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A NOTE ON CERTAIN TOPOLOGICAL INDICES OF THE DERIVED GRAPHS OF SUBDIVISION GRAPHS

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Abstract. In this note, we correct some errors in S. M. Hosamani et al. [TWMS J. App. Eng. Math., 6(2), (2016) pp. 324–332]. The derived graph $[G]^\dagger$ of a graph G is the graph having the same vertex set as G, with two vertices of $[G]^\dagger$ being adjacent if and only if their distance in G is two. Further, we compute generalized Randić, general Zagreb, general sum-connectivity, ABC, GA, ABC_4, and GA_5 indices of the derived graphs of subdivision graphs.

Keywords: derived graph, subdivision graph, line graph, topological index.

AMS Subject Classification: 05C90, 05C35, 05C12

1. Introduction

Let G be a simple, finite and undirected graph with vertex set $V(G)$ and edge set $E(G)$. The degree d_u of a vertex u in G is the number of edges incident to it in G and $S_u = \sum\limits_{v \in N_u} d_v$, where $N_u = \{v \in V(G) | uv \in E(G)\}$. N_u is also known as the set of neighbor vertices of u in G.

Li and Zhao introduced the first general Zagreb index in [14] as

$$M_\alpha(G) = \sum_{u \in V(G)} d_u^\alpha, \tag{1}$$

where α is a real number.

The *general Randić connectivity index* of G is defined in [15] as

$$R_\alpha(G) = \sum_{uv \in E(G)} (d_u d_v)^\alpha, \tag{2}$$

where α is a real number.

The *general sum-connectivity index* $\chi_\alpha(G)$ defined in [18] as

$$\chi_\alpha(G) = \sum_{uv \in E(G)} (d_u + d_v)^\alpha, \tag{3}$$

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where α is a real number.
The *atom-bond connectivity (ABC) index* of graph G is defined in [5] as

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}. \tag{4}$$

The *geometric arithmetic (GA) index* of a graph G is introduced by D. Vukicevic and B. Furtula in [17] and is defined as

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}. \tag{5}$$

The *fourth member of the class of ABC index* of a graph G is defined in [6] as

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}. \tag{6}$$

The *fifth geometric arithmetic index* of a graph G is defined in [7] as

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v}. \tag{7}$$

The *subdivision graph* $S(G)$ [9] is a graph with the vertex set $V(G) \cup E(G)$ such that two vertices of $S(G)$ are adjacent if and only if one corresponds to a vertex v of G and other to an edge e of G, and v is incident to e in G.

The *line graph* $L(G)$ [9] of graph G is the graph whose vertices are the edges of G, two vertices of $L(G)$ are adjacent if and only if the corresponding edges in G share a common vertex.

For undefined terminology we refer [9, 13]. For recent study on topological indices of transformation graphs, one can refer the articles [1, 2, 3, 4, 10, 16].

The *derived graph* $[G]^\dagger$ [8] of a graph G is the graph having the same vertex set as G, two vertices of $[G]^\dagger$ being adjacent if and only if their distance in G is two.

In [11], authors computed topological indices of derived graph of subdivision graph of tadpole graph $T_{n,k}$. But the expressions of $R_\alpha(T_{n,k})$ and $ABC(T_{n,k})$ does not holds for $k = 1$. The expressions of $R_\alpha(L(T_{n,k}))$, $\chi_\alpha(L(T_{n,k}))$, $ABC(L(T_{n,k}))$ and $GA(L(T_{n,k}))$, valid for $k > 2$. Also the expressions $GA(T_{n,k})$, $ABC_4(T_{n,k})$, $GA_5(T_{n,k})$, $M_\alpha(L(T_{n,k}))$, $ABC_4(L(T_{n,k}))$, $GA_5(L(T_{n,k}))$ are incorrect. In this paper, we correct all these errors. Further, we obtain topological indices of derived graph of subdivision graph of a crown graph CW_n, a gear graph G_n and a friendship graph $C_3^{(n)}$.

Theorem 1.1. [8] *Let G be any simple graph. Then $[S(G)]^\dagger \cong G \cup L(G)$.*

2. Tadpole graph

The tadpole graph $T_{n,k}$ [12] is the graph formed by joining the end point of a path of length k to a n-cycle.

