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EFFICIENT SENSING OF GROUND-BORNE VIBRATIONS INDUCED BY PILE DRIVING USING COMPRESSIVE SAMPLING

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Abstract. In this paper, we propose and discuss the applicability of the compressive sensing (CS) for the measurement and analysis of the ground-borne vibrations induced by pile driving. With this motivation, we consider two types of the most common pile driving techniques; i) impact pile driving and ii) vibratory pile driving. We show that ground-borne vibration parameters like longitudinal velocity (LV), transverse velocity (TV), vertical velocity (VV), force and inertia can be efficiently sensed by the CS for both of the two driving types mentioned above, since majority of these parameter's time series are sparse either in time or frequency domain. We show that time series of these parameters can be reconstructed using a far fewer number of samples than the classical Shannon's theorem states. Thus, when the CS technique is utilized, measurement times and hardware storage requirements of vibration sensing systems can be significantly reduced. Additionally, when the CS is used, such parameters can be exactly reconstructed even in the presence of missing data. CS based proposed in this paper can also be extended to measure and analyze other types of vibration data in soil dynamics and more generally to other vibration types encountered in engineering.

Keywords: Ground-borne vibrations, pile driving, compressive sensing.

AMS Subject Classification: 86-10, 86-11, 65K10

1. Introduction

Ground-borne vibrations are generated by different mechanisms such as the vehicles and railway trains. One other primary source of ground-borne vibrations is pile driving, which is a very complex process involving many parameters. The pile driver causes the pile to vibrate and this vibration is then transferred to the soil. Then ground-borne vibration may propagate in different layers of soils with different amplification and attenuation characteristics. Inevitably, ground-borne vibrations interact with existing ground and underground structures in the vicinity of the pile driving site. These vibrations affect nearby structures and may cause problems like discomfort of people, damaging of the

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structure itself or its foundation by excessive and/or differential settlement, just to name a few. However under certain circumstances, such as having very poor soil conditions at the site, construction of piles becomes inevitable, so that their installation can not be avoided.

The cost of damages and delays in the constructions due to geotechnical failures can lead to few billions of dollars in different countries [10]. Although exact amount is unknown, it is obvious that some of them caused by under- or overestimation of the pile driving effects. Therefore, measurement and analysis of the pile driving parameters is a must. These measurements and analysis need to be done using accurate, fast and cheap tools and methods. Compressive sensing (CS), has emerged has a key signal processing algorithm which can be used with these motivations [7], [8]. Some recent applications in civil structural health monitoring [1], [17], [18] and ocean engineering [13] can also be seen in the literature. The primary advantage of the CS based techniques proposed in these papers is their data and storage reduction capability. However, when hardware implementation of the CS is needed, the development of sensors working with this principle need to be developed. Although this brings some extra complications, it is obvious that with its great computational advantages, the CS has revolutionized the field of signal processing. With this motivation, in this study we propose the usage and discuss the advantages of the CS technique for measurement of the ground-borne vibrations induced by pile driving. With this motivation, we consider two types of common pile driving techniques; i) impact pile driving and ii) vibratory pile driving. We show that parameters like longitudinal (LV), transverse (TV) and vertical vibration velocities (VV), force and inertia used and generated due to resistance during driving process can be efficiently sensed by the CS since they are sparse signals in either time or Fourier domain. We show that most of the pile driving parameters, if not all, can be measured with great efficiency using the CS both for the impact and the vibratory pile driving. This technique can also be effectively used to measure and analyze associated parameters, not only during pile driving but also throughout their service life. When piles are subjected to impact and harmonic loadings, i.e. the offshore piles subjected to wave, wind and earthquake actions, the CS based technique proposed in this paper emerges as a useful method.

2. METHODOLOGY

2.1. **Review of the Compressive Sampling.**

Although introduced to the signal processing community less than two decades ago, compressive sensing (CS) has revolutionized the field of signal and image processing. Today it is a very widely used tool in many branches of engineering and applied mathematics [7], [8] . Introduction of the CS algorithm has paved the way to the development of the faster and accurate sensing devices such as single pixel cameras, analog-to-digital converters and CS based tomography devices, just to name a few. In this section we give a brief review of the compressive sampling (CS) algorithm described in [8].

