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FUZZY SOFT QUASI SEPARATION AXIOMS

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Abstract. In this work, we focus on fuzzy soft quasi separation axioms and give some new results about the concept of quasi coincidence with fuzzy soft sets defined in [17]. Further, we give relations between fuzzy soft quasi $T_{i-}(i = 0, 1, 2)$ spaces and fuzzy quasi $T_{i-}(i = 0, 1, 2)$ spaces.

Keywords: fuzzy soft set, fuzzy soft topology, quasi coincident, fuzzy soft quasi separation axioms

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1. Introduction

The set theory, which was initiated by George Cantor, has an important role for several branches of mathematics. In this theory, the sets are crisp and defined precisely by its elements and thus, it is clear if an element belongs to a set or not. However, if we aim to model a concept in real life by using the mathematical properties of Cantor‘s set theory, then we might run into various difficulties due to vagueness which exists in problems related to economics, engineering, medicine, etc. To fulfill this lack, many theories are developed such as fuzzy set theory [19], rough set theory [14], soft set theory [11] and recently the hybrid models [10].

The most popular theory for vagueness is undoubtedly the fuzzy set theory, which was first defined by Zadeh [19]. Fuzzy sets are specified by the membership function which identifies the belonging of an element to a set up to a degree. The rough set theory, which is defined by Pawlak [14], is another method to take the vagueness into account. It is based on the indiscernibility relations of elements of the finite universe and boundary region of a set. Moreover, Molodtsov [11] defined the soft set theory as a different approach to the doubtfulness and the theory has been used in the various branches of mathematics. He also showed that, the soft set theory is free from the parametrization inadequacy syndrome of the other theories developed for vagueness like fuzzy set theory, rough set theory and etc. The soft set theory has been studied by several researchers [2, 16]. As a further

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improvement, researchers combined the vague sets and these new hybrid sets are used in many studies [5, 10]. For instance, Maji et al. [10] defined fuzzy soft sets, which are a combination of fuzzy and soft sets. Many researchers have contributed to the theory of these hybrid models, see for example [1, 4, 7, 8, 9].
After the introduction of vague sets, it was natural to construct topological structures on those sets. For this purpose, Tanay and Kandemir [18] defined the fuzzy soft topology and obtained several results. Also, Roy and Samanta [15] worked on the topological structure on a fuzzy soft set. In another paper, we [17] defined the fuzzy soft topology over a fuzzy soft set with fixed parameter set and investigated the topological concepts as neighborhood systems, fuzzy soft interior and fuzzy soft closure points, quasi coincidence for fuzzy soft sets and etc. Furthermore, Atmaca and Zorlutuna [3] investigated the notions as fuzzy soft closure of a soft set, fuzzy soft base and fuzzy soft continuity in the fuzzy soft topological spaces.
In the present work, we first recall the well known definitions and results of fuzzy soft topology given in [3, 13, 15, 17, 18], we also, introduce fuzzy soft subspace and obtain some new results about fuzzy soft quasi coincidence points. Then, we define fuzzy soft quasi separation axioms and prove the properties of them. Further, we give relations between fuzzy soft quasi $T_i-(i = 0, 1, 2)$ spaces and fuzzy quasi $T_i-(i = 0, 1, 2)$ spaces.

2. Preliminaries

In this section, we present several preliminary definitions which are necessary in the process of defining our main results. For the sake of consistency, the following notations are used throughout the whole paper:

U	: the initial universe,
E	: the possible parameters for U,
$P(U)$	: the power set of U,
I^U	: the set of all fuzzy subsets of U,
(U, E)	: the universal set U and the parameter set E.

Definition 2.1. [19] *A fuzzy set A in U is a set of ordered pairs: $A = \{(x, \mu_A(x)) : x \in U\}$,where $\mu_A : U \longrightarrow [0, 1] = I$ is a mapping and $\mu_A(x)$ (or $A(x)$) states the grade of belonging of x in A.*

Definition 2.2. [11] *Let F be a mapping given by $F : A \to P(U)$ and $A \subseteq E$. Then, (F, A) is said to be soft set over U.*

Aktas and Cagman [2] showed that every fuzzy set is a soft set. That is, fuzzy sets are a special class of soft sets.