Table 1. The edge partition of tadpole graph $T_{n,k}$ when $k > 1$

(d_u, d_v) where $uv \in E(T_{n,k})$	(3, 2)	(2, 2)	(2, 1)
Number of edges	3	$n + k - 4$	1

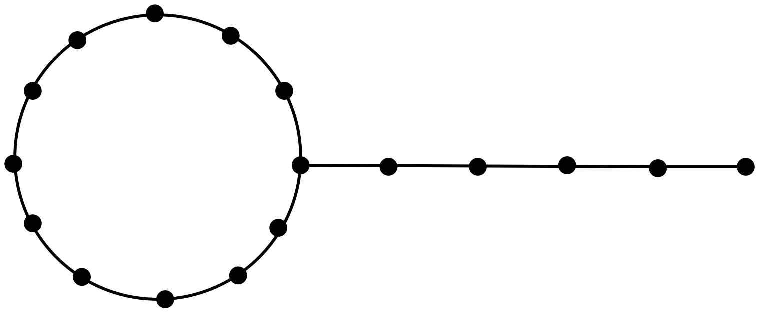

FIGURE 1. The tadpole graph $T_{12,5}$

TABLE 2. The edge partition of tadpole graph $T_{n,k}$ when $n, k > 3$

(S_u, S_v) where $uv \in E(T_{n,k})$	(6, 5)	(5, 4)	(4, 4)	(4, 3)	(3, 2)
Number of edges	3	3	$n+k-8$	1	1

Theorem 2.1. *Let $G = T_{n,k}$ be the tadpole graph. Then*

(1) $R_\alpha(G) = \begin{cases} (n+k-4)\cdot 4^\alpha + 3\cdot 6^\alpha + 2^\alpha & if\ k>1 \\ (n-2)\cdot 4^\alpha + 2\cdot 6^\alpha + 3^\alpha & if\ k=1\ ; \end{cases}$

(2) $ABC(G) = \begin{cases} \frac{1}{\sqrt{2}}(n+k) & if\ k>1 \\ \frac{n}{\sqrt{2}} + \sqrt{\frac{2}{3}} & if\ k=1; \end{cases}$

(3) $GA(G) = \begin{cases} (n+k-4) + \frac{6\sqrt{6}}{5} + \frac{2\sqrt{2}}{3} & if\ k>1 \\ (n-2) + \frac{4\sqrt{6}}{5} + \frac{\sqrt{3}}{2} & if\ k=1. \end{cases}$

Proof. The tadpole graph has $n+k$ vertices and $n+k$ edges, among them $n+k-2$ vertices are of degree two, one vertex of degree three and one vertex of degree one. Therefore we get the edge partition, based on the degrees of vertices as shown in Table 1. Using formulas (2), (4) and (5) to the information in Table 1, we obtain the required result. □

Theorem 2.2. [11] *Let $G = T_{n,k}$ be the tadpole graph. Then*

(1) $M_\alpha(G) = (n+k-2)\cdot 2^\alpha + 3^\alpha + 1^\alpha$;
(2) $\chi_\alpha(G) = (n+k-4)\cdot 4^\alpha + 3\cdot 5^\alpha + 3^\alpha$.

Theorem 2.3. *Let $G = T_{n,k}$ be the tadpole graph with $n, k > 3$. Then*

(1) $ABC_4(G) = (n+k-8)\sqrt{\frac{3}{8}} + 3\sqrt{\frac{3}{10}} + \frac{3}{2}\sqrt{\frac{7}{5}} + \sqrt{\frac{5}{12}} + \frac{1}{\sqrt{2}}$;

(2) $GA_5(G) = (n+k-8) + \frac{6\sqrt{30}}{11} + \frac{4\sqrt{3}}{7} + \frac{2\sqrt{6}}{5} + \frac{4\sqrt{5}}{3}$.

Proof. Using formulas (6) and (7) to the information in Table 2, we obtain the required result. □

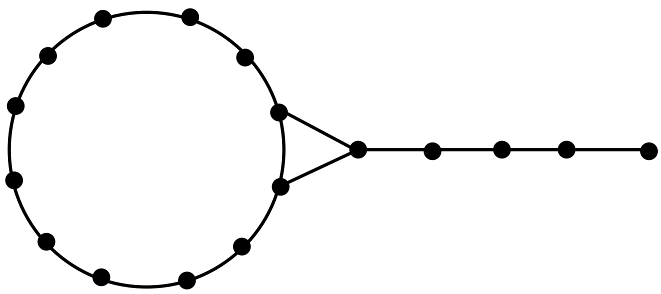

FIGURE 2. The line graph of tadpole graph $T_{12,5}$

TABLE 3. The edge partition of line graph of tadpole graph $L(T_{n,k})$ when $k > 2$

