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# A VIEW ON FUZZY PEANO SPACE AND ITS APPLICATION ON RAIL CONNECTIVITY NETWORK

K. SUGAPRIYA[1], B. AMUDHAMBIGAI[1], §

Abstract. By using fuzzy path connectedness of a fuzzy topological space (X, T), a wide class of a space named as the fuzzy Peano space has been defined in this paper. Some of the interesting properties of a Fuzzy Peano Space are discussed. As an application of the Fuzzy path connectedness, it is aimed to find the perfect shortest path to cover all the selected cities at the earliest time. In this connection the properties of Fuzzy path connectedness are used to determine the fuzzy Hamiltonian Cycle which is used in rail connectivity network to find the perfect shortest path.

Keywords: Fuzzy locally path connected spaces, universal fuzzy locally path connected spaces, fuzzy Peano spaces and rail connectivity network.

AMS Subject Classification: 54A40, 03E72.

## 1. Introduction

Zadeh [8] presented the idea of a Fuzzy set in 1965. In 1968, Chang [4] considered the idea of the fuzzy topological space. Fisher and Zastrow first characterized the idea of Generalized regular covering functions. Dydak, Brendon LaBuz and Atish Mitra[3] proves that for a topological space (X, T), we can form a new topology which is generated by all path components of (X, T). Fuzzy sets are commonly used in fuzzy graph theory, that is the fuzzy set can be visualized. The concept of fuzzy path plays an important role in fuzzy graphs. Fuzzy connectivity concepts like fuzzy edges, fuzzy vertex, fuzzy cut nodes and fuzzy bridges were established by Bhattacharya[2]. The concept of Hamiltonian cycles was introduced by Klas Markstrom[5]. By using fuzzy path connectedness of a fuzzy topological space (X, T), a wide class of a space named as the fuzzy Peano space has been defined in this paper. Some of the interesting properties of a Fuzzy Peano Space are discussed. As an application of the Fuzzy path connectedness, it is aimed to find the perfect shortest path to cover all the selected cities at the earliest time. In this connection the properties of Fuzzy path connectedness are used to determine the fuzzy Hamiltonian Cycle which is used in rail connectivity network to find the perfect shortest path.

[1] Department of Mathematics, Sri Sarada College for Women(Autonomous), Salem, Tamilnadu, India.
e-mail: nirmalapriya97@gmail.com; ORCID: https://orcid.org/0000-0001-6274-0432.
e-mail: rbamudha@yahoo.co.in; ORCID: https://orcid.org/0000-0003-3836-7660.
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## 2. Preliminaries

In this Section some basic definitions are discussed.

**Definition 2.1** (8)**.** Let $X$ be a non-empty set and $I$ be the unit interval [0, 1]. A fuzzy set in $X$ is an element of the set $I^X$ of all function from $X$ to $I$.

**Definition 2.2** (4)**.** A fuzzy topology is a family $\tau$ of fuzzy sets in $X$ which satisfies the following conditions :
(i) $0_X$, $1_X \in \tau$,
(ii) If $\lambda$, $\mu \in \tau$ then $\lambda \wedge \mu \in \tau$,
(iii) If $\lambda_i \in \tau$ for each $i \in J$ then $\vee\lambda_i \in \tau$. $\tau$ is called a fuzzy topology for $X$, and the pair (X, T) is a fuzzy topological space, or fts for short.

**Definition 2.3** (6)**.** A fuzzy set in X is called a fuzzy point iff it takes the value 0 for all y $\in$ X except one, say, x $\in$ X. If its value at x is $\lambda(0\leq \lambda \leq 1)$ we denote this fuzzy point by $x_\lambda$, where the point x is called its support.

**Definition 2.4** (6)**.** A fuzzy point P in X is a special fuzzy set with membership function defned by

$$P(x) = \begin{cases} \lambda & if x = y \\ 0 & if x \neq y \end{cases}$$

where $0 < \lambda \leq 1$. P is said to have support y, value $\lambda$ and is denoted by $P_y^\lambda$ or P(y, $\lambda$).

**Definition 2.5** (1)**.** Let (X, T) be a fuzzy topological space. A fuzzy topological space (X, T) is said to be fuzzy connected if it has no proper fuzzy clopen set. (A fuzzy set $\lambda$ in X is proper if $\lambda \neq 0$ and $\lambda \neq 1$).