Definition 2.3. [15] *Let $A \subseteq E$. (f_A, E) is defined to be a fuzzy soft set (briefly; fs-set) on (U, E) if f_A:$E \to I^U$ is a mapping defined by $f_A(e) = \mu^e_{f_A}$ where $\mu^e_{f_A} = \bar{O}$ if $e \in E - A$ and $\mu^e_{f_A} \neq \bar{O}$ if $e \in A$, where $\bar{O}(u) = 0$ for each $u \in U$.*

Definition 2.4. [15] *The complement of a fs-set (f_A, E) is a fs-set (f^c_A, E) on (U, E) which is denoted by $(f_A, E)^c$ and $f^c_A : E \to I^U$ is defined by $\mu^e_{f^c_A} = 1 - \mu^e_{f_A}$ if $e \in A$ and $\mu^e_{f^c_A} = \bar{1}$ if $e \in E \backslash A$, where $\bar{1}(u) = 1$ for each $u \in U$.*

Definition 2.5. [15] *The fs-set (f_Φ, E) on (U, E) is called null fs-set and is shown by Φ. $f_\Phi(e) = \bar{O}$ for every $e \in E$.*

Definition 2.6. [15] *The fs-set* (f_E, E) *on* (U, E) *is defined to be absolute fs-set and is shown by* $U_{\widetilde{E}}$. $U(e) = f_E(e) = \bar{1}$ *for every* $e \in E$.

Definition 2.7. [15] *Let* (f_A, E) *and* (g_B, E) *be two fs-sets on* (U, E). (f_A, E) *is called fs-subset of* (g_B, E) *if* $\mu^e_{f_A} \subseteq \mu^e_{g_B}$ *for all* $e \in E$, *i.e.,* $\mu^e_{f_A}(u) \le \mu^e_{g_B}(u)$ *for all* $u \in U$ *and for all* $e \in E$ *and is denoted by* $(f_A, E) \sqsubseteq (g_B, E)$.

Definition 2.8. [15] *Let* (f_A, E) *and* (g_B, E) *be two fs-sets on* (U, E). *The union of these two fs-sets is a fs-set* (h_C, E), *defined by* $h_C(e) = \mu^e_{h_C} = \mu^e_{f_A} \cup \mu^e_{g_B}$ *for all* $e \in E$ *where* $C = A \cup B$ *and is denoted by* $(h_C, E) = (f_A, E) \sqcup (g_B, E)$.

Definition 2.9. [15] *Let* (f_A, E) *and* (g_B, E) *be two fs-sets on* (U, E). *The intersection of these two fs-sets is a fs-set* (h_C, E), *defined by* $h_C(e) = \mu^e_{h_C} = \mu^e_{f_A} \cap \mu^e_{g_B}$ *for all* $e \in E$ *and where* $C = A \cap B$ *and is denoted by* $(h_C, E) = (f_A, E) \sqcap (g_B, E)$.

3. Fuzzy Soft Topology

Throughout this work U, E denote the universe and the parameter set, respectively and (f_A, E) is considered as a fs-set on (U, E).

Definition 3.1. [18, 15] *Let* τ_f *be the collection of fs-subsets of* $U_{\widetilde{E}}$. τ_f *is said to be a fuzzy soft topology (briefly; fs-topology) if*

1. $\Phi, U_{\widetilde{E}} \in \tau_f$,
2. *If* $(f_{i_A}, E) \in \tau_f$, *then* $\sqcup_i (f_{i_A}, E) \in \tau_f$,
3. *If* $(g_A, E), (h_A, E) \in \tau_f$, *then* $(g_A, E) \sqcap (h_A, E) \in \tau_f$.

The pair $(U_{\widetilde{E}}, \tau_f)$ *is said a fuzzy soft topological space (briefly; fst-space) over* $U_{\widetilde{E}}$. *Every member of* τ_f *is called the fuzzy soft open set (briefly; fs-open set). A fs-subset of* $U_{\widetilde{E}}$ *is called the fuzzy soft closed set (briefly; fs-closed set) if its complement is a member of* τ_f.

Theorem 3.1. [17] *Let* $(U_{\widetilde{E}}, \tau_f)$ *be a fst-space and* κ_f *denotes the collection of all fs-closed sets. Then,*

1. $\Phi, U_{\widetilde{E}}$ *are fs-closed sets,*
2. *The arbitrary intersection of fs-closed sets are fs-closed,*
3. *The union of two fs-closed sets is a fs-closed set.*

Theorem 3.2. [12] *If* $(U_{\widetilde{E}}, \tau_{f_1})$ *and* $(U_{\widetilde{E}}, \tau_{f_2})$ *are two fst-spaces then,* $(U_{\widetilde{E}}, \tau_{f_1} \cap \tau_{f_2})$ *is a fst-space.*