Lets consider a signal, η, and let it be a K-sparse signal with N elements. This means that K entries out of N are nonzero. Using an orthogonal transformation matrix $\mathbf{\Psi}$, η can be represented in terms of orthogonal basis functions. Typically used ones are the Fourier, wavelet or discrete cosine transforms. Therefore it is possible to represent the signal with $\eta = \mathbf{\Psi}\widehat{\eta}$. In here $\widehat{\eta}$ is a vector of coefficients. Removing the zero entries of η, we may obtain $\eta_s = \mathbf{\Psi}\widehat{\eta}_s$. In here η_s includes non-zero components only.

CS algorithm can reconstruct any K-sparse signal of length N using $M \geq C\mu^2(\mathbf{\Phi},\mathbf{\Psi})K \log \text{(N)}$ random measurements with a very high probability [8]. In here C is a positive constant.

$\mu^2(\Phi, \Psi)$ shows the mutual coherence between the sensing basis $\mathbf{\Phi}$ and transformed basis $\mathbf{\Psi}$ [8].

By taking M random projections and with the use of the sensing matrix $\mathbf{\Phi}$ we can reconstruct $g = \mathbf{\Phi}\eta$. Therefore the reconstruction problem can be written as

$$\min \|\widehat{\eta}\|_{l_1} \quad \text{under constraint} \quad g = \mathbf{\Phi}\mathbf{\Psi}\widehat{\eta} \tag{1}$$

where $\|\widehat{\eta}\|_{l_1} = \sum_i |\widehat{\eta}_i|$. So that among all possible signals which satisfy the constraints, the l_1 solution of this problem becomes $\eta_{CS} = \mathbf{\Psi}\widehat{\eta}$.

Some sparse signals can also be reconstructed using other optimization techniques such as greedy pursuit or re-weighted l_1 minimization algorithms [8], which is beyond the scope of this paper. A more comprehensive discussion of this revolutionary technique can be seen in [8].

2.2. **Proposed Soil Vibration Sensing Approach.** After a brief assessment of the pile driving data in existing literature, one can immediately recognize that majority of parameters of the impact pile driving are sparse signals. But, for vibratory pile driving data, it is not easy to do so since not their time series but their spectra are sparse, in general. Orthogonal transformations like Fourier or wavelet transforms are commonly used for spectral analysis purposes. Details of spectral techniques, their usage in computational mathematics and some their other uses can be seen in [2], [3], [9], [14]. Time series of vibratory pile driving data generally contains few fundamentals harmonics both in Fourier or wavelet space, therefore they can also be successfully reconstructed by the CS algorithm. In this paper, we propose to use CS in the following fashion; i) for impact pile driving parameters, M random measurements are taken in Fourier space and then l_1 minimization formulated above is applied for the reconstruction of the signal with $N > M$ components exactly; and ii) for vibratory pile driving data, M random measurements are taken in time domain and their sparse spectra can be reconstructed exactly using the l_1 minimization. We restrict ourselves to Fourier space in this work, but when some information about the phase of spectral changes are needed the method proposed here to wavelet space easily. We present the results of our analysis in the next part.

3. Results and Discussion

3.1. **Impact Pile Driving.** In order to show the applicability and usefulness of the proposed method, we first analyze the impact pile driving data. The piles are needed to improve the foundation support of soils of an ongoing tramway project in İstanbul, Turkey. The districts of Eminönü and Alibeyköy is planned to be interconnected via a tramway constructed at the ground surface level. When the construction is completed, the tramway line will be approximately $10km$ in length and will consist of 14 stations. Some stations and parts of the tramway line will be constructed over water at Haliç estuary, thus, pile foundations are needed. Soil conditions are very poor and vertical soil spring stiffness values of the clay, mud and fine grained sands at the site are as low as $k = 4125kN/m^2$ in some sections. According to stiffness based soil classification system, the foundation subsoil at the site can be classified as very soft soil [6]. Additionally, there are many historical buildings and a few schools and hospitals, which some are built by the Architect Sinan [12], in the vicinity of the proposed tramway line. Therefore monitoring and analyzing vibrations are of critical importance for the safety and serviceability of these buildings. Photos of the project site are shown in Fig. 1.