**Definition 2.6** (1)**.** Let (X, T) be a fuzzy topological space. A fuzzy connected component of (X, T) is a maximal fuzzy connected subset on (X, T). That is, a fuzzy connected subset which is not contained in any other(strictly) large fuzzy connected subset of (X, T).

**Definition 2.7** (1)**.** Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function f : (X, T)$\rightarrow$ (Y, S) is said to be fuzzy continuous if the inverse image of every fuzzy open set in $I^Y$ is fuzzy open in $I^X$.

**Definition 2.8** (1)**.** Let (X, T) be a fuzzy topological space. A continuous function f : I $\rightarrow$ X is said to be a fuzzy path in (X, T) if for any two fuzzy points p, q $\in$ X such that f(0) = p, f(1) = q.

**Definition 2.9** (1)**.** Any fuzzy topological space (X, T) is said to be a fuzzy path connected space iff for each pair of fuzzy point p, q $\in$ X there exists a fuzzy path f in X such that f(0) = p, f(1) = q.

**Definition 2.10** (1)**.** A fuzzy path component of a fuzzy point $x_1$ in a fuzzy topological space (X, T) is the maximal fuzzy path connected set in (X, T) that contains $x_1$ and denoted by $\mathcal{C}$.

**Definition 2.11** (1)**.** Any topological space (X, T) is said to be locally path connected at x if for every open set V containing x there exists a path connected open set U with x $\in$ U $\subset$ V. The space X is said to be locally path connected if it is locally path connected at x for all x in (X, T).

**Definition 2.12** (3)**.** A topological space X is an lpc-space if it is locally path-connected. X is a Peano space if it is locally path-connected and connected.

**Definition 2.13** (3)**.** Let $\tilde{X}$ and X be fuzzy topological spaces and p $\tilde{X}$: $\rightarrow$ X be a fuzzy continuous mapping. A fuzzy set U $\subset$ X is said to be evenly fuzzy covered by p if U is fuzzy connected and open fuzzy set, and each fuzzy component of $p^{-1}$(U) is an open fuzzy set that is mapped fuzzy homeomorphically onto U by p.

**Definition 2.14** (3)**.** A fuzzy covering mapping is a fuzzy continuous surjective mapping p $\tilde{X}$: $\rightarrow$ X such that $\tilde{X}$ is fuzzy path connected and locally fuzzy path connected, and every fuzzy point $a_\lambda \in$ X has an evenly fuzzy covered neighborhood.
If p $\tilde{X}$: $\rightarrow$ X is a fuzzy covering mapping, we call $\tilde{X}$ a fuzzy covering space of X.

**Definition 2.15** (1)**.** Let $\tilde{X}$, X be fuzzy topological spaces, p: $\tilde{X} \rightarrow$ X be a fuzzy covering mapping and $\phi$ : B $\rightarrow$ X be any fuzzy continuous mapping. If the mapping $\tilde{\phi}$: B $\rightarrow \tilde{X}$ is fuzzy continuous such that p $\circ\ \tilde{\phi} = \phi$, then $\tilde{\phi}$ is called a fuzzy lifting of $\phi$.

**Definition 2.16** (1)**.** Given a topological space X its universal lpc-space lpc(X) is an lpc-space together with a continuous map (called the universal Peano map) $\pi$ : lpc(X)$\rightarrow$ X satisfying the following universality condition: For any map f : Y $\rightarrow$ X from an lpc-space Y there is a unique continuous lift g : Y $\rightarrow$ lpc(X) of f (i.e., $\pi\circ$g = f).

**Definition 2.17** (2)**.** A fuzzy graph is a pair of functions $\sigma$ : V $\rightarrow$ [0, 1] and $\mu$ : V $\times$ V $\rightarrow$ [0, 1], where for all u, v in V we have $\mu$(u, v) $\leq \sigma$(u)$\wedge$ $\sigma$(v).

**Definition 2.18** (2)**.** A fuzzy Hamiltonian path is a path that passes through each of the vertices in a fuzzy graph exactly once.

**Definition 2.19** (2)**.** A fuzzy Hamiltonian cycle is a cycle that visits every vertex in a fuzzy graph once with no repeats and being a fuzzy Hamiltonian cycle it must start and end at the same vertex.

