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EDGE DOMINATION IN SOME BRICK PRODUCT GRAPHS

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Abstract. Let $G = (V, E)$ be a simple connected and undirected graph. A set F of edges in G is called an edge dominating set if every edge e in $E - F$ is adjacent to at least one edge in F. The edge domination number $\gamma^{'}(G)$ of G is the minimum cardinality of an edge dominating set of G. The shadow graph of G, denoted $D_2(G)$ is the graph constructed from G by taking two copies of G, say G itself and $G^{'}$ and joining each vertex u in G to the neighbors of the corresponding vertex $u^{'}$ in $G^{'}$. Let D be the set of all distinct pairs of vertices in G and let D_s (called the distance set) be a subset of D. The distance graph of G, denoted by $D(G, D_s)$ is the graph having the same vertex set as that of G and two vertices u and v are adjacent in $D(G, D_s)$ whenever $d(u, v) \in D_s$. In this paper, we determine the edge domination number of the shadow distance graph of the brick product graph $C(2n, m, r)$.

Keywords: Dominating set, Brick product graph, Edge domination number, Minimal edge dominating set, Shadow distance graph.

AMS Subject Classification: 05C69

1. Introduction

By a graph $G = (V, E)$ we mean a finite undirected graph without loops and multiple edges. A subset S of V is called a dominating set of G if every vertex not in S is adjacent to some vertex in S. The domination number of G denoted by $\gamma(G)$ is the minimal cardinality taken over all dominating sets of G. A subset F of E is called an edge dominating set if each edge in E is either in F or is adjacent to an edge in F. An edge dominating set F is called minimal if no proper subset of F is an edge dominating set. The edge domination number of G denoted by $\gamma^{'}(G)$ is the minimum cardinality taken over all edge dominating sets of G.

The open neighbourhood of an edge $e \in E$ denoted by $N(e)$ is the set of all edges adjacent to e in G. If $e = (u, v)$ is an edge in G, the degree of e denoted by $deg(e)$ is

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defined as $deg(e) = deg(u) + deg(v) - 2$. The maximum degree of an edge in G is denoted by $\triangle^{'}(G)$.

Let m, n and r be a positive integers. Let $C_{2n} = a_0, a_1, a_2, ..., a_{2n-1}, a_0$ denote a cycle order $2n$. The (m, r) - brick product of C_{2n}, [1] denoted by $C(2n, m, r)$, is defined in two cases as follows.