(d_u, d_v) where $uv \in E(L(T_{n,k}))$	(3, 3)	(3, 2)	(2, 2)	(2, 1)
Number of edges	3	3	$n+k-6$	1

TABLE 4. The edge partition of line graph of tadpole graph $L(T_{n,k})$ when $n, k > 4$

(S_u, S_v) where $uv \in E(L(T_{n,k}))$	(8, 8)	(8, 5)	(5, 4)	(4, 4)	(4, 3)	(3, 2)
Number of edges	3	3	3	$n+k-10$	1	1

Theorem 2.4. *Let $H = L(T_{n,k})$ be the line graph of tadpole graph with $k > 2$. Then*

(1) $R_\alpha(H) = (n+k-6) \cdot 4^\alpha + 3 \cdot 6^\alpha + 3 \cdot 9^\alpha + 2^\alpha$;
(2) $\chi_\alpha(H) = (n+k-6) \cdot 4^\alpha + 3 \cdot 5^\alpha + 3 \cdot 6^\alpha + 3^\alpha$;
(3) $M_\alpha(H) = (n+k-4) \cdot 2^\alpha + 3^{\alpha+1} + 1^\alpha$;
(4) $ABC(H) = (n+k)\frac{1}{\sqrt{2}} + (2-\sqrt{2})$;
(5) $GA(H) = (n+k-3) + \frac{6\sqrt{6}}{5} + \frac{2\sqrt{2}}{3}$.

Proof. The line graph of tadpole graph $T_{n,k}$ with $k > 2$ has $n+k$ vertices and $n+k+1$ edges, among them $n+k-4$ vertices are of degree two, three vertices of degree three and one vertex of degree one. Therefore we get the edge partition, based on the degrees of vertices as shown in Table 3. Using formulas (1)–(5) to the information in Table 3, we obtain the required result. □

Theorem 2.5. *Let $H = L(T_{n,k})$ be the line graph of tadpole graph with $n, k > 4$. Then*

(1) $ABC_4(H) = (n+k-10)\sqrt{\frac{3}{8}} + \frac{3\sqrt{14}}{8} + 3\sqrt{\frac{11}{40}} + \frac{3}{2}\sqrt{\frac{7}{5}} + \sqrt{\frac{5}{12}} + \frac{1}{\sqrt{2}}$;
(2) $GA_5(H) = (n+k-7) + \frac{12\sqrt{10}}{13} + \frac{4\sqrt{3}}{7} + \frac{4\sqrt{5}}{3} + \frac{2\sqrt{6}}{5}$.

Proof. Using formulas (6) and (7) to the information in Table 4, we obtain the required result. □

Theorem 2.6. *Let $[S(G)]^\dagger$ be the derived graph of subdivision graph of tadpole graph $G = T_{n,k}$ with $k > 2$. Then*

(1) $R_\alpha([S(G)]^\dagger) = 2(n+k-5) \cdot 4^\alpha + 6^{\alpha+1} + 2^{\alpha+1} + 3 \cdot 9^\alpha$;

(2) $\chi_\alpha([S(G)]^\dagger) = 2(n+k-5)\cdot 4^\alpha + 6\cdot 5^\alpha + 2\cdot 3^\alpha + 3\cdot 6^\alpha$;
(3) $M_\alpha([S(G)]^\dagger) = (n+k-3)\cdot 2^{\alpha+1} + 4\cdot 3^\alpha + 2\cdot 1^\alpha$;
(4) $ABC([S(G)]^\dagger) = \sqrt{2}(n+k-1)+2$;
(5) $GA([S(G)]^\dagger) = 2(n+k) - 7 + \frac{12\sqrt{6}}{5} + \frac{4\sqrt{2}}{3}$.

Proof. From Theorem 1.1, we have $[S(G)]^\dagger \cong G \cup L(G)$. Then by using the informations in Theorems 2.1, 2.2 and 2.4 we obtain the desired result. □

Theorem 2.7. *Let* $[S(G)]^\dagger$ *be the derived graph of subdivision graph of tadpole graph* $G = T_{n,k}$ *with* $n, k > 4$. *Then*

(1) $ABC_4([S(G)]^\dagger) = 2(n+k-9)\sqrt{\frac{3}{8}} + \frac{3\sqrt{14}}{8} + 3\sqrt{\frac{11}{40}} + 3\sqrt{\frac{7}{5}} + \sqrt{\frac{5}{3}} + 3\sqrt{\frac{3}{10}} + \sqrt{2}$;
(1) $GA_5([S(G)]^\dagger) = 2(n+k) - 15 + \frac{12\sqrt{10}}{13} + \frac{6\sqrt{30}}{11} + \frac{8\sqrt{3}}{7} + \frac{4\sqrt{6}}{5} + \frac{8\sqrt{5}}{3}$.