Remark 3.1. [12] *The union of two fs-topologies may not be a fs-topology as seen in the following example.*

Example 3.1. *Let* $U = \{x, y, z\}$ *be the universe set,* $E = \{e_1, e_2, e_3\}$ *be the parameter set,* $A = \{e_1, e_2\}$ *and* $\tau_{f_1} = \{\Phi, U_{\widetilde{E}}, (f_{1_A}, E), (f_{2_A}, E), (f_{3_A}, E), (f_{4_A}, E)\}$ *and* $\tau_{f_2} = \{\Phi, U_{\widetilde{E}}, (g_{1_A}, E), (g_{2_A}, E)\}$ *be the fs-topologies on* $U_{\widetilde{E}}$ *where,*

$$
\begin{aligned}
(f_{1_A}, E) &= \{e_1 = \{x_{0.2}, y_{0.4}, z_{0.7}\}, e_2 = \{x_{0.1}, y_{0.5}, z_{0.2}\}\},\\
(f_{2_A}, E) &= \{e_1 = \{x_{0.5}, y_{0.3}, z_{0.5}\}, e_2 = \{x_{0.4}, y_{0.8}, z_{0.6}\}\},\\
(f_{3_A}, E) &= \{e_1 = \{x_{0.2}, y_{0.3}, z_{0.5}\}, e_2 = \{x_{0.1}, y_{0.5}, z_{0.2}\}\},\\
(f_{4_A}, E) &= \{e_1 = \{x_{0.5}, y_{0.4}, z_{0.7}\}, e_2 = \{x_{0.4}, y_{0.8}, z_{0.6}\}\},\\
(g_{1_A}, E) &= \{e_1 = \{x_{0.4}, y_{0.6}, z_{0.5}\}, e_2 = \{x_{0.3}, y_{0.6}, z_{0.2}\}\},\\
(g_{2_A}, E) &= \{e_1 = \{x_{0.3}, y_{0.4}, z_{0.3}\}, e_2 = \{x_{0.1}, y_{0.3}, z_{0.2}\}\}.
\end{aligned}
$$

It is easily seen that, τ_{f_1}, τ_{f_2} are fs-topologies on $U_E^{\sim}$ but $\tau_{f_1} \cup \tau_{f_2}$ is not a fs-topology since $(f_{1_A}, E) \sqcup (g_{1_A}, E)$ is not a member of $\tau_{f_1} \cup \tau_{f_2}$.

Definition 3.2. [15] *Let P_x^λ be a fuzzy point in I^U. Then, (P_x^λ, E) is a fs-set on (U, E) where $P_x^\lambda(e) = \mu^e_{P_x^\lambda}$, $\mu^e_{P_x^\lambda}(u) = \lambda$, if $u = x$ and $\mu^e_{P_x^\lambda}(u) = 0$ if $u \neq x$ for every $e \in E$ and every $u \in U$.*

Definition 3.3. [15] *Let $P_x^\lambda (x \in U, \lambda \in (0, 1])$ be a fuzzy point in I^U. If $\lambda \leq \mu^e_{f_A}(x)$, for every $e \in A$, then, P_x^λ belongs to (f_A, E) and it is denoted by $P_x^\lambda \in^\sim (f_A, E)$.*

Definition 3.4. [15] *Let $(U_E^{\sim}, \tau_f)$ be a fst-space, (g_A, E) be a fs-subset of $U_E^{\sim}$. The intersection of all fs-closed sets containing (g_A, E) is called the fuzzy soft closure of (g_A, E). $(g_A, E)^- = \sqcap\{(h_A, E) : (g_A, E) \sqsubseteq (h_A, E)$ and (h_A, E) is fs-closed$\}$*

Theorem 3.3. [13] *Let $(U_E^{\sim}, \tau_f)$ be a fst-space. Then, the collection*

$$\tau_{f_e} = \{g_A(e) : (g_A, E) \in \tau_f\}$$

for each $e \in E$ defines a fuzzy topology over $U(e)$.