**Algorithm [7]**
Step 1: Form the adjacency matrix A(G).
Step 2: Search for a minimum non-zero entry in A(G), say $a_{ij}$. (If there are repetitions, then choose any one).
Step 3: If the minimum value chosen does not permit for a fuzzy Hamiltonian path, then the next higher minimum value is selected.
Step 4: Identify the row and column, say $u_i$ and $u_j$ respectively where the minimum entry appears.
Step 5: Search for a minimum non-zero entry in the row of $u_j$ , such that,
(i) it forms a fuzzy Hamiltonian path. (no repetition of vertices in the path)
(ii) if the minimum value occurs more than once, then an appropriate entry is selected to get a fuzzy Hamiltonian path.
Step 6: Repeat Step 3 through Step 4 row-wise until a fuzzy Hamiltonian path is found with all n vertices of G. Else conclude, there is no fuzzy Hamiltonian path and go to Step 2 or Step 3 as required. If fuzzy Hamiltonian path is true, then at this stage only one row will be left out with no entries marked.
Step 7: Select a non-zero entry from that row to form a fuzzy Hamiltonian cycle, if exists.

## 3. Fuzzy Peano Space

**Definition 3.1.** Let (X, T) be a fuzzy path connected space and $\lambda \in I^X$ be a fuzzy open set in (X, T). Then the fuzzy points $x_t$, $y_t \in \mathcal{FP}$(X) is said to be a fuzzy path connected points if there exists a fuzzy path p : I $\rightarrow \mathcal{FP}$(X) in (X, T) such that p(0) = $x_t$ and p(1) = $y_t$ and $x_t$, $y_t \leq \lambda$.

**Notation 3.1.** The collection of all fuzzy path connected points in (X, T) is denoted by $\mathcal{FPCP}$(X).

**Theorem 3.2.** Let (X, T) be a fuzzy topological space and $x_t$, $y_t$, $z_t \in \mathcal{FPCP}$(X) and suppose that $p_1$ : [0, 1] $\rightarrow \mathcal{FPCP}$(X) is a fuzzy path from $x_t$ to $y_t$ and $p_2$ : [0, 1] $\rightarrow \mathcal{FPCP}$(X) is a fuzzy path from $y_t$ to $z_t$. Then there exists a fuzzy path $p_3$ : [0, 1] $\rightarrow \mathcal{FPCP}$(X) from $x_t$ to $z_t$.

*Proof.* Let $x_t$, $y_t$, $z_t \in \mathcal{FPCP}$(X) and let $p_1$ : [0, 1] $\rightarrow \mathcal{FPCP}$(X) be a fuzzy path from $x_t$ to $y_t$ such that $p_1(0) = x_t$, $p_1(1) = y_t$ and $p_2$ : [0, 1] $\rightarrow \mathcal{FPCP}$(X) be a fuzzy path from $y_t$ to $z_t$ such that $p_2(0) = y_t$ and $p_2(1) = z_t$. Now define $p_3$ : [0, 1] $\rightarrow \mathcal{FPCP}$(X) by

$$p_3(t) = \left\{ \begin{array}{cc} p_1(2t) & 0 \leq t \leq \frac{1}{2} \\ p_2(2t-1) & \frac{1}{2} \leq t \leq 1 \end{array} \right\}$$

Then $p_3(0) = p_1(0) = x_t$ and $p_3(1) = p_2(1) = z_t$. Hence $p_3$ is a fuzzy path from $x_t$ to $z_t$. □

**Definition 3.3.** Let (X, T) be a fuzzy path connected space and $\lambda \in I^X$ be any fuzzy open set. If for any two fuzzy points $x_t$, $y_t \in \mathcal{FP}$(X) and $x_t$, $y_t \leq \lambda$ there exists a fuzzy path p : [0, 1] $\rightarrow \mathcal{FP}$(X) such that p(0) = $x_t$, p(1) = $y_t$. Then $\lambda$ is said to be a fuzzy path connected open set in (X, T).

**Notation 3.2.** *The collection of all fuzzy path connected open set in (X, T) is denoted by FPCO(X) and fuzzy path connected closed set in (X, T) is denoted by FPCC(X).*

**Definition 3.4.** Let (X, T) be a fuzzy path connected space. Then for any $\lambda \in I^X$, the fuzzy path connected interior of $\lambda$ is denoted and defined as $FPC_{int}(\lambda) = \vee$ { $\mu \in I^X$ : $\mu \leq \lambda$ and $\mu$ is fuzzy path connected open }.