Proof. From Theorem 1.1, we have $[S(G)]^\dagger \cong G \cup L(G)$. Then by using the information in Theorems 2.3 and 2.5, we obtain the required result. □

3. Crown graph

A cycle C_n with a pendent edge attached at each vertex is called a *crown graph* CW_n [12]. Equivalently $CW_n = C_n \circ K_1$.

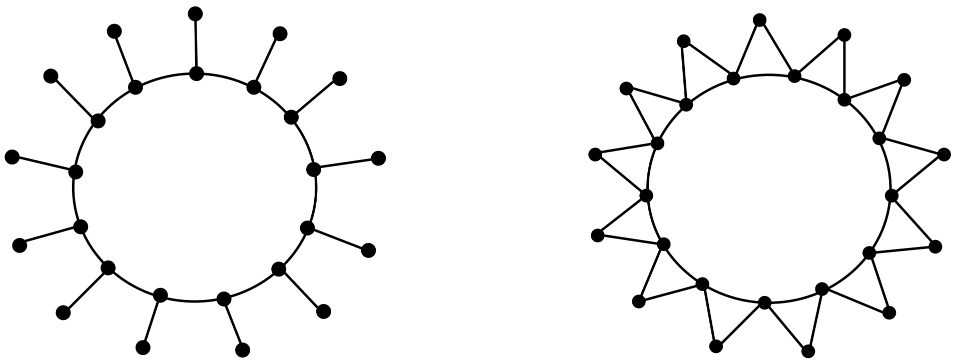

Figure 3. The crown graph CW_{13} and its line graph $L(CW_{13})$

Table 5. The edge partition of crown graph

(d_u, d_v) where $uv \in E(CW_n)$	(1, 3)	(3, 3)
Number of edges	n	n

Table 6. The edge partition of crown graph

(S_u, S_v) where $uv \in E(CW_n)$	(3, 7)	(7, 7)
Number of edges	n	n

Table 7. The edge partition of line graph of crown graph

(d_u, d_v) where $uv \in E(L(CW_n))$	(2, 4)	(4, 4)
Number of edges	$2n$	n

Table 8. The edge partition of line graph of crown graph

(S_u, S_v) where $uv \in E(L(CW_n))$	(8, 12)	(12, 12)
Number of edges	$2n$	n

Theorem 3.1. *Let $G = CW_n$ be the crown graph. Then*

(1) $M_\alpha(G) = n[1^\alpha + 3^\alpha]$;
(2) $R_\alpha(G) = n \cdot 3^\alpha[1 + 3^\alpha]$;
(3) $\chi_\alpha(G) = n \cdot 2^\alpha[2^\alpha + 3^\alpha]$;
(4) $ABC(G) = n\sqrt{\frac{2}{3}}\left[1 + \sqrt{\frac{2}{3}}\right]$;
(5) $GA(G) = n\left[1 + \frac{\sqrt{3}}{2}\right]$;
(6) $ABC_4(G) = 2n\left[\sqrt{\frac{2}{21}} + \frac{\sqrt{3}}{7}\right]$;
(7) $GA_5(G) = n\left[1 + \frac{\sqrt{21}}{5}\right]$.

Proof. The crown graph has $2n$ vertices and $2n$ edges, among them n vertices are of degree one and n vertices of degree three. Therefore we get the edge partition, based on the degrees of vertices as shown in Tables 5 and 6. Using formulae (1)–(7) to the information in Table 5 and 6, we obtain the desired result. □

Theorem 3.2. *Let $H = L(CW_n)$ be the line graph of crown graph. Then*

(1) $M_\alpha(H) = n \cdot 2^\alpha[2 + 2^\alpha]$;
(2) $R_\alpha(H) = n \cdot 8^\alpha[2 + 2^\alpha]$;
(3) $\chi_\alpha(H) = n \cdot 2^\alpha[2 \cdot 3^\alpha + 4^\alpha]$;
(4) $ABC(H) = n\left[\sqrt{2} + \sqrt{\frac{3}{8}}\right]$;
(5) $GA(H) = n\left[1 + \frac{4\sqrt{2}}{3}\right]$;
(6) $ABC_4(H) = \frac{n}{2}\left[\sqrt{3} + \frac{\sqrt{22}}{6}\right]$;
(7) $GA_5(H) = n\left[1 + \frac{4\sqrt{6}}{5}\right]$.