Example 3.2. *The converse inclusion of Theorem 3.3., does not hold generally.*
Let $U = \{x, y, z\}$, $E = \{e_1, e_2\}$.

$$f_{1_A}(e_1) = \{x_{0.3}, y_{0.5}, z_{0.8}\}, f_{1_A}(e_2) = \{x_{0.1}, y_{0.6}, z_{0.3}\}$$
$$f_{2_A}(e_1) = \{x_{0.2}, y_{0.8}, z_{0.5}\}, f_{2_A}(e_2) = \{x_{0.2}, y_{0.4}, z_{0.5}\}$$
$$f_{3_A}(e_1) = \{x_{0.3}, y_{0.8}, z_{0.8}\}, f_{3_A}(e_2) = \{x_{0.1}, y_{0.4}, z_{0.3}\}$$
$$f_{4_A}(e_1) = \{x_{0.2}, y_{0.5}, z_{0.5}\}, f_{4_A}(e_2) = \{x_{0.2}, y_{0.6}, z_{0.5}\}$$

Then, $\tau_f = \{\Phi, U_E^{\sim}, (f_{1_A}, E), (f_{2_A}, E), (f_{3_A}, E), (f_{4_A}, E)\}$ is not a fs-topology since $(f_{1_A}, E) \sqcup (f_{2_A}, E)$ is not a member of τ_f. But $\tau_{f_{e_1}}$ and $\tau_{f_{e_2}}$ are fuzzy topologies over the fuzzy sets $U(e_1)$ and $U(e_2)$, respectively.

Definition 3.5. *Let V be a subset of U . Then, the sub fs-set of (f_A, E) over (V, E) is denoted by $({}^V f_A, E)$ and is defined as follows:*

$$({}^V f_A, E) = V_E^{\sim} \sqcap (f_A, E)$$

where the symbol $V_E^{\sim}$ denotes the absolute fs-set on (V, E).

Definition 3.6. *Let $(U_E^{\sim}, \tau_f)$ be a fst-space and $V \subseteq U$. Then*

$$\tau_{f_V} = \left\{({}^V f_A, E) : (f_A, E) \in \tau_f\right\}$$

is said to be soft relative topology on ${}^V U_E^{\sim}$ and $({}^V U_E^{\sim}, \tau_{f_V})$ is called a fs-subspace of $(U_E^{\sim}, \tau_f)$.

Definition 3.7. [3] *Let P_x^λ be a fuzzy point in I^U. P_x^λ is said to be quasi-coincident (briefly; q-coincident) with (f_A, E), denoted by $P_x^\lambda q (f_A, E)$ if $\lambda + \mu^e_{f_A}(x) > 1$ for any $e \in A$.*

Definition 3.8. [3] *Let (f_A, E) and (g_A, E) be two fs-sets on (U, E). (f_A, E) is said to be q-coincident with (g_A, E), denoted by $(f_A, E) q (g_A, E)$, if there exists $u \in U$ such that $\mu^e_{f_A}(u) + \mu^e_{g_A}(u) > 1$, for any $e \in A$. If this is true, we can say that (f_A, E) and (g_A, E) is q-coincident at u.*

Theorem 3.4. [3] *Let $(g_A, E), (h_A, E)$ be two fs-sets. If $(g_A, E) \sqcap (h_A, E) = \Phi$ then (g_A, E) is not q-coincident with (h_A, E).*

Theorem 3.5. *Let $(g_A, E), (h_A, E)$ be two fs-sets on (U, E). If $(g_A, E)^c q(h_A, E)$ at u then, (g_A, E) is not q-coincident with $(h_A, E)^c$ at u.*

Proof. Let $(g_A, E)^c q(h_A, E)$ at u. Then, $(1 - \mu^e_{g_A}(u)) + \mu^e_{h_A}(u) > 1$ for any $e \in A$. Hence, $1 > (1 - \mu^e_{h_A}(u)) + \mu^e_{g_A}(u)$. Thus, (g_A, E) is not q-coincident with $(h_A, E)^c$ at u. $\square$

4. Fuzzy Soft Quasi Separation Axioms

Definition 4.1. *Let $(U^{\sim}_E, \tau_f)$ be a fst-space. If, for any fuzzy points P^λ_x, P^μ_y $(x, y \in U, x \neq y)$ in I^U there exists $(g_A, E) \in \tau_f$ such that $P^\lambda_x q(g_A, E) \sqsubseteq (P^\mu_y, A)^c$ or $P^\mu_y q(g_A, E) \sqsubseteq (P^\lambda_x, A)^c$ then, $(U^{\sim}_E, \tau_f)$ is called fuzzy soft quasi T_{0-} space (briefly; fsq-T_{0-} space).*

Lemma 4.1. *If, $P^\lambda_x q(f_A, E)$ then $P^\lambda_x \notin^{\sim} (f_A, E)^c$.*