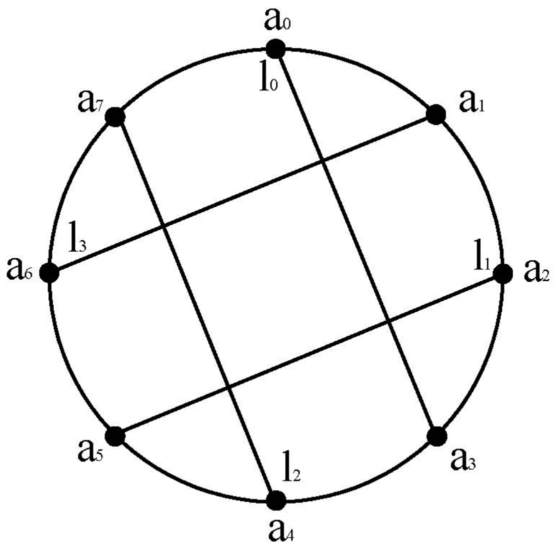

FIGURE 1. The brick product graph $C(8, 1, 3)$

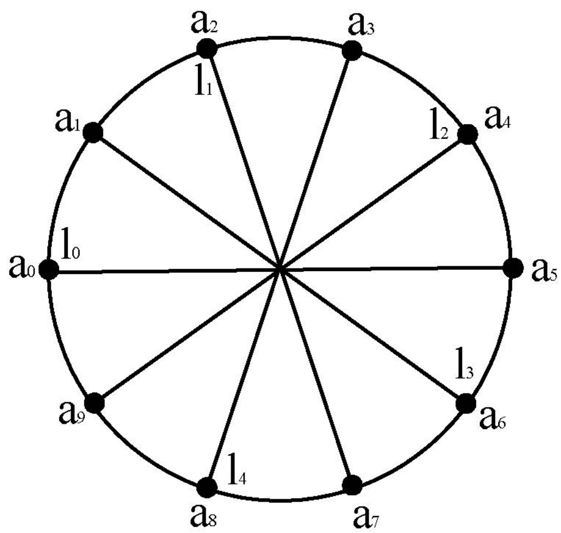

FIGURE 2. The brick product graph $C(10, 1, 5)$

(1) For m=1, we require that r be odd and greater than 1. Then, C(2n,m,r) is obtained from C_{2n} by adding chords $a_{2k}a_{2k+r}$, k = 1,2,...,n, where the computation is performed modulo 2n.
(2) For m >1, we require that $m + r$ be even. Then, C(2n,m,r) is obtained by first taking the disjoint union of m copies of C_{2n}, namely $C_{2n}(1), C_{2n}(2), ..., C_{2n}(m)$, where for each $i = 1, 2, ..., m$, $C_{2n}(i) = (i, 0)(i, 1)...(i, 2n)$. Next, for each odd i

$= 1, 2, ...m-1$ and each even $k = 0, 1, 2, ...2n-2$, an edge (called a brick edge) is drawn to join(a_i, a_k) to (a_{i+1}, a_k), whereas, for each even $i = 1, 2, ..., m-1$ and each odd $k = 1, 2, ..., 2n-1$, an edge (also called a brick edge) is drawn to join (a_i, a_k) to (a_{i+1}, a_k). Finally, for each odd $k = 1, 2, .., 2n-1$, an edge (called a hooking edge) is drawn to join (a_1, a_k) to (a_m, a_{k+r}). An edge in C(2n,m,r) which is neither a brick edge nor a hooking edge is called a flat edge.

The shadow graph of G, denoted by $D_2(G)$ is the graph constructed from G by taking two copies of G, namely G itself and $G^{'}$ and by joining each vertex u in G to the neighbors of the corresponding vertex $u^{'}$ in $G^{'}$.

Let D be the set of all distances between distinct pairs of vertices in G and let D_s (called the distance set) be a subset of D. The distance graph of G denoted by $D(G, D_s)$ is the graph having the same vertex set as that of G and two vertices u and v are adjacent in $D(G, D_s)$ whenever $d(u, v) \in D_s$.

The shadow distance graph of G, denoted by $D_{sd}(G, D_s)$ is constructed from G with the following conditions:

(1) consider two copies of G say G itself and $G^{'}$
(2) if $u \in V(G)$ (first copy) then we denote the corresponding vertex as $u^{'} \in V(G^{'})$ (second copy)
(3) the vertex set of $D_{sd}(G, D_s)$ is $V(G) \cup V(G^{'})$
(4) the edge set of $D_{sd}(G, D_s)$ is $E(G) \cup E(G^{'}) \cup E_{ds}$ where E_{ds} is the set of all edges (called the shadow distance edges) between two distinct vertices $u \in V(G)$ and $v^{'} \in V(G^{'})$ that satisfy the condition $d(u, v) \in D_s$ in G.

The applications of domination in graph structures lies in various fields like social networks, radio stations, communication networks etc. In particular, applications of edge domination are well known and available in literature. Two problems of interest with regard to an arbitrary graph G are (a) Determining a Hamiltonian cycle in G and (b) constructing an efficient algorithm to generate a Hamiltonian cycle in G. An alternative to the construction of an algorithm, which leads to a reconstruction problem, is to determine a spanning tree T such that G is the distance graph of T.

2. Main results

We recall the following results related to the edge domination number of a graph.

Theorem 2.1. [7] $\gamma^{'}(C_p) = \lceil \frac{p}{3} \rceil$ *for* $p \geq 3$.

Theorem 2.2. [6] *An edge dominating set F is minimal if and only if for each edge* $e \in G$, *one of the following two conditions holds:*

(1) $N(e) \cap F = \phi$
(2) *there exists an edge* $e \in E - F$ *such that* $N(e) \cap F = \{e\}$.