Proof. The line graph of crown graph has $2n$ vertices and $3n$ edges, among them n vertices are of degree four and $2n$ vertices of degree two. Therefore we get the edge partition, based on the degrees of vertices as shown in Tables 7 and 8. Using formulae (1)–(7) to the information in Table 7 and 8, we obtain the desired result. □

Theorem 3.3. *Let $[S(G)]^\dagger$ be the derived graph of subdivision graph of crown graph $G = CW_n$. Then*

(1) $M_\alpha([S(G)]^\dagger) = n[1^\alpha + 2^{\alpha+1} + 3^\alpha + 4^\alpha]$;
(2) $R_\alpha([S(G)]^\dagger) = n[2 \cdot 8^\alpha + 16^\alpha + 3^\alpha + 9^\alpha]$;
(3) $\chi_\alpha([S(G)]^\dagger) = n \cdot 2^\alpha[3^{\alpha+1} + 4^\alpha + 2^\alpha]$;
(4) $ABC([S(G)]^\dagger) = n\left[\sqrt{\frac{2}{3}} + \frac{2}{3} + \sqrt{2} + \sqrt{\frac{3}{8}}\right]$;
(5) $GA([S(G)]^\dagger) = n\left[2 + \frac{\sqrt{3}}{2} + \frac{4\sqrt{2}}{3}\right]$;
(6) $ABC_4([S(G)]^\dagger) = n\left[2\sqrt{\frac{2}{21}} + \frac{2\sqrt{3}}{7} + \frac{\sqrt{3}}{2} + \frac{\sqrt{22}}{12}\right]$;
(7) $GA_5([S(G)]^\dagger) = \frac{n}{5}\left[10 + 4\sqrt{6} + \sqrt{21}\right]$.

Proof. From Theorem 1.1, we have $[S(G)]^\dagger \cong L(G) \cup G$. Then by using the information in Theorems 3.1 and 3.2, we obtain the desired result. □

4. Gear graph

The *gear graph* G_n [12] is obtained from the wheel W_n by adding a vertex between every pair of adjacent vertices of the n-cycle.

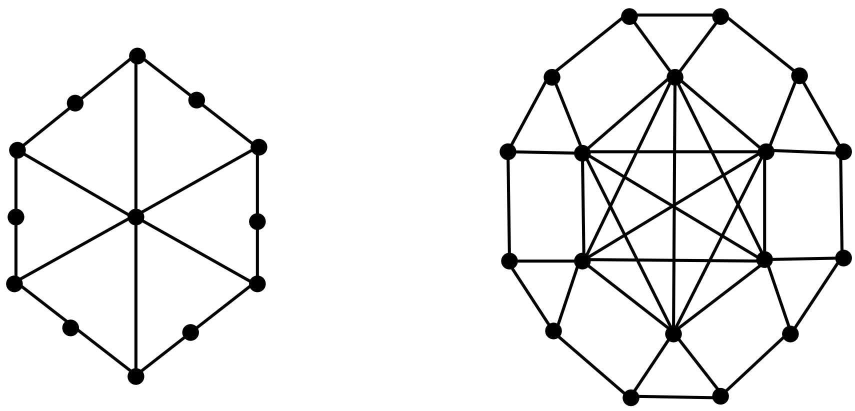

FIGURE 4. The gear graph G_6 and its line graph $L(G_6)$

TABLE 9. The edge partition of gear graph

(d_u, d_v) where $uv \in E(G_n)$	(2, 3)	$(3, n)$
Number of edges	$2n$	n

TABLE 10. The edge partition of gear graph

(S_u, S_v) where $uv \in E(G_n)$	$(6, n+4)$	$(3n, n+4)$
Number of edges	$2n$	n

TABLE 11. The edge partition of line graph of gear graph

(d_u, d_v) where $uv \in E(L(G_n))$	$(3, 3)$	$(3, n+1)$	$(n+1, n+1)$
Number of edges	$2n$	$2n$	$\binom{n}{2}$

TABLE 12. The edge partition of line graph of gear graph

(S_u, S_v) where $uv \in E(L(G_n))$	$(n+7, n+7)$	$(n+7, n^2+5)$	(n^2+5, n^2+5)
Number of edges	$2n$	$2n$	$\binom{n}{2}$