Some of the piles at the site are already installed and others are planned to be driven into the ground in a 3 row layout. The driven steel piles are $60m$ in length and $914mm$ in diameter with a thickness of $11.9mm$. These piles are driven into the soil with an

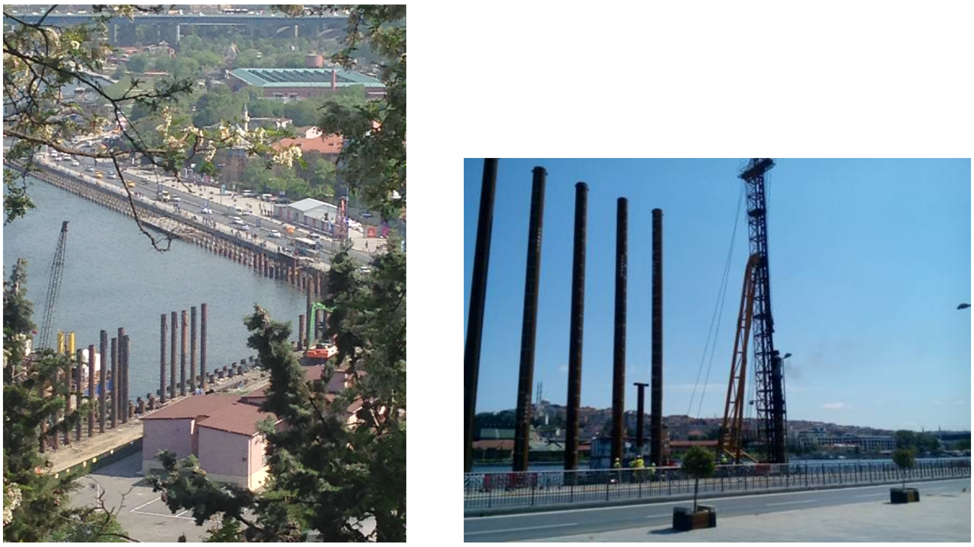

Figure 1. View of the pile driving site in Eyüp-İstanbul a) Layout view from Pierre Loti hill b) Piles being driven by hammer

approximately 9 tons hammer. First $15m$ of driving is easily done by the dead weight of the pile, for the $15m-50m$ section moderate effort is enough, but for the last $50m-60m$ section higher driving effort is needed. The impact data are measured at a spot $77m$ away from the nearest pile that is driven into the ground, where a historic soup kitchen is located. The impact data is measured using SYSCOM-MR3000TR vibration measurement system at a high rate of 100 Hz and the peak value over a set time range of $5s$ is recorded for approximately 2 hours. Time series data of the vertical ground-borne vibration velocities are depicted in Fig. 2-a. Driven pile for this case did not penetrate into stiff soil.

Checking Fig. 2 one can realize that vertical vibration velocity is more critical than longitudinal and transverse velocities with a peak about $1.1m/s$, and in some cases depending on the distance of the measurement point from the source and due to soil amplification effect, this value would certainly change. In this paper we are not concentrating on the magnitude of such vibration nor their effect on building. Therefore we turn our attention to efficiently sensing and analyzing of time series of the pile driving parameters.