**Definition 3.5.** Let (X, T) be a fuzzy path connected space. Then for any $\lambda \in I^X$, the fuzzy path connected closure of $\lambda$ is denoted and defined as $FPC_{cl}(\lambda) = \cup$ { $\mu \in I^X$ : $\lambda \leq \mu$ and $\mu$ is fuzzy path connected closed }.

**Definition 3.6.** Any fuzzy path connected space (X, T) is said to be a fuzzy locally path connected space (briefly, flpc-space) if for any fuzzy path connected point $x_t \in \mathcal{FPCP}$(X) and for any fuzzy open set $\lambda \in I^X$ with $x_t \leq \lambda$, there exists a fuzzy path connected open set $\mu \in I^X$ such that $x_t \leq \mu \leq \lambda$.
In otherwords, A fuzzy path connected space (X, T) is fuzzy locally path connected at a fuzzy path connected point $x_t \in \mathcal{FPCP}$(X), if for every fuzzy neighborhood $\lambda \in I^X$, there exists a fuzzy path connected neighborhood $\mu$ of $x_t$ such that $x_t \leq \mu \leq \lambda$.

**Definition 3.7.** Let (X, T) be a fuzzy locally path connected space. Then the fuzzy path component of a fuzzy point $x_t \in \mathcal{FP}$(X) is defined as $\mathcal{FPC}$(X) $= \vee_{i=1}^{n} \lambda_i$, where $\lambda \in$ FPCO(X), $x_t \leq \lambda$ and $\lambda \neq 1_X$.

**Theorem 3.8.** Let (X, T) be a fuzzy locally path connected space and let $\lambda \in I^X$ be a fuzzy open set and $\mu \in I^X$ be a fuzzy path connected open set. If $\gamma \in I^X$ is a fuzzy path component of $x_t$ and $\mu \leq \lambda$ then $\mu \leq \gamma$.

**Definition 3.9.** A fuzzy topological space (X, T) is said to be a fuzzy Peano space if it is a flpc space and a fuzzy connected space.

**Theorem 3.10.** Every fuzzy Peano space is a fuzzy Connected space.
*Proof.* Let (X, T) be a fuzzy Peano space. Then from Definition 3.2, it is clear that (X, T) is a fuzzy connected space . □

**Theorem 3.11.** Any fuzzy locally path connected space (X,T) is a fuzzy peano space if and only if for every fuzzy path connected open set $\lambda \in I^X$ there exists a fuzzy point $x_t$ $\in \mathcal{FPCP}$ (X), such that the fuzzy path component of $x_t$ is fuzzy path connected open in (X,T).

*Proof.* Assume that (X,T) is a fuzzy peano space, then by Definition , it is clear that (X,T) is a fuzzy locally path connected space. Let $\lambda \in I^X$ be a fuzzy path component of $x_t$. Since $\lambda$ is a fuzzy neighbourhood of $x_t$, choose a fuzzy path connected neighbourhood $\mu$ of $x_t$ such that $\mu \leq \lambda$. Since $\mu$ is a fuzzy path connected open set, By Proposition 2.0.2, $x_t \leq \mu \leq \lambda$. Therefore $\lambda$ is fuzzy path connected open in (X,T). □

**Definition 3.12.** *Let (X, T) and (Y, S) be any two fuzzy locally path connected space. A function f : (X, T) → (Y, S) is called fuzzy flpc-continuous if the inverse image of every fuzzy path connected open set in $I^Y$ is a fuzzy path connected open set in $I^X$.*

**Theorem 3.13.** Let (X, T) and (Y, S) be any two fuzzy locally path connected spaces. Then the function f: (Y, S) → (X, T) satisfy the following equivalent condition:

1. f is flpc-continuous
2. f(FPC$_{cl}(\lambda)) \leq$ FPC$_{cl}$f($\lambda$), for each $\lambda \in I^X$.
3. FPC$_{cl}(f^{-1}(\mu)) \leq f^{-1}($FPC$_{cl}(\mu))$, for each $\mu \in I^Y$.
4. $f^{-1}($FPC$_{int}(\mu)) \leq$ FPC$_{int}(f^{-1}(\mu))$, for each $\mu \in I^Y$.