Theorem 4.1. *Let $G = G_n$ be the gear graph. Then*

(1) $M_\alpha(G) = n \cdot 3^\alpha + n \cdot 2^\alpha + n^\alpha$;

(2) $R_\alpha(G) = 2n \cdot 6^\alpha + 3^\alpha \cdot n^{\alpha+1}$;

(3) $\chi_\alpha(G) = 2n \cdot 5^\alpha + n \cdot (n+3)^\alpha$;

(4) $ABC(G) = n\left[\sqrt{2} + \sqrt{\frac{n+1}{3n}}\right]$;

(5) $GA(G) = 2n\left[\frac{2\sqrt{6}}{5} + \frac{\sqrt{3n}}{n+3}\right]$;

(6) $ABC_4(G) = 2n\left[\sqrt{\frac{n+8}{6(n+4)}} + \sqrt{\frac{2n+1}{6n(n+4)}}\right]$;

(7) $GA_5(G) = \frac{4n\sqrt{6(n+4)}}{n+10} + \frac{n\sqrt{3n(n+4)}}{2(n+1)}$.

Proof. The gear graph has $2n+1$ vertices and $3n$ edges, among them n vertices are of degree two, n vertices of degree three and one vertex of degree n. Therefore we get the edge partition, based on the degrees of vertices as shown in Tables 9 and 10. Using formulas (1)–(7) to the information in Tables 9 and 10, we obtain the desired result. □

Theorem 4.2. *Let $H = L(G_n)$ be the line graph of gear graph. Then*

(1) $M_\alpha(H) = 2n \cdot 3^\alpha + n(n+1)^\alpha$;

(2) $R_\alpha(H) = 2n \cdot 9^\alpha + \binom{n}{2}(n+1)^{2\alpha} + 2n[3(n+1)]^\alpha$;

(3) $\chi_\alpha(H) = 2n \cdot 6^\alpha + \binom{n}{2}[2(n+1)]^\alpha + 2n(n+4)^\alpha$;

(4) $ABC(H) = \binom{n}{2}\frac{\sqrt{2n}}{n+1} + 2n\sqrt{\frac{n+2}{3(n+1)}} + \frac{4n}{3}$;

(5) $GA(H) = \frac{n(n+3)}{2} + \frac{4n\sqrt{3(n+1)}}{n+4}$;

(6) $ABC_4(H) = \binom{n}{2}\frac{\sqrt{2n^2+8}}{n^2+5} + 2n\left[\sqrt{\frac{n^2+n+10}{(n^2+5)(n+7)}} + \frac{\sqrt{2(n+6)}}{n+7}\right]$;

(7) $GA_5(H) = \frac{n(n+3)}{2} + \frac{4n\sqrt{(n^2+5)(n+7)}}{n^2+n+12}$.

Proof. The line graph of gear graph has $3n$ vertices and $\frac{n(n+7)}{2}$ edges, among them $2n$ vertices are of degree three and n vertices of degree $n+1$. Therefore we get the edge partition, based on the degrees of vertices as shown in Tables 11 and 12. Using formulae (1)–(7) to the information in Table 11 and 12, we obtain the desired result. □

Theorem 4.3. *Let $[S(G)]^\dagger$ be the derived graph of subdivision graph of gear graph $G = G_n$. Then*

(1) $M_\alpha([S(G)]^\dagger) = n[2^\alpha + 3^{\alpha+1} + (n+1)^\alpha] + n^\alpha$;

(2) $R_\alpha([S(G)]^\dagger) = 2n(6^\alpha + 9^\alpha) + n\cdot(3n)^\alpha + 2n[3(n+1)]^\alpha + \binom{n}{2}(n+1)^{2\alpha}$;

(3) $\chi_\alpha([S(G)]^\dagger) = 2n[5^\alpha + 6^\alpha + (n+4)^\alpha] + n(n+3)^\alpha + \binom{n}{2}[2(n+1)]^\alpha$;

(4) $ABC([S(G)]^\dagger) = n\left[\sqrt{2} + \frac{4}{3} + \sqrt{\frac{n+1}{3n}} + 2\sqrt{\frac{n+2}{3(n+1)}}\right] + \binom{n}{2}\frac{\sqrt{2n}}{n+1}$;

(5) $GA([S(G)]^\dagger) = 2n\left[\frac{2\sqrt{6}}{5} + \frac{\sqrt{3n}}{n+3}\right] + n\left[\frac{(n+3)}{2} + \frac{4\sqrt{3(n+1)}}{n+4}\right]$;