All three time series in Fig. 2 can be treated as sparse signals in time domain since majority of the entries are zero. In the case of small ambient environmental noise, a threshold can be applied to filter it. Treating these vibration time series as sparse signals in time domain, one can think the CS algorithm as an efficient tool to sense and analyze them. Since they are sparse in time domain, $M = 64$ random measurements are taken in frequency domain and the original signal with $N = 1024$ elements are reconstructed exactly using l_1 minimization of the CS, which are depicted in Fig. 2-b. If the instantaneous measurements in each second are taken rather than the peaks in $5s$ intervals, then the sparsity of the time series would be greater, that is more entries in the time series would be zero. In that case, CS could be used to reconstruct the signal even with a better efficiency, that is with a higher undersampling ratio. Depending on the number of entries of the vibration velocity time series (N) and its sparsity parameter (K), then number of random observations (M) can change significantly. Also it is possible to find a sparse representation of time series of pile driving data in different known or unknown orthogonal domains.

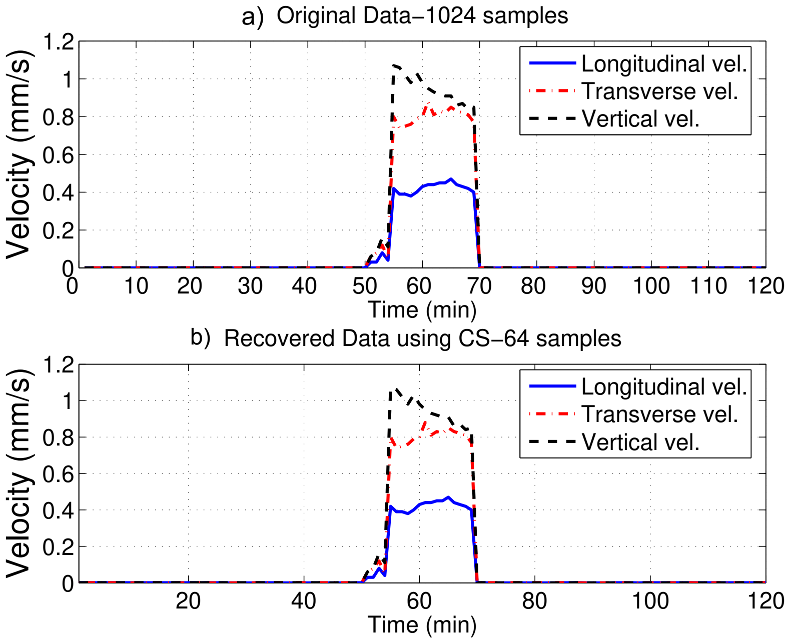

FIGURE 2. Reconstruction of the first time series of ground borne vibration parameters measured during impact pile driving a) using classical sampling b) using compressive sampling.

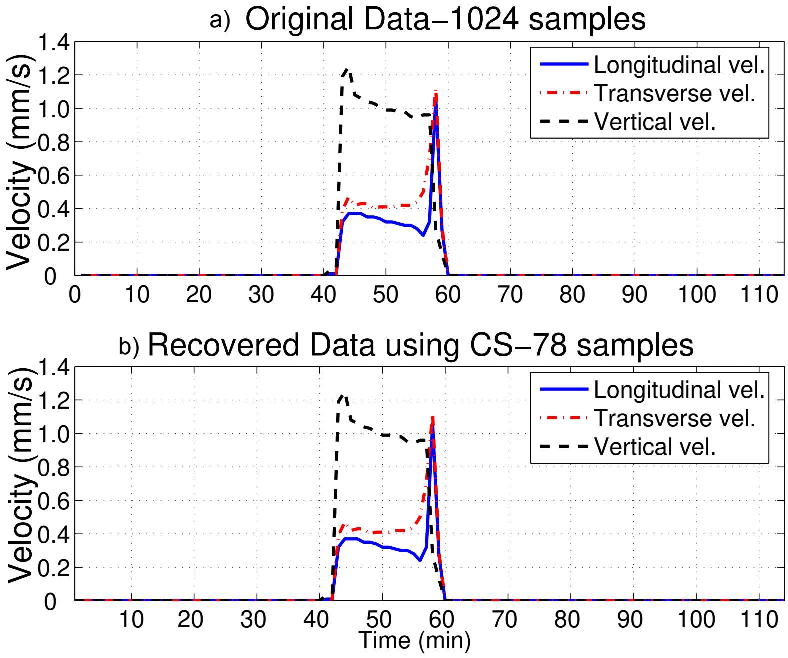

FIGURE 3. Reconstruction of the second time series of ground borne vibration parameters measured during impact pile driving a) using classical sampling b) using compressive sampling.