Proof. Let $P^\lambda_x q(f_A, E)$. Then, for any $e \in A$, $\lambda + \mu^e_{f_A}(x) > 1$ and hence ,$\lambda > 1 - \mu^e_{f_A}(x)$ and $\lambda > \mu^e_{f^c_A}(x)$. Therefore, $P^\lambda_x \notin^{\sim} (f_A, E)^c$. $\square$

Theorem 4.1. *If $(U^{\sim}_E, \tau_f)$ is a fsq-T_{0-} space then, for any pair of fuzzy points P^λ_x, P^μ_y $(x, y \in U, x \neq y)$ in I^U $P^\lambda_x \notin (P^\mu_y, A)^-$ or $P^\mu_y \notin (P^\lambda_x, A)^-$.*

Proof. Let P^λ_x, P^μ_y $(x, y \in U, x \neq y)$ be a pair of fuzzy points in I^U. Since $(U^{\sim}_E, \tau_f)$ is a fsq-T_{0-} space, there exists $(g_A, E) \in \tau_f$ such that $P^\lambda_x q(g_A, E) \sqsubseteq (P^\mu_y, A)^c$ or $P^\mu_y q(g_A, E) \sqsubseteq (P^\lambda_x, A)^c$. We consider the first state. By Lemma 4.1., $P^\lambda_x \notin^{\sim} (g_A, E)^c$ and $(P^\mu_y, A) \sqsubseteq (g_A, E)^c$. Since $(g_A, E)^c$ is a fs-closed set, $(P^\mu_y, A)^- \sqsubseteq (g_A, E)^c$. Therefore we get that $P^\lambda_x \notin (P^\mu_y, A)^-$. The proof can be done for P^μ_y similarly. $\square$

Definition 4.2. [6] *A fuzzy topological space (X, τ) is said to be fuzzy quasi T_0 (briefly; fq-T_0) iff for every pair of fuzzy points $P^\lambda_x, P^\mu_y \in X$ such that $x \neq y$ there exists $U \in \tau$ such that $P^\lambda_x qU \leq (P^\mu_y)^c$ or $P^\mu_y qU \leq (P^\lambda_x)^c$.*

Theorem 4.2. *If $(U^{\sim}_E, \tau_f)$ is a fsq-T_{0-} space then, for any $e \in E$, $(U(e), \tau_{f_e})$ is fq-T_0.*

Proof. Let P^λ_x, P^μ_y $(x, y \in U, x \neq y)$ be two fuzzy points in I^U. Then, there exists $(g_A, E) \in \tau_f$ such that $P^\lambda_x q(g_A, E) \sqsubseteq (P^\mu_y, A)^c$ or $P^\mu_y q(g_A, E) \sqsubseteq (P^\lambda_x, A)^c$. Since $(U^{\sim}_E, \tau_f)$ is a fsq-T_{0-} space, then $P^\lambda_x q(g_A, E) \sqsubseteq (P^\mu_y, A)^c$ or $P^\mu_y q(g_A, E) \sqsubseteq (P^\lambda_x, A)^c$. Hence, $P^\lambda_x q g_A(e) \leq (P^\mu_y)^c$ or $P^\mu_y q g_A(e) \leq (P^\lambda_x)^c$ for any $e \in E$. This shows that $(f_A(e), \tau_{f_e})$ is fq-T_0. $\square$

Theorem 4.3. *If $(U^{\sim}_E, \tau_f)$ is a fsq-T_{0-} space then, $({}^V U^{\sim}_E, \tau_{f_V})$ is fsq-T_0.*

Proof. Let $(U^{\sim}_E, \tau_f)$ be a fsq-T_{0-} space and P^λ_x, P^μ_y $(x, y \in V, x \neq y)$ be two fuzzy points. Then, there exists a fs-open set (g_A, E) such that $P^\lambda_x q(g_A, E) \sqsubseteq (P^\mu_y, A)^c$ or $P^\mu_y q(g_A, E) \sqsubseteq (P^\lambda_x, A)^c$. We consider the first state. Since $x \in V$, we obtain that $P^\lambda_x q(V^{\sim}_E \sqcap (g_A, E)) \sqsubseteq (P^\mu_y, A)^c$. The proof for the second case can be done in a similar way. $\square$

Definition 4.3. *Let $(U^{\sim}_E, \tau_f)$ be a fst-space. If for any fuzzy points P^λ_x, P^μ_y $(x, y \in U, x \neq y)$ of I^U there exist fs-open sets $(g_A, E), (h_A, E)$ such that $P^\lambda_x q(g_A, E) \sqsubseteq (P^\mu_y, A)^c$ and $P^\mu_y q(h_A, E) \sqsubseteq (P^\lambda_x, A)^c$ then $(U^{\sim}_E, \tau_f)$ is called fuzzy soft quasi T_{1-} space (briefly; fsq-T_{1-} space).*