We begin our results with the edge domination in brick product graphs

Theorem 2.3. *Let* $G = C(2n, 1, 3)$. *Then* $\gamma^{'}(G) = \lceil \frac{2n}{3} \rceil$, *for* $n > 3$, *where* $2n \equiv k(mod3)$ *and* $k = 1, 2$.

Proof. We consider the vertex set of G as $V(G) = \{a_0, a_1, a_2, ...a_{2n-1}\}$ and the edge set of G as $E(G) = E_1 \cup E_2$, where $E_1 = \{e_{i+1} | e_{i+1} = (a_i, a_{i+1})\}$, $i = 0, 1, 2, ...2n-1$, modulo $2n$ and $E_2 = \{l_i | l_i = (a_{2k}, a_{2k+r})\}$, $i = 1, 2, ..., n$, modulo $2n$.
For $n = 4$, the set $F = \{e_1, e_4, e_6\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G) = 3$.

For $n = 5$, the set $F = \{e_1, e_4, e_6, e_9\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G)= 4$.
For $n = 7$, the set $F = \{e_1, e_4, e_6\}\cup\{l_4, l_5\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G)= 5$.
For $n = 8$, the set $F = \{e_1, e_4, e_6, e_9\} \cup \{l_4, l_5\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G)= 6$.
case(i): Let $n = 3p + 7$, where $p= 1, 2, 3, ...$

Consider the set $F = \{e_1, e_4, e_6\}\cup\{e_{12j+4}, e_{12j+6}\}\cup\{l_{6k-2}, l_{6k-1}\}$ where $1 \leq j \leq \lceil \frac{n}{6} \rceil - 1$ when n is even, $1 \leq j \leq \lceil \frac{n}{6} \rceil - 2$ when n is odd and $1 \leq k \leq \lceil \frac{n}{6} \rceil - 1$
Case (ii) Let $n = 6q + 8$, $q = 1, 2, 3, ...$

Consider the set $F = \{e_1, e_4, e_6\} \cup \{e_{12j+4}, e_{12j+6}\} \cup \{e_{2n-1}\} \cup \{l_{6k-2}, l_{6k-1}\}$ where $1 \leq j \leq \lceil \frac{n-4}{6} \rceil - 1$, $1 \leq k \leq \lceil \frac{n-4}{6} \rceil$
Case (iii) Let $n = 6t + 5$, $t = 1, 2, 3, ...$

Consider the set $F = \{e_1, e_4, e_6\} \cup \{e_{12j+4}, e_{12j+6}\} \cup \{e_{2n-2}\} \cup \{l_{6k-2}, l_{6k-1}\}$ where $1 \leq j, k \leq \lceil \frac{n}{6} \rceil - 1$.

The set F in cases $(i), (ii)$and (iii) is a minimal edge dominating set with minimum cardinality since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G. Hence, any set containing edges less than that of F cannot be a dominating set of G. Also G is regular of degree 3 and each edge of G is of degree 4 and an edge of G can dominate atmost 5 distinct edges of G including itself.

This implies that the set F described above is of minimum cardinality and since $|F| = \lceil \frac{2n}{3} \rceil$, it follows that $\gamma^{'}(C(2n, 1, 3) = \lceil \frac{2n}{3} \rceil$

Hence the proof. □

Theorem 2.4. *Let $G = C(2n, 1, r)$. Then $\gamma^{'}(G) = \lceil \frac{2n}{3} \rceil$ for*
(i) $r = 5$ and $n > 3$
(ii) $r= 7$ and $n > 4$
(iii) $r = n$ and $n > 4$ where $2n \equiv k(mod 3)$ and $k= 1, 2$

Proof. The vertex set and edge set of G are as in theorem 2.3.
Consider the set $F = \{e_1, e_4, e_7, ...e_{3j-2}\}$, where $1 \leq j \leq \lceil \frac{2n}{3} \rceil$

This set F is a minimal edge dominating set with minimum cardinality since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G. Hence, any set containing edges less than that of F cannot be a dominating set of G. Also G is regular of degree 3 and each edge of G is of degree 4 and an edge of G can dominate atmost 5 distinct edges of G including itself.