Fourier, discrete cosine and wavelet domains are known and is suitable transforms to sense majority of the vibration parameter, if not all.

Now we turn our attention to Fig. 3, where second time series of vibration velocities are depicted. Again the data is recorded during an approximately 2 hour process and again the peak values are recorded in $5s$ intervals until the pile has penetrated into stiff soil. Similar to the previous case, all three time series in Fig. 3 can also be treated as sparse signals in time domain, therefore they can be reconstructed by taking random measurements in frequency domain. In this case $M = 78$ random measurements were enough to exactly reconstruct the original time series with $N = 1024$. Again, depending on N and K, the required number of random measurements M can significantly change making the reconstruction easier or harder. The advantage of using the CS is obvious, one needs fewer number samples to reconstruct the vibration parameters' time series which means less memory and less acquisition time compared to the classical sampling. CS would also be used to recover the signals exactly in the case of missing temporal or spectral data, therefore it can be used as a vibration data analysis tools for hindcasting, forecasting and post-processing purposes. In order to illustrate the possible usage of the CS in this regard, we depict Fig. 4.

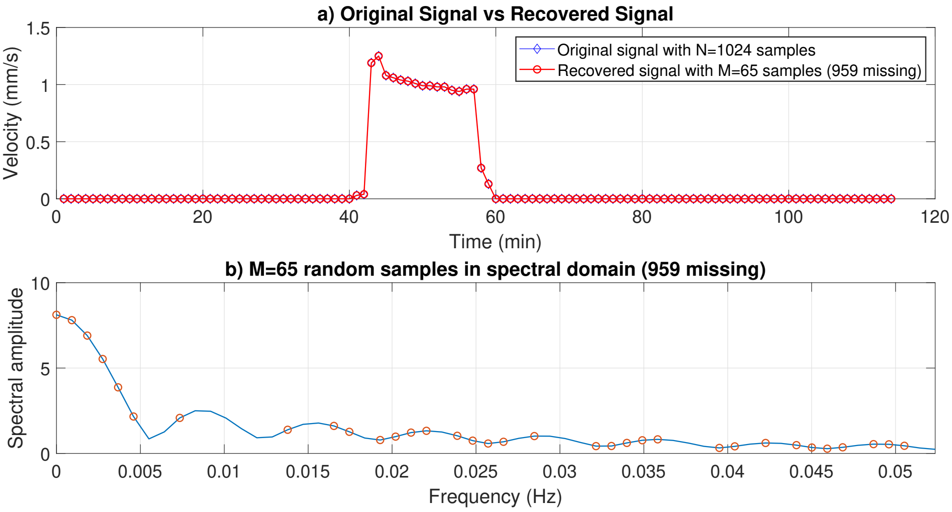

FIGURE 4. Reconstruction of the ground borne vibration velocity time series with missing data a) classical samples (N=1024) and compressive samples (M=65) b) random M=65 samples (959 missing) taken in spectral domain.

In Fig. 4, we use one of the ground-borne vertical vibration time series measured during pile installation. The original time series displayed in the first subplot of Fig. 4 has $N = 1024$ classical samples. The one-sided Fourier spectrum for this time series is displayed in the second subplot of this figure. Since the original time series is sparse in the temporal domain, we take random samples in the spectral domain and indicate them in the second subplot as circular sample points. In order to simulate a situation in which the majority of the spectral data is missing, we select the number of compressive samples as $M = 65$. This means that 1024-65=959 of the spectral data are missing which gives a considerable undersampling ration. Then, using these compressive samples and utilizing the CS algorithm, it is possible to reconstruct the original time series with the majority of the spectral data are missing as the comparison in the first subplot of Fig. 4 confirms.