Theorem 4.4. *$(U^{\sim}_E, \tau_f)$ is fsq-T_{1-} space if (P^1_x, A) is fs-closed for any $x \in U$.*

Proof. Let P^λ_x, P^μ_y $(x, y \in U, x \neq y)$ be two fuzzy points of I^U. Since (P^1_x, A), (P^1_y, A) are fs-closed sets, $(P^1_x, A)^c$,$(P^1_y, A)^c \in \tau_f$. It is easy to see that $P^\lambda_x q(P^1_y, A)^c$ and $P^\mu_y q(P^1_x, A)^c$. Moreover, $(P^1_y, A)^c \sqsubset (P^\mu_y, A)^c$ and $(P^1_x, A)^c \sqsubset (P^\lambda_x, A)^c$. Therefore, $(U^{\sim}_E, \tau_f)$ is a fsq-T_{1-} space. $\square$

Definition 4.4. [6] *A fst-space (X, τ) is said to be fuzzy quasi T_1 (briefly; fq-T_1) iff for every pair of fuzzy points $P_x^\lambda, P_y^\mu \in X$ such that $x \neq y$ there exist $U, V \in \tau$ such that $P_x^\lambda qU \leq (P_y^\mu)^c$ and $P_y^\mu qV \leq (P_x^\lambda)^c$.*

Theorem 4.5. *If $(U_E^\sim, \tau_f)$ is a fsq-T_{1-} space then, for any $e \in E$ $(f_A(e), \tau_{f_e})$ is fq-T_1.*

Proof. Let P_x^λ, P_y^μ $(x, y \in U, x \neq y)$ be two fuzzy points. Since $(U_E^\sim, \tau_f)$ is fsq-T_1, there exist (g_A, E), $(h_A, E) \in \tau_f$ such that $P_x^\lambda q(g_A, E) \sqsubseteq (P_y^\mu, A)^c$ and $P_y^\mu q(h_A, E) \sqsubseteq (P_x^\lambda, A)^c$. Hence, $P_x^\lambda q g_A(e) \leq (P_y^\mu)^c$ and $P_y^\mu q h_A(e) \leq (P_x^\lambda)^c$ for any $e \in E$. This shows that, $(U_E^\sim, \tau_{f_e})$ is fq-T_1. □

Theorem 4.6. *If $(U_E^\sim, \tau_f)$ is a fsq-T_{1-} space then, $({}^V U_E^\sim, \tau_{f_V})$ is fsq-T_1.*

Proof. Similar to Theorem 4.3. □

Remark 4.1. *Every fsq-T_{1-} space is a fsq-T_{0-} space, but the converse is not true generally as seen the following example:*

Example 4.1. *Let $U = \{x, y\}$ and $E = \{e_1, e_2, e_3\}$ and $\tau_f = \{\Phi, U_E^\sim, (f_{1_A}, E)\}$ be a fst-space where,*

$$(f_{1_A}, E) = \{e_1 = \{x_1, y_0\}, e_2 = \{x_1, y_0\}\}.$$

Then, $(U_E^\sim, \tau_f)$ is a fsq-T_{0-} space but not fsq-T_{1-} space.

Definition 4.5. *Let $(U_E^\sim, \tau_f)$ be a fst-space. If for any fuzzy points P_x^λ, P_y^μ $(x, y \in U, x \neq y)$ of I^U there exist fs-open sets $(g_A, E), (h_A, E)$ such that $P_x^\lambda q(g_A, E) \sqsubseteq (P_y^\mu, A)^c$ and $P_y^\mu q(h_A, E) \sqsubseteq (P_x^\lambda, A)^c$ and (g_A, E) is not q-coincident with (h_A, E) then $(U_E^\sim, \tau_f)$ is called fuzzy soft quasi T_{2-} space (briefly; fsq-T_{2-} space).*

Example 4.2. *Let $U = \{x, y\}$, $E = \{e_1, e_2, e_3\}$, $A = \{e_1, e_2\}$ and $\tau_f = \{\Phi, E^\sim, (f_{1_A}, E), (f_{2_A}, E)\}$ where,*

$$(f_{1_A}, E) = \{e_1 = \{x_0, y_1\}, e_2 = \{x_0, y_1\},$$
$$(f_{2_A}, E) = \{e_1 = \{x_1, y_0\}, e_2 = \{x_1, y_0\}.$$

Then, (f_A, E, τ_f) is a fsq-T_{2-} space.

