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STATUS CONNECTIVITY INDICES OF CARTESIAN PRODUCT OF GRAPHS

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ABSTRACT. In this paper, we establish one of the recent topological indices called the first status connectivity index $S_1(G) = \sum_{uv\in E(G)} [\sigma_G(u) + \sigma_G(v)]$ and second status connectivity index $S_2(G) = \sum_{uv\in E(G)} [\sigma_G(u)\sigma_G(v)]$ of Cartesian product of two simple graphs are determined. Also these indices are computed for nanotube, nanotorus, grid and cartesian product of complete graphs.

Keywords: Distance in graph, status connectivity indices, Cartesian product, Molecular graph.

AMS Subject Classification: 05C12, 92E10

1. Introduction

Graph theory has successfully provided chemists with a variety of useful tools [4, 9, 11, 12], among which are the topological indices. In theoretical chemistry, assigning a numerical value to the molecular structure that will closely correlate with the physical quantities and activities. Molecular structure descriptors (also called topological indices) are used for modeling physicochemical, pharmacologic, toxicologic, biological and other properties of chemical compounds. Many of these descriptors are defined in terms of degrees and distance of a graph (for details see [14, 8, 30, 15, 32, 23, 24]). The oldest well known distance based graph parameter is the Wiener index which was used to study the chemical properties of parafins [31]. Recently, Ramane and Yalnaik [27], introduced the status connectivity indices based on the distances and correlated it with the boiling point of benzenoid hydrocarbons. In this extension [28], Ramane et al. defined status co-indices and obtained the relations between status connectivity indices and status co-indices. Also they computed these indices for intersection graph, hypercube, Kneser graph and achiral polyhex nanotorus. Adiga et al. [1] defined degree status connectivity index and obtained its value for certain standard graphs. Recently, many authors studied various topological indices using different products of graphs such as cartesian product, lexicographic product, strong product and corona of two graphs. For details see[26, 33, 29, 25]. To this continuity in this paper we obtain the status connectivity indices for Cartesian product of connected

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graphs. Further we compute these status indices for C_4- nanotube, C_4- nanotorus, grid and cartesian product of complete graphs. Let $G = (V(G), E(G))$ be graph with vertex set $V(G)$ and edge set $E(G)$. The *distance* between the vertices u and v is the length of the shortest path joining u and v and is denoted by $d_G(u, v)$. All the graphs considered in this paper are simple and connected.

The *status of a vertex* $u \in V(G)$*, denoted by* $\sigma_G(u)$ *is defined as*[16]*,*

$$\sigma_G(u) = \sum_{v\in V(G)} d_G(u, v).$$

The Wiener index $W(G)$ *of a connected graph* G *is defined as* [31]*,*

$$W(G) = \sum_{\{u,v\}\subseteq V(G)} d_G(u, v) = \frac{1}{2} \sum_{u\in V(G)} \sigma_G(u).$$

The first Zagreb index and second Zagreb index are defined as [10]

$$M_1(G) = \sum_{uv\in E(G)} (d_G(u) + d_G(v)) \text{ and } M_2(G) = \sum_{uv\in E(G)} (d_G(u)d_G(v)).$$

where $d_G(u)$ *denote the degree of the vertex* u *in* G*. The Zagreb indices are found to have appilications in QSPR and QSAR studies as well, see* [7]*. For more results on the Zagreb indices see* [5, 13, 19]*.*

The eccentric connectivity indices of a connected graph G *are defined as* [2]

$$\xi_1(G) = \sum_{uv\in E(G)} (e_G(u) + e_G(v)) \text{ and } \xi_2(G) = \sum_{uv\in E(G)} (e_G(u)e_G(v)),$$

where $e_G(u) = max\{d(u, v)|v \in V(G)\}$*. Details of its applications can be found in* [3, 6, 18]*. Motivated by these indices, Ramane and Yalnaik* [27] *introduced first status connectivity index* $S_1(G)$ *and second status connectivity index* $S_2(G)$ *of a connected graph* G *as:*

$$S_1(G) = \sum_{uv\in E(G)} [\sigma_G(u) + \sigma_G(v)] \text{ and } S_2(G) = \sum_{uv\in E(G)} [\sigma_G(u)\sigma_G(v)].$$