(6) $ABC_4([S(G)]^\dagger) = 2n\left[\sqrt{\frac{n+8}{6(n+4)}} + \sqrt{\frac{2n+1}{6n(n+4)}} + \sqrt{\frac{n^2+n+10}{(n^2+5)(n+7)}} + \frac{\sqrt{2(n+6)}}{n+7}\right] + \binom{n}{2}\frac{\sqrt{2n^2+8}}{n^2+5}$;

(7) $GA_5([S(G)]^\dagger) = n\left[\frac{(n+3)}{2} + \frac{\sqrt{3n(n+4)}}{2(n+1)}\right] + 4n\left[\frac{\sqrt{6(n+4)}}{n+10} + \frac{\sqrt{(n^2+5)(n+7)}}{n^2+n+12}\right]$.

Proof. From Theorem 1.1, we have $[S(G)]^\dagger \cong L(G) \cup G$. Then by using the information in Theorems 4.1 and 4.2, we obtain the required result. □

5. Friendship graph

Let $C_t^{(n)}$ denote the one-point union of n cycles of length t. The graph $C_3^{(n)}$ is called a *friendship graph* [12].

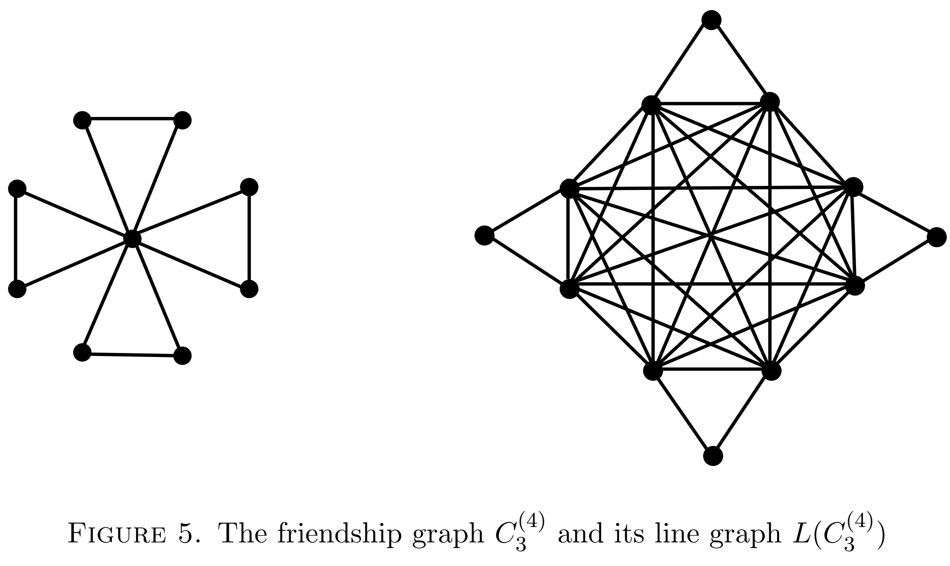

Figure 5. The friendship graph $C_3^{(4)}$ and its line graph $L(C_3^{(4)})$

Table 13. The edge partition of friendship graph

(d_u, d_v) where $uv \in E(C_3^{(n)})$	$(2, 2)$	$(2, 2n)$
Number of edges	n	$2n$

Table 14. The edge partition of friendship graph

(S_u, S_v) where $uv \in E(C_3^{(n)})$	$(2(n+1), 2(n+1))$	$(4n, 2(n+1))$
Number of edges	n	$2n$

Table 15. The edge partition of line graph of friendship graph

(d_u, d_v) where $uv \in E(L(C_3^{(n)}))$	$(2n, 2n)$	$(2n, 2)$
Number of edges	$n(2n-1)$	$2n$

Table 16. The edge partition of line graph of friendship graph

(S_u, S_v) where $uv \in E(L(C_3^{(n)}))$	$(4n, 2[n(2n-1)+1])$	$(2[n(2n-1)+1], 2[n(2n-1)+1])$
Number of edges	$2n$	$n(2n-1)$

Theorem 5.1. *Let $G = C_3^{(n)}$ be the friendship graph. Then*

(1) $M_\alpha(G) = 2^\alpha[2n + n^\alpha]$;

(2) $R_\alpha(G) = 4^\alpha n[1 + 2 \cdot n^\alpha]$;

(3) $\chi_\alpha(G) = n[4^\alpha + 2^{\alpha+1}(n+1)^\alpha]$;

(4) $ABC(G) = \dfrac{3n}{\sqrt{2}}$;

(5) $GA(G) = n\left[1 + \dfrac{4\sqrt{n}}{n+1}\right]$;

(6) $ABC_4(G) = \dfrac{n}{n+1}\sqrt{\dfrac{2n+1}{2}} + n\sqrt{\dfrac{3}{n+1}}$;

(7) $GA_5(G) = n\left[1 + \dfrac{4\sqrt{2n(n+1)}}{3n+1}\right]$.

