 2, 3, \dots, n$, $-2(\lambda_i(H) + 2)$ ($i = 2, 3, \dots, m$), 0 repeating $m-1$ times and the four roots of the polynomial*

$$[x^2 - (-r_1 + 3m + 3n - 2)x - 3m(r_1 + 2)] \times [x^2 + (r_1 + 2r_2 - m - n + 6)x + (2r_2 - m + 4)r_1 + (-2n + 4)r_2 - 2m - 4n + 8].$$

Theorem 2.4. *Let G be an r_1 -regular graph on n vertices and H be an r_2 -regular graph on m vertices. Suppose $N_G(x) - N_G(y) \neq \{y\}$ for all adjacent vertices x and y in G , then the distance spectrum of $\Theta_3(G, H)$ consists of $-2(\lambda_i(G) + 2)$ $i = 2, 3, \dots, n$, $-2(\lambda_i(H) + 2)$ ($i = 2, 3, \dots, m$), 0 repeating $n + m - 2$ times and the four roots of the polynomial $[x^2 - (-2r_2 + 5m + 3n - 4)x - 6n(r_2 - m + 2)][x^2 + (2r_1 + m - n + 4)x + 2m(r_1 - n + 2)]$.*

3. CONSTRUCTION OF DISTANCE EQUIENERGETIC GRAPHS OF DIAMETER 3

Let H_i ($i = 1, 2$) be a cubic graph on 6 vertices as depicted in Fig. 2. Let G_1 and G_2 be the line graphs of H_1 and H_2 . Clearly G_i ($i = 1, 2$) is a 4-regular graph on 9 vertices. By direct computation, it can be seen that the spectrum of G_1 consists of 4, 1 and -2 with last two eigenvalues repeating four times and the spectrum of G_2 consists of 4, 2, 1 and -1 with last two eigenvalues repeating twice, -2 repeating thrice.

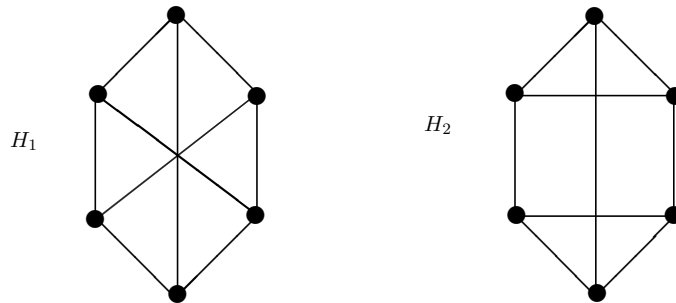


FIGURE 2. Cubic graphs on 6 vertices.

With these information and from Theorems 2.2, 2.3 and 2.4, we have the following table which give pairs of distance equienergetic graphs of diameter 3 on $18 + 2n$ vertices.

TABLE 1. Some families of distance equienergetic graphs on $18+2n$ vertices and their distance energy.

Distance non cospectral and Distance equienergetic pairs on $18+2n$ vertices	Distance energy
$\Theta_1(nK_1, G_1)$ and $\Theta_1(nK_1, G_2)$	$7n + 51 + \sqrt{(n + 1)(n + 49)}, n \geq 4$
$\Theta_1(G_1, nK_1)$ and $\Theta_1(G_2, nK_1)$	$9n + 37 + \sqrt{n^2 + 42n + 9}, n \geq 4$
$\Theta_2(nK_1, G_1)$ and $\Theta_2(nK_1, G_2)$	$4n + \sqrt{9n^2 + 150n + 841} + \sqrt{n^2 + 38n + 1} + 20$
$\Theta_2(G_1, nK_1)$ and $\Theta_2(G_2, nK_1)$	$4n + 2\sqrt{n^2 + 22n + 49} + 20$
$\Theta_3(nK_1, G_1)$ and $\Theta_3(nK_1, G_2)$	$7n + \sqrt{n^2 + 46n + 25} + 53, n \geq 2$
$\Theta_3(G_1, nK_1)$ and $\Theta_3(G_2, nK_1)$	$9n + \sqrt{n^2 + 30n + 9} + 43$

Acknowledgement The authors would like to thank the referees for their comments which have improved the overall presentation of the paper.

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