Remark 4.2. *Every fsq-T_{2-} space is a fsq-T_{1-} space.*

Definition 4.6. [6] *A fuzzy topological space (X, τ) is said to be fuzzy quasi T_2 (briefly; fq-T_2) iff for every pair of fuzzy points $P_x^\lambda, P_y^\mu \in X$ such that $x \neq y$ there exist $U, V \in \tau$ such that $P_x^\lambda qU \leq (P_y^\mu)^c$, $P_y^\mu qV \leq (P_x^\lambda)^c$ and U is not q-coincident with V.*

Theorem 4.7. *If $(U_E^\sim, \tau_f)$ is a fsq-T_{2-} space then, for any $e \in E$, $(f_A(e), \tau_{f_e})$ is fq-T_{2-} space.*

Theorem 4.8. *If $(U_E^\sim, \tau_f)$ is a fsq-T_{2-} space then, $({}^V U_E^\sim, \tau_{f_V})$ is fsq-T_2.*

Proof. Similar to Theorem 4.3. □

Definition 4.7. *Let $(U_E^\sim, \tau_f)$ be a fst-space P_x^λ be a fuzzy point of I^U and (g_A, E) be a fs-closed set such that $P_x^\lambda q(g_A, E)^c$. If there exist fs-open sets $(s_A, E), (h_A, E)$ such that $P_x^\lambda q(s_A, E)$, $(g_A, E)q(h_A, E)$ and $(s_A, E)q(h_A, E)^c$ then, $(U_E^\sim, \tau_f)$ is called fuzzy soft quasi regular space (briefly; fsq-regular space).*

Example 4.3. *Let $U = \{x, y\}$, $E = \{e_1, e_2, e_3\}$ and*

$$\tau_f = \{\Phi, U_E^\sim, (f_{1_A}, E), (f_{2_A}, E), (f_{3_A}, E), (f_{4_A}, E), (f_{5_A}, E),$$
$$(f_{6_A}, E), (f_{7_A}, E), (f_{8_A}, E), (f_{9_A}, E), (f_{10_A}, E), (f_{11_A}, E)\}$$

be a fs-topology where,

$$\begin{aligned}
(f_{1_A}, E) &= \{e_1 = \{x_{0.6}, y_{0.7}\}, e_2 = \{x_{0.6}, y_{0.8}\}\},\\
(f_{2_A}, E) &= \{e_1 = \{x_0, y_{0.9}\}, e_2 = \{x_0, y_{0.8}\}\},\\
(f_{3_A}, E) &= \{e_1 = \{x_{0.8}, y_0\}, e_2 = \{x_{0.8}, y_0\}\},\\
(f_{4_A}, E) &= \{e_1 = \{x_{0.9}, y_{0.9}\}, e_2 = \{x_{0.8}, y_{0.8}\}\},\\
(f_{5_A}, E) &= \{e_1 = \{x_{0.6}, y_{0.9}\}, e_2 = \{x_{0.6}, y_{0.8}\}\},\\
(f_{6_A}, E) &= \{e_1 = \{x_0, y_{0.7}\}, e_2 = \{x_0, y_{0.8}\}\},\\
(f_{7_A}, E) &= \{e_1 = \{x_{0.8}, y_{0.7}\}, e_2 = \{x_{0.8}, y_{0.8}\}\},\\
(f_{8_A}, E) &= \{e_1 = \{x_{0.6}, y_0\}, e_2 = \{x_{0.6}, y_0\}\},\\
(f_{9_A}, E) &= \{e_1 = \{x_{0.8}, y_{0.9}\}, e_2 = \{x_{0.8}, y_{0.8}\}\},\\
(f_{10_A}, E) &= \{e_1 = \{x_1, y_{0.9}\}, e_2 = \{x_1, y_{0.8}\}\},\\
(f_{11_A}, E) &= \{e_1 = \{x_{0.9}, y_1\}, e_2 = \{x_{0.8}, y_1\}\}.
\end{aligned}$$

Then, $(U_E^{\sim}, \tau_f)$ *is a fsq-regular space.*

Definition 4.8. *If* $(U_E^{\sim}, \tau_f)$ *is both fsq-regular and fsq-*T_1 *then it is called fuzzy soft quasi* T_{3-} *space (briefly; fsq-*T_{3-} *space).*