Also they observed the status connectivity indices has good correlation with the boiling point of benzenoid hydrocarbons. In fact, one can rewrite the first status connectivity index as

$$S_1(G) = \sum_{u\in V(G)} d(u)\sigma_G(u).$$

Graph operations play an important role in the study of graph decompositions into isomorphic subgraphs. It is well known that many graphs arise from simpler graphs via various graph operations and one can study the properties of smaller graphs and deriving with it some information about larger graphs. Hence it is important to understand how certain invariants of such product graphs are related to the corresponding invariants of the original graphs. One of the most studied graph product is Cartesian product. Various topological indices are studied using Cartesian product of graphs see [17, 20, 21, 22]*. The Cartesian product of* G *and* H *is a graph, denoted by* $G\,\square\,H$*, with the vertex set* $V(G\,\square\,H)\;\{(u, v)|\,u \in V(G), v \in V(H)\}$ *and* $(u, x)(v, y)$ *is an edge of* $G\,\square\,H$ *if* $u = v$ *and* $xy \in E(H)$ *or,* $uv \in E(G)$ *and* $x = y$*, For example see Figure.1. For the convenience, let* $V(G) = \{u_1, u_2, \ldots, u_{n_1}\}$ *and let* $V(H) = \{v_1, v_2, \ldots, v_{n_2}\}$ *and any* $r-th$ *vertex in a graph* $G\,\square\,H$*, is denoted by* $x_r = \{u_r, v_r\}$*.*

2. Status connectivity indices of Cartesian product of graphs

From the structure of the Cartesian product G *and* H*, one can easily observe the following lemma and corollary.*

Lemma 2.1. *Let* G *and* H *be two connected graph with* n_1 *and* n_2 *vertices, respectively. Then the status of any vertex* $x_r \in V(G\,\square\,H)$ *is* $n_2\sigma_G(u_r) + n_1\sigma_H(v_r)$*.*

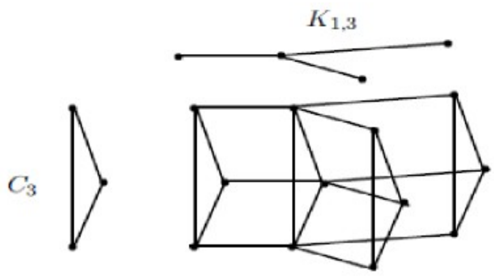

Figure 1. Cartesian product of cycle C_3 and star $K_{1,3}$

Proof. For any vertex $x_r \in V(G \,\square\, H)$, one can easily observe from the structure of $G \,\square\, H$, that

$$\begin{aligned}
\sigma_{G \,\square\, H}(x_r) &= \sum_{z_s \in V(G \,\square\, H))} d(x_r, z_s) \\
&= \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} [d(u_r, u_i) + d(v_r, v_j)] \\
&= n_2 \sum_{i=1}^{n_1} d(u_r, u_i) + n_1 \sum_{j=1}^{n_2} d(v_r, v_j) \\
&= n_2 \sigma_G(u_r) \; + \; n_1 \sigma_H(v_r)
\end{aligned}$$

□

Theorem 2.1. *Let G and H be two connected graphs with n_1 and n_2 vertices, respectively. Then $S_1(G \,\square\, H) = n_2^2 S_1(G) + 4n_1 m_1 W(H) + 4n_2 m_2 W(G) + n_1^2 S_1(H)$.*

Proof. From the definition of first status connectivity index, we have

$$\begin{aligned}
S_1(G\square H) &= \sum_{x_r\in V(G\square H)} d(x_r)\sigma_{G\square H}(x_r)\\
&= \sum_{\substack{u_r\in V(G)\\ v_r\in V(H)}} (d_G(u_r)+d_H(v_r))(n_2\sigma_G(u_r)\ +\ n_1\sigma_H(v_r))\\
&\text{ since } d_{G\square H}(.) = d_G(.)+d_H(.) \text{ and using lemma 2.1}\\
&= n_2 \sum_{\substack{u_r\in V(G)\\ v_r\in V(H)}} d_G(u_r)\sigma_G(u_r)\ +\ n_1 \sum_{\substack{u_r\in V(G)\\ v_r\in V(H)}} d_G(u_r)\sigma_H(v_r)\\
&\quad + n_2 \sum_{\substack{u_r\in V(G)\\ v_r\in V(H)}} d_H(v_r)\sigma_G(u_r)\ +\ n_1 \sum_{\substack{u_r\in V(G)\\ v_r\in V(H)}} d_H(v_r)\sigma_H(v_r)\\
&= n_2^2 \sum_{u_r\in V(G)} d_G(u_r)\sigma_G(u_r)\ +\ n_1 \sum_{u_r\in V(G)} d_G(u_r) \sum_{v_r\in V(H)} \sigma_H(v_r)\\
&\quad + n_2 \sum_{v_r\in V(H)} d_H(v_r) \sum_{u_r\in V(G)} \sigma_G(u_r)\ +\ n_1^2 \sum_{v_r\in V(H)} d_H(v_r)\sigma_H(v_r)\\
&= n_2^2 S_1(G)\ +\ n_1(2m_1) \sum_{v_r\in V(H)} \sigma_H(v_r)\\
&\quad + n_2(2m_2) \sum_{u_r\in V(G)} \sigma_G(u_r)\ +\ n_1^2 S_1(H)\\
&= n_2^2 S_1(G)\ +\ 4n_1m_1W(H)\ +\ 4n_2m_2W(G)\ +\ n_1^2 S_1(H)
\end{aligned}$$