3.2. **Vibratory Pile Driving.** Our next attention would be the vibratory pile driving, which is another type of pile driving technique. Vibratory pile driving is performed using vibration process, that is the piles are driven into the ground using by a vibratory hammer. Currently, we do not have vibratory pile driving data and in order to show the advantages of the CS for vibratory pile driving analysis we reproduced two sets of measurements. The first set of measurements reproduced are reported in [15]. This vibratory pile driving data is recorded during installation of the first $20+m$ section of $60+m$ long bearing piles for I-90 Innerbelt Bridge in Cleveland, OH, USA [15]. The soil types at the site are loose sands and then hard clays. An ICE Model $66-80$ vibratory hammer with $26.7Hz$ rated frequency and $749Nm$ eccentric moment is used in the installation. A comprehensive discussion of this setup can be seen in [15].

The data given in [15] has been reproduced and depicted in Fig. 5-a. The parameters plotted here are the time series of vibration velocity and force, both of which show a harmonic behavior including few Fourier modes. Therefore one can treat these time series as sparse signals in Fourier space. Taking $M = 92$ random samples in time domain and applying the l_1 reconstruction of the CS, one can recover these time series with $N = 1024$ components exactly, as depicted in Fig. 5-b. If the vibratory pile driving data consists of fewer number of harmonics or if it is a perfect sinusoid, then the reconstruction with smaller number of samples would be sufficient. This method can also be applied to recover other vibratory pile driving data such as accelerations and inertia, that is mass times acceleration of the pile. Additionally CS can be used to fore- or hindcast the data including post-processing phase, i.e. even if the majority of the data is missing in time domain, CS would still exactly recover the time series.

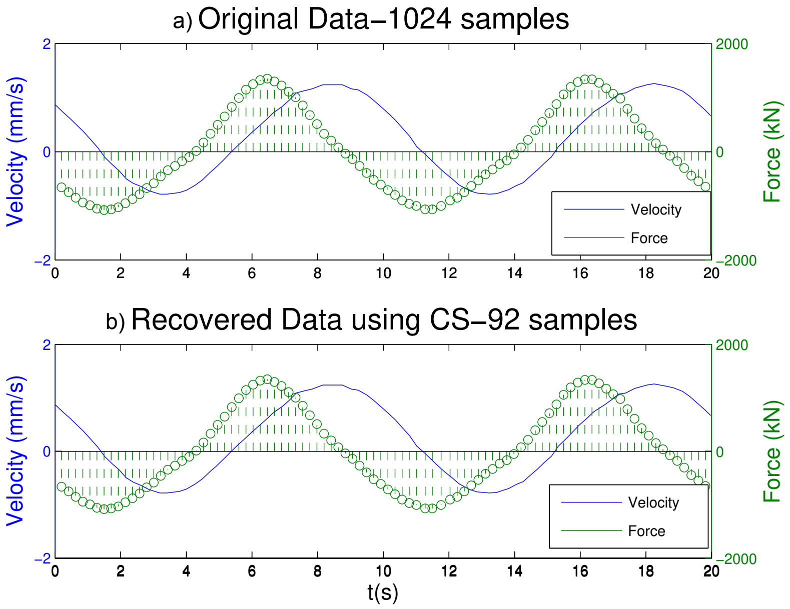

FIGURE 5. A time series of ground borne vibration parameters measured during vibratory pile driving. a) Original data b) Same data recovered using compressive sampling.

Next, we turn our attention to another set of measurements taken during vibratory sheet pile installation reported in [10], [11]. During the installation reported in [10], the $12m$ long sheet piles are driven for a depth of about $11m$ into the ground [10]. The installation is performed into sand and silt sandy soil using a vibrator model of Dieseko $2316VM$ using a frequency of $28Hz$ [10]. The ground-borne parameters of vertical velocity (VV), longitudinal velocity (LV) and transverse velocity (TV) are recorded using geophones [10]. Details of the test setup which is used to measure the time series depicted in Fig. 6-a can be seen in [10].