This implies that the set F described above is of minimum cardinality and since $|F| = \lceil \frac{2n}{3} \rceil$, it follows that $\gamma^{'}(C(2n, 1, r) = \lceil \frac{2n}{3} \rceil$

Hence the proof. □

We now investigate the edge domination number of some shadow distance graphs associated with brick product graphs.

Theorem 2.5. *Let $G = C(2n, 1, 3)$. Then* $\gamma^{'}(D_{sd}\{G, \{2\}\}) = \begin{cases} 2n-2, & n = 4,5,7 \\ 2n-4, & n \geq 8 \end{cases}$
where $2n \equiv k(mod 3)$ and $k= 1, 2$.

Proof. Consider two copies of G namely G itself and $G^{'}$. In the first copy, let $V(G) = \{(a_0)_1, (a_1)_1, (a_2)_1, ..., (a_{2n-1})_1\}$ and $E(G)= (E_1)_1\cup(E_2)_1$ where $(E_1)_1= \{(e_{i+1})_1|(e_{i+1})_1 = ((a_i)_1, (a_{i+1})_1)\}$, $i = 0, 1, 2, ...2n-1$, modulo $2n$ and $(E_2)_1 = \{(l_i)_1|(l_i)_1 = ((a_{2k})_1, (a_{2k+r})_1)\}$, $i = 1, 2, ..., n$, modulo $2n$. In the second copy, let $V(G^{'}) = \{(a_0)_2, (a_1)_2, (a_2)_2, ..., (a_{2n-1})_2\}$

and $E(G^{'})= (E_1)_2 \cup (E_2)_2$ where $(E_1)_2 = \{(e_{i+1})_2 | (e_{i+1})_2 = ((a_i)_2, (a_{i+1})_2)\}$, $i = 0, 1, 2, ...2n-1$, modulo $2n$ and $(E_2)_2 = \{(l_i)_2 | (l_i)_2 = ((a_{2k})_2, (a_{2k+r})_2)\}$, $i = 1, 2, ..., n$, modulo $2n$.
Let $G_1 = (D_{sd}\{G, \{2\}\}$. Then $V(G_1) = V(G) \cup V(G^{'})$ and $E(G_1) = E(G) \cup E(G^{'}) \cup E_3$, where E_3 are the shadow distance edges.

For $n = 4$, the set $F = \{e_2, e_4, e_8, e_2^{'}, e_4^{'}, e_8^{'}\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G_1) = 6$. $(= 2n - 2)$