□

Now, to obtain the second status connectivity index of Cartesian product of graphs, the procedure is as follows.

Theorem 2.2. *Let G and H be two connected graphs with n_1,n_2 vertices, respectively. Then $S_2(G\square H) = n_2^3 S_2(G) + 2n_1n_2S_1(G)W(H) + n_1^2 m_1 \sum_{v_r\in V(H)} \sigma_H^2(v_r)\ +\ n_1^3 S_2(H) + 2n_1n_2S_1(H)W(G) + n_2^2 m_2 \sum_{u_r\in V(G)} \sigma_G^2(u_r)$.*

Proof. From the definition of second status connectivity index, we have

$$\begin{aligned}
S_2(G\,\square\,H) &= \sum_{x_ry_r\in E(G\,\square\,H)} \sigma_{G\,\square\,H}(x_r)\sigma_{G\,\square\,H}(y_r)\\
&= \sum_{\substack{k=1\\ u_iu_j\in E(G),\ v_k\in V(H)}}^{n_2} (n_2\sigma_G(u_i)\,+\,n_1\sigma_H(v_k))(n_2\sigma_G(u_j)\,+\,n_1\sigma_H(v_k))\\
&+ \sum_{\substack{s=1\\ v_iv_j\in E(H),\ u_s\in V(G)}}^{n_1} (n_2\sigma_G(u_s)\,+\,n_1\sigma_H(v_i))(n_2\sigma_G(u_s)\,+\,n_1\sigma_H(v_j))\\
&\quad \text{since by lemma 2.1}\\
&= \sum_{\substack{k=1\\ u_iu_j\in E(G),v_k\in V(H)}}^{n_2} n_2^2\sigma_G(u_i)\sigma_G(u_j)\\
&+ \sum_{\substack{k=1\\ u_iu_j\in E(G),\ v_k\in V(H)}}^{n_2} n_1n_2\sigma_G(u_i)\sigma_H(v_k)\\
&+ \sum_{\substack{k=1\\ u_iu_j\in E(G),\ v_k\in V(H)}}^{n_2} n_1n_2\sigma_G(u_j)\sigma_H(v_k)\,+\,\sum_{\substack{k=1\\ u_iu_j\in E(G),\ v_k\in V(H)}}^{n_2} n_1^2\sigma_H^2(v_k)\\
&+ \sum_{\substack{s=1\\ v_iv_j\in E(H),\ u_s\in V(G)}}^{n_1} n_1^2\sigma_H(v_i)\sigma_H(v_j)\\
&+ \sum_{\substack{s=1\\ v_iv_j\in E(H),\ u_s\in V(G)}}^{n_1} n_1n_2\sigma_G(u_s)\sigma_H(v_i)\\
&+ \sum_{\substack{s=1\\ v_iv_j\in E(H),\ u_s\in V(G)}}^{n_1} n_1n_2\sigma_G(u_s)\sigma_H(v_j)+\sum_{\substack{s=1\\ v_iv_j\in E(H),\ u_s\in V(G)}}^{n_1} n_2^2\sigma_G^2(u_s)\\
&= n_2^3S_2(G)+2n_1n_2S_1(G)W(H)+n_1^2m_1\sum_{v_r\in V(H)}\sigma_H^2(v_r)\,+\,n_1^3S_2(H)\\
&+ 2n_1n_2S_1(H)W(G)+n_2^2m_2\sum_{u_r\in V(G)}\sigma_G^2(u_r).
\end{aligned}$$

Since by the definition of first connectivity, second connectivity and Wiener index of graph. □

Theorem 2.3. [27] *Let G be a connected graph with n vertices and m edges and $diam(G)\leq 2$. Then $S_1(G)=4m(n-1)-M_1(G)$ and $S_2(G)=4m(n-1)^2-2(n-1)M_1(G)+M_2(G)$.*

The proof of the following corollaries are the direct consequence of Theorems 2.1 to 2.3.