*Proof.* (1)$\Longrightarrow$(2)
For any fuzzy set $\lambda \in I^X$, $1_X$-fcl(f($\lambda$)) is fuzzy path connected open. Since, f is a flpc-continuous function, $f^{-1}(1_X$ - fcl(f($\lambda$)) is fuzzy path connected open and so $f^{-1}$(fcl(f($\lambda$))) is fuzzy path connected closed. Since $\lambda \leq f^{-1}$(f($\lambda$))$\leq f^{-1}$ (fcl(f($\lambda$))), it follows that FPC$_{cl}(\lambda) \leq$ FPC$_{cl}(f^{-1}$(fcl(f($\lambda$))) $\leq f^{-1}$(fcl(f($\lambda$))). Hence, f(FPC$_{cl}(\lambda$))) $\leq$ fcl(f($\lambda$)), for each $\lambda \in I^X$.
(2)$\Longrightarrow$(3)
Let $\mu \in I^Y$. By (2), f(FPC$_{cl}(f^{-1}(\mu))) \leq$ fcl(f($f^{-1}(\mu)) \leq$ fcl($\mu$). Hence FPC$_{cl}(f^{-1}(\mu)) \leq f^{-1}$(fcl($\mu$)), for each $\mu \in I^Y$
(3)$\Longrightarrow$(4)
Let $\mu \in I^Y$. By (3), $f^{-1}$ (fcl($1_Y$ - $\mu$))$\geq$ FPC$_{cl}(f^{-1}(1_Y$ - $\mu$)) = FPC$_{cl}(1_X$ - $f^{-1}(\mu)$). Hence $f^{-1}$(fint($\mu$)) $\leq$ FPC$_{int}(f^{-1}(\mu))$, for each $\mu \in I^Y$
(4)$\Longrightarrow$(1)
Let $\lambda \in I^Y$ be fuzzy path connected open set. Then fint($\lambda$) = $\lambda$. Now $f^{-1}$ ($\lambda$) = $f^{-1}$ (fint($\lambda$)) $\leq$ FPC$_{int}(f^{-1}(\lambda))$. Thus $f^{-1}(\lambda) \leq$ FPC$_{int}(f^{-1}(\lambda))$. It is always true that $f^{-1}(\lambda) \geq$ FPC$_{int}(f^{-1}(\lambda))$. Thus, $f^{-1}(\lambda)$ = FPC$_{int}(f^{-1}(\lambda))$. Hence, $f^{-1}(\lambda)$ is a fuzzy path connected open set, which implies that f is a flpc-continuous function. □

**Theorem 3.14.** Let (X, T) and (Y, S) be any two fuzzy locally path connected spaces. Then the function f: (Y, S) → (X, T) satisfy the following equivalent condition:

1. f is flpc-continuous
2. For $\lambda \in I^Y$ be a fuzzy path connected open set in (Y, S) and let $y_t \in \mathcal{FPCP}$(Y) be a fuzzy path connected point with $y_t \leq \lambda$, then f(FPC($y_t$)) $\leq$ FPC($f(y_t)$), for each FPC($y_t$) $\in I^Y$.
3. For $\mu \in I^X$ be a fuzzy path connected open set in (X, T) and let $x_t \in \mathcal{FPCP}$(X) be a fuzzy path connected point with $x_t \leq \mu$, then FPC($f^{-1}(x_t)$) $\leq f^{-1}$(FPC($x_t$)), for each FPC($x_t$) $\in I^X$.
4. For $\mu \in I^X$ be a fuzzy path connected open set in (X, T) and let $x_t \in \mathcal{FPCP}$(X) be a fuzzy path connected point with $x_t \leq \mu$, then f(FPC($x_t$)) $\leq$ FPC(f($x_t$)), for each FPC($x_t$) $\in I^X$.