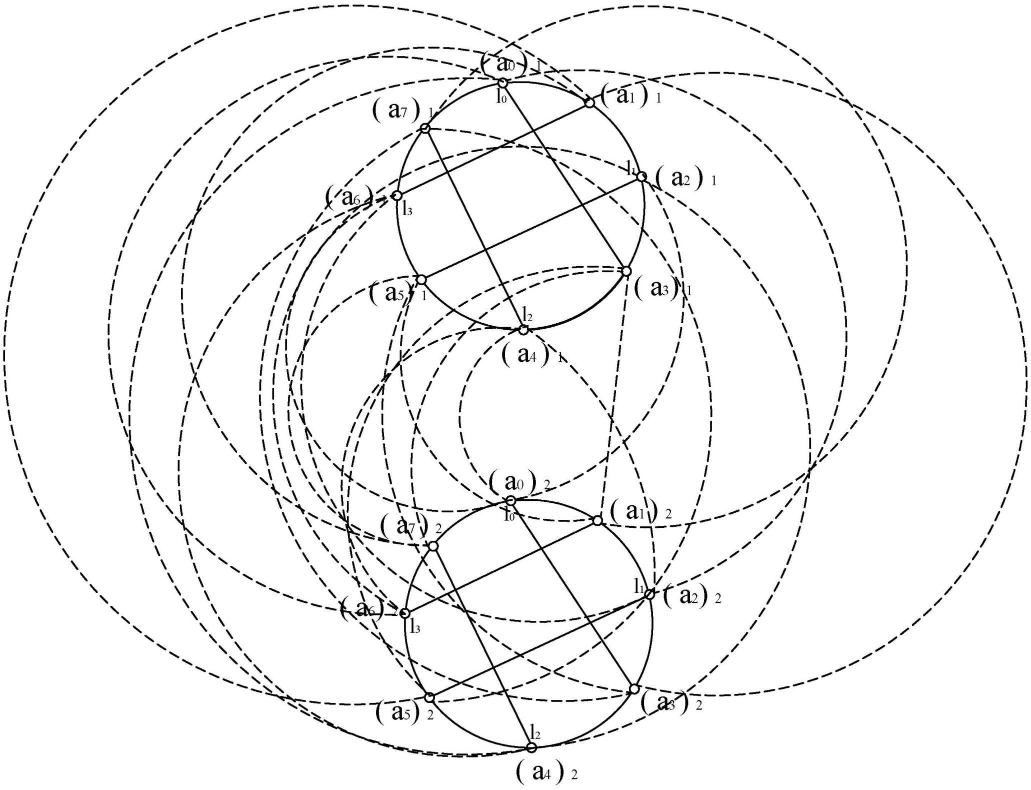

FIGURE 3. The brick product graph $\gamma^{'}(D_{sd}\{C(8, 1, 3), \{2\}\})$

For $n = 5$, the set $F = \{e_2, e_4, e_7, e_{10}, e_2^{'}, e_4^{'}, e_7^{'}, e_{10}^{'}\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G_1) = 8$. $(= 2n - 2)$

For $n = 7$, the set $F = \{e_2, e_4, e_7, e_{10}, e_{12}, e_{14}, e_2^{'}, e_4^{'}, e_7^{'}, e_{10}^{'}, e_{12}^{'}, e_{14}^{'}\}$ is minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G_1) = 12$. $(= 2n - 2)$

For $n = 8$, the set $F = \{e_2, e_4, e_7, e_{10}, e_{12}, e_{15}, e_2^{'}, e_4^{'}, e_7^{'}, e_{10}^{'}, e_{12}^{'}, e_{15}^{'}\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G_1) = 12$. $(= 2n - 4)$

Let $n \geq 10$.

Consider the set $F = \{e_2, e_4, e_7, e_{10}, e_{12}, e_2^{'}, e_4^{'}, e_7^{'}, e_{10}^{'}, e_{12}^{'}, \} \cup \{e_{2j+13}, e_{2j+13}^{'}\} \cup \{e_0, e_0^{'}\}$, where $1 \leq j \leq n - 8$.

This set F is a minimal edge dominating set with minimum cardinality since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G_1. Hence, any set containing edges less than that of F cannot be a dominating set of G_1. Also G_1 is regular of degree 7 and each edge of G_1 is of degree 12 and an edge of G_1 can dominate atmost 13 distinct edges of G_1 including itself.

This implies that the set F described above is of minimum cardinality and since $|F|$ $=2n-4$, it follows that $\gamma^{'}(D_{sd}\{G,\{2\}\}) = 2n-4$

Hence the proof. □

Theorem 2.6. *Let $G = C(2n, 1, 3)$. Then $\gamma^{'}(D_{sd}\{G,\{3\}\}) = 2n$, where $2n \equiv k(mod3)$ and $k= 1, 2$.*

Proof. Let $G_1 = (D_{sd}\{G,\{3\}\}$. The vertex set and edge set of G_1 are as in theorem 2.5.

Let $n \geq 4$.

Consider the set $F = F_1 \cup F_2$, where $F_1 = \{e_{2j-1}\}$ and $F_2 = \{e^{'}_{2j-1}\}$, $1 \leq j \leq n$.

This set F is a minimal edge dominating set with minimum cardinality since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G_1. Hence, any set containing edges less than that of F cannot be a dominating set of G_1. Also for $4 \leq n \leq 5$, G_1 is regular of degree n, each edge of G_1 is of degree $2(n-1)$ and an edge of G_1 can dominate atmost $2n$ distinct edges of G_1 including itself. Further , for $n \geq 7$, G_1 is regular of degree 7, each edge of G_1 is of degree 12 and an edge of G_1 can dominate atmost 13 distinct edges of G_1 including itself.