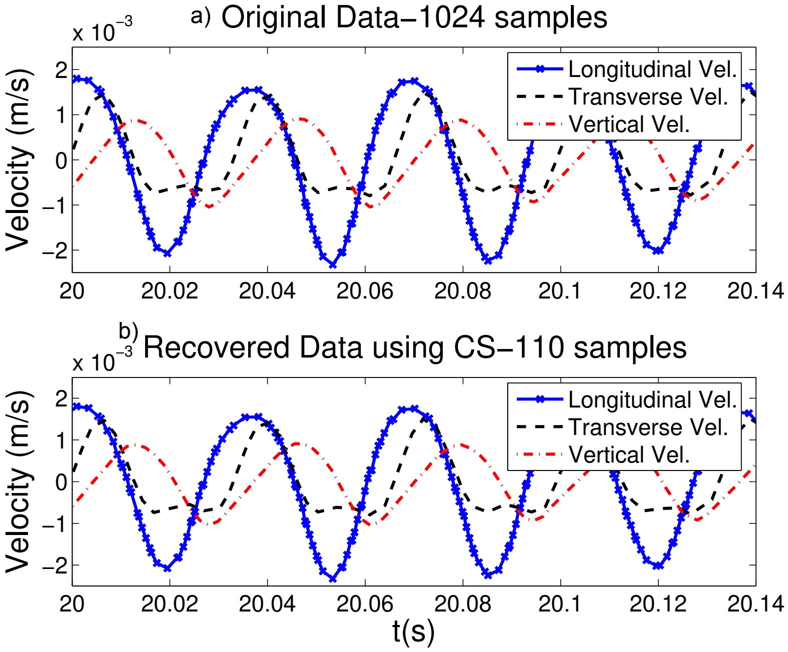

Figure 6. Another time series of ground borne vibration parameters measured during vibratory pile driving. a) Original data b) Same data recovered using compressive sampling.

Checking the VV, LV and VV time series depicted in Fig. 6-a, it can be conceived that CS is an sensing algorithm since they show sinusoidal behaviors including few harmonics only. Therefore by using Fast Fourier transform in order to get their representation in Fourier space, one can come up with a sparse spectra. Therefore by taking some random measurements in time domain and again using the l_1 minimization, these time series can be reconstructed exactly. An example reconstruction is done using $M = 110$ random temporal measurements, and the recovered exact time series are plotted in Fig. 6-b. As mentioned above, the CS would also be very beneficial in the case of missing temporal or spectral data and could be used to perform exact reconstruction of the VV, LV and TV time series, as well. Depending on the sparsity of the spectra of these vibration parameters, exact reconstruction may be possible using even smaller number of measurements. Additionally, CS can be used to measure and analyze similar ground-borne vibrations generated by vehicles and trains, such as the ones discussed in [4], [5], [16].

4. Conclusion

In this paper, we showed the advantages of using the compressive sampling technique to measure and analyze ground-borne vibrations generated by pile driving. We showed that the vibration velocities and forces in all directions, can be efficiently measured by using far fewer number of measurements compared to the classical Shannon's theorem states when compressive sampling is utilized, both for the impact and vibratory pile driving. This technique can also be efficiently used to measure and analyze parameters including but are not limited to accelerations, inertia, and settlements. It is possible to utilize the CS based measurement technique not only during pile driving process but also during their entire service lifetime. When piles are subjected to harmonic or impact loadings, i.e. the offshore piles subjected to earthquake and harmonic waves or structures transversely oscillating due to Von Karman vortices etc., the CS would be very efficient for the measurement, analysis and completion of pile vibration parameters. Using compressive sensing, measurement times and hardware storage requirements of the sensing systems can be dramatically reduced. Additionally, compressive sensing can be used to sense such parameters exactly in the presence of missing data, thus it enables forecasting and hindcasting of pile driving data. Of course, in general, it is possible to extend this advantageous technique to sense other types of ground-borne vibrations in soil dynamics as well as to other types of mechanical and structural vibrations.

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Cihan Bayındır for the photography and short autobiography, see TWMS J. App. and Eng. Math. V.5, N.2.

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