*Proof.* (1)$\Longrightarrow$(2)
Let $\lambda$ = FPC($y_t$) be a fuzzy path component of (Y, S) and let f(FPC(($y_t$))) be any fuzzy path component of (X, T). Since f is flpc-continuous, $f^{-1}$f((FPC(($y_t$)))) = $f^{-1}$(FPC($f(y_t)$)) is fuzzy open in (Y, S). Since FPC($y_t$) is the smallest fuzzy path component of (Y, S), FPC($y_t$) $\leq f^{-1}$(FPC($f(y_t)$)). Hence f(FPC($y_t$)) $\leq$ FPC($f(y_t)$).
(2)$\Longrightarrow$(3)
Let FPC($x_t$) be any fuzzy path component of (X, T). Then by (2), f(FPC($f^{-1}(x_t)$)) $\leq$ FPC(f($f^{-1}(x_t)$)). Hence FPC($f^{-1}(x_t)$) $\leq f^{-1}$(FPC($x_t$)).
(3)$\Longrightarrow$(4)
For FPC($x_t$) $\in I^X$, then by (3), FPC($f^{-1}(x_t)$) $\leq$ FPC(f($y_t$))$\leq$ FPC($f^{-1}$(f(f($y_t$)))) $\leq$ $f^{-1}$(FPC(f(f($y_t$)))) $\leq f^{-1}$FPC(f($x_t$)). Hence f(FPC($f^{-1}(x_t)$)) $\leq$ FPC(f($x_t$)).
(4)$\Longrightarrow$(1)
Let FPC($x_t$) be any fuzzy path component of (Y, S) which is fuzzy open and let FPC($f^{-1}(x_t)$) be any fuzzy path component of (X, T) which is also fuzzy open. Then by (4), f(FPC($f^{-1}(x_t)$)) $\leq$ FPC($x_t$). That is, $f^{-1}$(FPC($x_t$)) $\geq$ FPC($f^{-1}(x_t)$). Similarly, $f^{-1}$(FPC($x_t$)) $\leq$ FPC($f^{-1}(x_t)$). Hence $f^{-1}$(FPC($x_t$))= FPC($f^{-1}(x_t)$).Thus f is flpc-continuous. $\square$

**Theorem 3.15.** If (X, T) is a fuzzy Peano space, then (X, T) is a fuzzy path connected space.

*Proof.* Let (X, T) be a fuzzy Peano space. For $x_t \in \mathcal{FPCP}$(X), let $\Im = \{y_t$ : where $y_t$ is the end fuzzy point in a fuzzy path$\}$. Suppose $y_t \in \mathcal{FPCP}$(X). Let f : [0, 1] $\to \mathcal{FPCP}$(X) be a fuzzy path from $x_t$ to $y_t$ such that f(0) = $x_t$, f(1) = $y_t$. Since (X, T) is a fuzzy Peano space, then for any fuzzy path connected point $x_t \in \mathcal{FPCP}$(X) and for any fuzzy open set $\lambda \in I^X$ with $x_t \leq \lambda$, there exists a fuzzy path connected open set $\mu \in I^X$ such that $x_t \leq \mu \leq \lambda$. If $z_t \leq \mu$, there exists a fuzzy path g : [0, 1] $\to \mathcal{FPCP}$(X) from $y_t$ to $z_t$ such that g(0) = $y_t$, g(1) = $z_t$. By Proposition 2.0.1, there exists a fuzzy path h : [0, 1] $\to \mathcal{FPCP}$(X) from $x_t$ to $z_t$ such that h(0) = $x_t$, h(1) = $z_t$. so $z_t \in \Im$. Therefore $\lambda \in \Im$. This is true for all $\lambda \in \Im$. Hence $\Im$ is fuzzy open. Similarly, we can prove $\Im$ is fuzzy closed. Since (X, T) is fuzzy connected and $\Im$ is a fuzzy clopen set implies (X, T) is fuzzy path connected. $\square$

**Definition 3.16.** Let (X, T) be a fuzzy locally path connected space. A flpc-continuous function f : flpc(X, T) $\to$ (X, T) is said to be a universal fuzzy locally path connected space if it satisfies the following condition:
For any fuzzy locally path connected space (Y, S) and for any function g : (Y, S) $\to$ (X, T) there exists a unique fuzzy lift h : (Y, S) $\to$ flpc(X, T) of g such that f$\circ$h = g.

**Definition 3.17.** Let (X, T) ba a fuzzy locally path connected space. Then the fuzzy locally path connected space (Y, S) is said to be a generated fuzzy locally path connected space if it is generated by all fuzzy path components of fuzzy open sets of $I^X$.

**Theorem 3.18.** Let (X, T) be a fuzzy locally path connected space. Every space (X, T) is a universal fuzzy locally path connected space if f : (X, T) $\to$ (Y, S) is fuzzy homeomorphic where (Y, S) is a generated fuzzy topological space.