This implies that the set F described above is of minimum cardinality and since $|F|$ $=2n$, it follows that $\gamma^{'}(D_{sd}\{G,\{3\}\})= 2n$

Hence the proof □

Theorem 2.7. *Let $G = C(2n, 1, 5)$. Then $\gamma^{'}(D_{sd}\{G,\{2\}\})= 2n-2$, for $n > 3$*

Proof. Let $G_1 = (D_{sd}\{G,\{2\}\}$. The vertex set and edge set of G_1 are as theorem 2.5.

For $n = 4$, the set $F = \{e_2, e_4, e_8, e^{'}_2, e^{'}_4, e^{'}_8\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G_1) = 6$ $(= 2n-2)$.

For $n = 5$, the set $F = \{e_2, e_4, e_7, e_{10}, e^{'}_2, e^{'}_4, e^{'}_7, e^{'}_{10}\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G_1) = 8$ $(= 2n-2)$.

For $n = 6$, the set $F = \{e_2, e_4, e_7, e_9, e_{12}, e^{'}_2, e^{'}_4, e^{'}_7, e^{'}_9, e^{'}_{12}\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G_1) = 10(= 2n-2)$.

Let $n \geq 7$.

Consider set $F = \{e_2, e_4, e_7, e_9, e_{11}\} \cup F_1 \cup \{e^{'}_2, e^{'}_4, e^{'}_7, e^{'}_9, e^{'}_{11}\} \cup F_2$, where $F_1 = \{e_{2j+12}\}$ and $F_2 = \{e^{'}_{2j+12}\}$, $1 \leq j \leq n-6$.

This set F is a minimal edge dominating set with minimum cardinality since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G_1. Hence, any set containing edges less than that of F cannot be a dominating set of G_1. Also G_1 is regular of degree 9 and each edge of G_1 is of degree 16 and an edge of G_1 can dominate atmost 17 distinct edges of G_1 including itself.

This implies that the set F described above is of minimum cardinality and since $|F|$ $=2n-2$, it follows that $\gamma^{'}(D_{sd}\{G,\{2\}\})= 2n-2$

Hence the proof. □

Theorem 2.8. *Let $G = C(2n, 1, 5)$. Then $\gamma^{'}(D_{sd}\{G,\{3\}\})= 2n$, for $n > 3$*

Proof. Let $G_1 = (D_{sd}\{G,\{3\}\})$. The vertex set and edge set of G_1 are as in theorem 2.5.

For $n = 4$, the set $F = \{e_1, e_3, e_5, e_8, e^{'}_1, e^{'}_3, e^{'}_5, e^{'}_8\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G_b) = 8 = 2n$.

Let $n \geq 5$.

Consider set $F = F_1 \cup F_2$, where $F_1= \{e_{2j-1}\}$ and $F_2 = \{e^{'}_{2j-1}\}$, $1 \leq j \leq n$.

This set F is a minimal edge dominating set with minimum cardinality since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G_1. Hence, any set containing edges less than that of F cannot be a dominating set of G_1. Also for $4 \leq n \leq 9$, G_1 is regular of degree n, each edge of G_1 is of degree $2(n-1)$ and an edge of G_1 can dominate atmost $2n$ distinct edges of G_1 including itself. Further, for $n \geq 10$, G_1 is regular of degree 10, each edge of G_1 is of degree 18 and an edge of G_1 can dominate atmost 19 distinct edges of G_1 including itself.

This implies that the set F described above is of minimum cardinality and since $|F|$ $=2n$, it follows that $\gamma^{'}(D_{sd}\{G,\{3\}\})= 2n$

Hence the proof. □

Theorem 2.9. *Let $G = C(2n, 1, 7)$. Then $\gamma^{'}(D_{sd}\{G,\{2\}\})= 2n-2$, for $n > 4$*

Proof. Let $G_1 = (D_{sd}\{G,\{2\}\})$. The vertex set and edge set of G_1 are as in theorem 2.5.