Corollary 2.1. *Let G and H be a connected graph on n_1 and n_2 vertices and m_1 and m_2 edges, respectively. Let $diam(G)\leq 2$ and $diam(H)\leq 2$. Then $S_1(G\,\square\,H)=4m_1n_2^2(n_1-1)-n_2^2M_1(G)+4n_1m_1W(H)+4n_2m_2W(G)+4m_2n_1^2(n_2-1)-n_1^2M_1(H)$ and $S_2(G\,\square\,H)=n_2^3(4m_1(n_1-1)^2+M_2(G))+n_1^3(4m_2(n_2-1)^2+M_2(H))+8n_1n_2(m_1(n_1-1)W(H)+m_2(n_2-$*

$1)W(G)) + n_1^2 m_1 \sum_{v_r \in V(H)} \sigma_H^2(v_r) + n_2^2 m_2 \sum_{u_r \in V(G)} \sigma_G^2(u_r) - (2n_1^3(n_2-1)$
$+ 2n_1n_2W(G))M_1(H) - (2n_2^3(n_1-1) + 2n_1n_2W(H))M_1(G).$

Corollary 2.2. *Let G and H be a connected $r-$ regular graph with n_1 and n_2 vertices and m_1 and m_2 edges, respectively. Let $diam(G) \leq 2$ and $diam(H) \leq 2$. Then* $S_1(G\,\Box\, H) = 2m_1n_2^2(2(n_1-1)-r_1) + 4(n_1m_1W(H) + 4n_2m_2W(G)) + 2m_2n_1^2((n_2-1)-r_2)$ *and* $S_2(G\,\Box\, H) = n_2^3(4m_1(n_1-1)^2 + m_1r_1^2) + n_1^3(4m_2(n_2-1)^2 + m_2r_2^2) + 8n_1n_2(m_1(n_1-1)W(H) + m_2(n_2-1)W(G)) + n_1^2m_1 \sum_{v_r\in V(H)} \sigma_H^2(v_r) + +n_2^2m_2 \sum_{u_r \in V(G)} \sigma_G^2(u_r) - 2(2n_1^3(n_2-1) + 2n_1n_2W(G))m_2r_2 - 2(2n_2^3(n_1-1) + 2n_1n_2W(H))m_1r_1.$

3. Examples

There are several molecular graphs that can be realized as a product of graphs, for instance nanotorous as $C_n\,\Box\, C_m$, nanotubes as $P_n\,\Box\, C_m$, grid as $P_n\,\Box\, P_m$. In this section we compute the first status connectivity index and second status connectivity index for such molecular structures and Cartesian product of complete graphs. It is well known that for path, cycle and complete graph the following indices are $W(P_n) = \frac{n(n^2-1)}{6}$,

$W(C_n) = \begin{cases} \frac{n^3}{8} & n \text{ is even} \\ \frac{n(n^2-1)}{8} & n \text{ is odd.} \end{cases}$, $W(K_n) = \frac{n(n-1)}{2}$, $S_1(P_n) = \frac{n(n-1)(2n-1)}{3}$,

$S_1(C_n) = \begin{cases} \frac{n^3}{2} & n \text{ is even} \\ \frac{n(n^2-1)}{2} & n \text{ is odd} \end{cases}$, $S_1(K_n) = n(n-1)^2$, $S_2(P_n) = \frac{(n-1)(n^4-n^2)}{4} - \frac{n(n-1)(n^3-n)}{2} + \frac{n(n-1)(2n-1)(2n^2-1)}{6} - \frac{n^3(n-1)^2}{2} + \frac{6(n-1)^5+15(n-1)^4+10(n-1)^3-(n-1)}{30}$,

$S_2(C_n) = \begin{cases} \frac{n^5}{16} & n \text{ is even} \\ \frac{n(n^2-1)^2}{16} & n\text{- odd} \end{cases}$ *and* $S_2(K_n) = \frac{n(n-1)^3}{2}$. *Using these facts, in Theorems 2.2, 2.3 and Corollary 2.6, we obtain the following examples.*