Proof. The friendship graph has $2n+1$ vertices and $3n$ edges, among them $2n$ vertices are of degree two and one vertex of degree $2n$. Therefore we get the edge partition, based on the degrees of vertices as shown in Tables 13 and 14. Using formulas (1)–(7) to the information in Table 13 and 14, we obtain the desired result. □

Theorem 5.2. *Let $H = L(C_3^{(n)})$ be the line graph of friendship graph. Then*

(1) $M_\alpha(H) = n \cdot 2^\alpha + (2n)^{\alpha+1}$;

(2) $R_\alpha(H) = 4^\alpha n\left[n^{2\alpha}(2n-1) + 2 \cdot n^\alpha\right]$;

(3) $\chi_\alpha(H) = 4^\alpha n^{\alpha+1}(2n-1) + 2^{\alpha+1}n(n+1)^\alpha$;

(4) $ABC(H) = n\sqrt{2} + \dfrac{(2n-1)\sqrt{(2n-1)}}{\sqrt{2}}$;

(5) $GA(H) = n(2n-1) + \dfrac{4n\sqrt{n}}{n+1}$;

(6) $ABC_4(H) = n\left[\sqrt{\dfrac{(2n+1)}{2n^2-n+1}} + \dfrac{(2n-1)\sqrt{8n^2-4n+2}}{4n^2-2n+2}\right]$;

(7) $GA_5(H) = n(2n-1) + \dfrac{4n\sqrt{2n[n(2n-1)+1]}}{n(2n+1)+1}$.

Proof. The line graph of friendship graph has $3n$ vertices and $n(2n+1)$ edges, among them n vertices are of degree two and $2n$ vertices of degree $2n$. Therefore we get the edge partition, based on the degrees of vertices as shown in Tables 15 and 16. Using formulas (1)–(7) to the information in Table 15 and 16, we obtain the desired result. □

Theorem 5.3. *Let $[S(G)]^\dagger$ be the derived graph of subdivision graph of friendship graph $G = C_3^{(n)}$. Then*

(1) $M_\alpha([S(G)]^\dagger) = (1+2n)(2n)^\alpha + 3n \cdot 2^\alpha$;
(2) $R_\alpha([S(G)]^\dagger) = n \cdot 4^\alpha + (4n)^{\alpha+1} + n(2n-1)(2n)^{2\alpha}$;
(3) $\chi_\alpha([S(G)]^\dagger) = n \cdot 4^\alpha + n(2n-1)(4n)^\alpha + 4n[2(n+1)]^\alpha$;
(4) $ABC([S(G)]^\dagger) = \dfrac{5n}{\sqrt{2}} + \dfrac{(2n-1)\sqrt{(2n-1)}}{\sqrt{2}}$;
(5) $GA([S(G)]^\dagger) = 2n^2 + \dfrac{8n\sqrt{n}}{n+1}$;
(6) $ABC_4([S(G)]^\dagger) = n\left[\dfrac{1}{n+1}\sqrt{\dfrac{2n+1}{2}} + \sqrt{\dfrac{3}{n+1}} + \sqrt{\dfrac{(2n+1)}{2n^2-n+1}} + \dfrac{(2n-1)\sqrt{8n^2-4n+2}}{4n^2-2n+2}\right]$;
(7) $GA_5([S(G)]^\dagger) = 2n^2 + \dfrac{4\sqrt{2n(n+1)}}{3n+1} + \dfrac{4n\sqrt{2n[n(2n-1)+1]}}{n(2n+1)+1}$.

Proof. From Theorem 1.1, we have $[S(G)]^\dagger \cong L(G) \cup G$. Then by using the information in Theorems 5.1 and 5.2, we obtain the required result. □

6. Conclusion

In this paper, we have computed expression for some topological indices of derived graph of subdivision graph of a tadpole graph, a crown graph, a gear graph and a friendship graph.

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