For $n = 5$, the set $F = \{e_1, e_4, e_7, e_9, e^{'}_1, e^{'}_4, e^{'}_7, e^{'}_9\}$ is a minimal edge dominating set with minimum cardinality and hence $\gamma^{'}(G_b) = 8$.

Let $n \geq 6$.

Consider set $F = \{e_2, e_4\} \cup F_1 \cup \{e^{'}_2, e^{'}_4\} \cup F_2$, where $F_1 = \{e_{2j+5}\}$ and $F_2 = \{e^{'}_{2j+5}\}$, $1 \leq j \leq n-3$.

This set F is a minimal edge dominating set with minimum cardinality since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G_1. Hence, any set containing edges less than that of F cannot be a dominating set of G_1. Also for $n > 8$, G_1 is regular of degree 9 and each edge of G_1 is of degree 16 and an edge of G_1 can dominate atmost 17 distinct edges of G_1 including itself.

This implies that the set F described above is of minimum cardinality and since $|F|$ $=2n-2$, it follows that $\gamma^{'}(D_{sd}\{G,\{2\}\})= 2n-2$

Hence the proof. □

Theorem 2.10. *Let $G = C(2n, 1, 7)$. Then $\gamma^{'}(D_{sd}\{G,\{3\}\})= 2n$, for $n > 4$*

Proof. Let $G_1 = (D_{sd}\{G,\{3\}\})$. The vertex set and edge set of G_1 are as theorem 2.5.

Let $n \geq 4$.

Consider set $F = F_1 \cup F_2$, where F_1= $\{e_{2j-1}\}$ and $F_2 = \{e^{'}_{2j-1}\}$, $1 \leq j \leq n$.

This set F is a minimal edge dominating set with minimum cardinality since for any edge $e_i \in F$, $F - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G_1. Hence, any set containing edges less than that of F cannot be a dominating set of G_1. Also for $4 \leq n \leq 9$, G_1 is regular of degree n, each edge of G_1 is of degree $2(n-1)$ and an edge of G_1 can dominate atmost $2n$ distinct edges of G_1 including itself. Further, for $n \geq 10$, G_1 is regular of degree 10, each edge of G_1 is of degree 18 and an edge of G_1 can dominate atmost 19 distinct edges of G_1 including itself.

This implies that the set F described above is of minimum cardinality and since $|F|$ $=2n$, it follows that $\gamma^{'}(D_{sd}\{G,\{3\}\})= 2n$

Hence the proof. □

3. Conclusions

In this paper, the edge domination number $\gamma^{'}(G)$ of some shadow distance graphs (introduced in [3]) associated with the brick product graphs of even cycles C_{2n} are determined.

References

[1] Brian Alspach, C.C. Chen, Kevin McAvaney., (1996), On a class of Hamiltonian laceable 3-regular graphs,Discrete Mathematics, 151, pp. 19-38. .

[2] S.K.Vaidya and R.M.Pandit., (2014), Edge domination in some path and cycle related graphs, Hindawi Publishing, ISRN Discrete Mathematics, 975812, pp.1-5.
[3] U. Vijaya Chandra Kumar and R. Murali., (2016), Edge Domination in Shadow distance Graphs , International journal of Mathematics and its applications, pp . 125 - 130.
[4] U. Vijaya Chandra Kumar and R. Murali., (2017), Edge Domination in Shadow distance Graph of some star related graphs, Annals of Pure and Applied Mathematics and its applications, pp. 33-40.
[5] S.T.Hedetniemi and R.C.Laskar., (1990), Bibliography on domination in graphs and some basic definitions of domination parameters, Discrete Mathematics, pp.257277.
[6] V.R.Kulli., (2013), Theory of domination in graphs, Vishwa International Publications.
[7] S.R.Jayaram., (1987), Line domination in graphs, Graphs Combin.3, pp. 357-363.
[8] Frank Harary., (1969), Graph Theory, Addison - Wesley Publications.

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