Example 3.1. *Let P_k and P_l be a path on k and l vertex respectively, with $k, l \geq 2$. Then*
$S_1(P_k\,\Box\, P_l) = \frac{kl}{3}\Big((k-1)(2(l^2+kl-1)-l) + (l-1)(2(k^2+kl-1)-k)\Big)$ *and*
$S_2(P_k\,\Box\, P_l) = l^3\big(\frac{(k-1)(k^4-k^2)}{4} - \frac{k(k-1)(k^3-k)}{2} + \frac{k(k-1)(2k-1)(2k^2-1)}{6} - \frac{k^3(k-1)^2}{2}$
$+ \frac{6(k-1)^5+15(k-1)^4+10(k-1)^3-(k-1)}{30}\big) + \frac{k^2l^2\big[(k-1)(2k-1)(l^2-1)+(l-1)(2l-1)(k^2-1)\big]}{9}$
$+ k^2(k-1) \sum_{v_r \in V(P_l)} \sigma_{P_l}^2(v_r) + k^3\big(\frac{(l-1)(l^4-l^2)}{4} - \frac{l(l-1)(l^3-l)}{2} + \frac{l(l-1)(2l-1)(2l^2-1)}{6} - \frac{l^3(l-1)^2}{2}$
$+ \frac{6(l-1)^5+15(l-1)^4+10(l-1)^3-(l-1)}{30}\big) + l^2(l-1) \sum_{u_r \in V(P_k)} \sigma_{P_k}^2(u_r).$

Example 3.2. *Let C_k be a cycle with $k \geq 2$ vertex and P_l be a path on $l \geq 2$ vertex. Then*

$$S_1(C_k\,\Box\, P_l) = \begin{cases} lk^2\Big(\frac{(2l-1)k}{2} + \frac{(l-1)(4l+1)}{3}\Big), & \text{if } k \text{ is even} \\ lk\Big(\frac{(2l-1)(k^2-1)}{2} + \frac{k(l-1)(4l+1)}{3}\Big), & \text{if } k \text{ is odd} \end{cases}.$$

For even k
$S_2(C_k\,\Box\, P_l) = \frac{l^3k^5}{16} + \frac{l^2k^4(l^2-1)}{6} + \frac{l^2k^4(l-1)(2l-1)}{12} + k^3 \sum_{v_r \in V(P_l)} \sigma_{P_l}^2(v_r)$
$+ l^2(l-1) \sum_{u_r \in V(C_k)} \sigma_{C_k}^2(u_r) + k^3\big[\frac{(l-1)(l^4-l^2)}{4} - \frac{l(l-1)(l^3-l)}{2} + \frac{l(l-1)(2l-1)(2l^2-1)}{6} - \frac{l^3(l-1)^2}{2}$
$+ \frac{6(l-1)^5+15(l-1)^4+10(l-1)^3-(l-1)}{30}\big],$
and for odd k

$$S_2(C_k \square P_l) = \frac{l^3k(k^2-1)^2}{16} + \frac{l^2k^2(k^2-1)(l-1)(4l+1)}{12} + k^3 \sum_{v_r \in V(P_l)} \sigma^2_{P_l}(v_r) + l^2(l-1) \sum_{u_r \in V(C_k)} \sigma^2_{C_k}(u_r)$$
$$+ k^3 [\frac{(l-1)(l^4-l^2)}{4} - \frac{l(l-1)(l^3-l)}{2} + \frac{l(l-1)(2l-1)(2l^2-1)}{6} - \frac{l^3(l-1)^2}{2} + \frac{6(l-1)^5+15(l-1)^4+10(l-1)^3-(l-1)}{30}]$$

Example 3.3. *Let C_k and C_l be a cycle on k and l vertex respectively, with $k, l \geq 3$. Then*

$$S_1(C_k \square C_l) = \begin{cases} kl(k+l)(kl-1), & \text{if } l,k \text{ are odd} \\ kl^{\,2}\big(k(k+l)-1\big), & \text{if } k \text{ is odd and } l \text{ is even} \\ (kl)^2(k+l), & \text{if } l,k \text{ are even} \end{cases}.$$

Example 3.4. *Let K_k and K_l be a complete graph on k and l vertex respectively. Then* $S_1(K_k \square K_l) = kl\left(l(k-1)^2 + k(l-1)^2 + (k-1)(l-1)(k+l)\right)$ *and*

$$S_2(K_k \square K_l) = \frac{5kl}{2}\left(l^2(k-1)^3 + k^2(l-1)^3\right) - kl^2(k-1)^2\left(2k(k-1) + k(l-1)\right)$$
$$- lk^2(l-1)^2\left(2k(l-1) + l(k-1)\right) + 2k^2l^2(k-1)(l-1)(k+l-2)$$
$$+ \frac{k^3(k-1)}{2} \sum_{v_r \in V(K_l)} \sigma^2_{K-l}(v_r) + \frac{l^3(l-1)}{2} \sum_{u_r \in V(K_k)} \sigma^2_{K_k}(u_r).$$

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