Remark 4.3. *If* $(U_E^{\sim}, \tau_f)$ *is a fsq-regular space then* $({}^V U_E^{\sim}, \tau_{f_V})$ *may not be fsq-regular space as seen in the following example.*

Example 4.4. *We consider example 4.3.* $({}^V U_E^{\sim}, \tau_{f_V})$ *where* $V = \{x\}$ *is not a fsq-regular space. Because, for the fuzzy point* $P_x^{0.3}$ *and the fs-closed set* $(f_{10_A}, E)^c$ *such that* $P_x^{0.3} q (f_{10_A}, E)$ *there do not exist any fs-open sets* $(g_A, E), (h_A, E)$ *such that* $P_x^{0.3} q (g_A, E)$ *and* $(f_{10_A}, E)^c q (h_A, E)$ *and* $(g_A, E)^c q (h_A, E)$.

Definition 4.9. *Let* $(U_E^{\sim}, \tau_f)$ *be a fst-space and* $(g_A, E), (s_A, E)$ *be fs-closed sets such that* $(g_A, E) q (s_A, E)^c$*. If there exist fs-open sets* $(h_A, E), (k_A, E) \in \tau_f$ *such that* $(g_A, E) q (h_A, E)$, $(s_A, E) q (k_A, E)$ *and* $(h_A, E) q (k_A, E)^c$ *then* $(U_E^{\sim}, \tau_f)$ *is called fuzzy soft quasi normal space (briefly; fsq-normal space).*

Example 4.5. *Let* $U = \{x, y\}$, $E = \{e_1, e_2, e_3\}$ *and* $\tau_f = \{\Phi, U_E^{\sim}, (f_{1_A}, E), (f_{2_A}, E), (f_{3_A}, E), (f_{4_A}, E), (f_{5_A}, E), (f_{6_A}, E)\}$ *be a fs-topology where,*

$$\begin{aligned}
(f_{1_A}, E) &= \{e_1 = \{x_{0.3}, y_{0.5}\}, e_2 = \{x_{0.5}, y_{0.4}\}\},\\
(f_{2_A}, E) &= \{e_1 = \{x_{0.6}, y_{0.4}\}, e_2 = \{x_{0.2}, y_{0.7}\}\},\\
(f_{3_A}, E) &= \{e_1 = \{x_{0.6}, y_{0.5}\}, e_2 = \{x_{0.5}, y_{0.7}\}\},\\
(f_{4_A}, E) &= \{e_1 = \{x_{0.3}, y_{0.4}\}, e_2 = \{x_{0.2}, y_{0.4}\}\},\\
(f_{5_A}, E) &= \{e_1 = \{x_{0.9}, y_{0.9}\}, e_2 = \{x_{0.8}, y_{0.8}\}\},\\
(f_{6_A}, E) &= \{e_1 = \{x_1, y_{0.4}\}, e_2 = \{x_1, y_{0.5}\}\}.
\end{aligned}$$

Then $(U_E^{\sim}, \tau_f)$ *is a fsq-normal space.*

Definition 4.10. *If* $(U_E^{\sim}, \tau_f)$ *is both fsq-normal and fsq-*T_1 *then it is called fuzzy soft quasi* T_{4-} *space (briefly; fsq-*T_{4-} *space).*

Remark 4.4. *If* $(U_E^{\sim}, \tau_f)$ *is a fsq-normal space then* $({}^V U_E^{\sim}, \tau_{f_V})$ *may not be fsq-normal space as seen in the following example.*

Example 4.6. *We consider example 4.5.* $({}^{V}U_{\widetilde{E}}, \tau_{f_V})$ *where* $V = \{x\}$ *is not a fsq-normal space. Because, for the fs-closed sets* $(f_{1_A}, E)^c$ *and* $(f_{5_A}, E)^c$ *such that* $(f_{1_A}, E)^c q(f_{5_A}, E)$ *there do not exist any fs-open sets* $(g_A, E), (h_A, E)$ *such that* $(f_{1_A}, E)^c q(g_A, E)$*,* $(f_{5_A}, E)^c q(h_A, E)$ *and* $(g_A, E)^c q(h_A, E)$*.*

5. Conflict of Interests

There is no conflict of interests regarding the publication of this paper.

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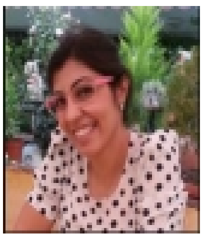

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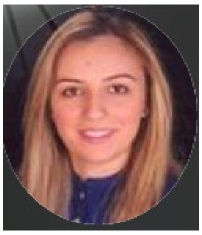

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