*Proof.* Let $\lambda \in I^X$ and $\mu \in I^X$ be any two fuzzy path component of $x_t$ and $y_t$ respectively. Assume $\lambda$ and $\mu$ are fuzzy open. Since $z_t \leq \lambda \wedge \mu$ implies FPC($z_t$, $\lambda \wedge \mu$) $\leq \lambda \wedge \mu$, the family $\{\lambda\}$ of fuzzy path components of $x_t \leq \lambda$ forms a basis for all $\lambda \in$ (X, T). Now the generating fuzzy topology will be denoted by (Y, S) which is fuzzy locally path connected space. Consider a function f : (Y, S) $\to$ (X, T) defined as $f^{-1}$(FPC($x_t$))= FPC($f^{-1}(x_t)$). Then by Theorem 3.14, f is fuzzy continuous. Hence (X, T) is a Universal fuzzy locally path connected-space. $\square$

**Definition 3.19.** Let (X, T) be a fuzzy locally path connected space then the sub collection $\Lambda$ of T is said to countable fuzzy neighbourhood base if for a fuzzy path connected point $x_t \in \mathcal{FPCP}$(X) with $x_t \leq \mu$ there exists $\lambda \in I^X$ with $\mu \leq \lambda$ such that $x_t \leq \mu \leq \lambda$.

**Definition 3.20.** A fuzzy locally path space (X, T) is said to be a first countable fuzzy locally path connected space if every fuzzy path connected point $x_t \in \mathcal{FPCP}$(X) has a countable fuzzy neighbourhood base. A first countable fuzzy locally path connected space is also said to be a space satisfying the first axiom of countability.

**Theorem 3.21.** Let (X, T) be any fuzzy locally path connected space. Suppose f : (Y, S) $\rightarrow$ (X, T) is a function from a first countable fuzzy locally path connected space to (X, T). If f $\circ$ g is flpc-continuous for every fuzzy path g : I $\rightarrow I^Y$ then f is flpc-continuous.

*Proof.* Suppose $\lambda \in I^X$ be the fuzzy path component of $x_t$ which is fuzzy open in (X, T). We have to prove that if there exists a fuzzy path component $\mu \in I^Y$ of $y_t$ then $f^{-1}$ ($\lambda$) $\leq \mu$. Pick a countable fuzzy neighbourhood basis $\{\mu_{t_n}\}$, n $\geq$ 1 for all $\mu_n \in$ (Y, S) and for all $y_{t_n} \leq \mu_n$. Assume that for each fuzzy neighbourhoods there is a fuzzy path $g_n$ in ($\mu_{t_n}$) joining $\mu_t$ to $\mu_{t_n} \neq f^{-1}$ ($\lambda$). These fuzzy paths can be joined to one fuzzy path g from $\mu$ to $\mu_{t_1}$ and going through all fuzzy path components $\mu_n$, n $\geq$ 2. Since f$\circ$ g is fuzzy continuous, f $\circ$ g starts from f($\mu$) and goes through all fuzzy path components f($\mu_{t_n}$), n $\geq$ 1. However $\lambda$ is fuzzy open in (X, T). Then $\lambda \geq$ f($\mu_{t_n}$). Hence $\mu_{t_n} \leq f^{-1}$ ($\lambda$), which is contradiction. □

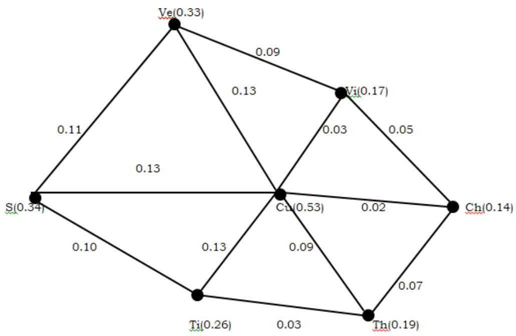
**Theorem 3.22.** Suppose (X, T) is a fuzzy locally path connected space and $\wp$ is a class of fuzzy Peano spaces. The family $\Im$ of fuzzy sets $\lambda \in I^X$ such that $f^{-1}(\lambda)$ is fuzzy open in P $\in \wp$ for any function f : (P, S) $\rightarrow$ (X, T) in the original fuzzy topology, is a fuzzy topology and $\wp$(X) = (X, T) is a fuzzy Peano space.

*Proof.* The family $\Im$ is a fuzzy topology on $I^X$ since fuzzy sets $\lambda$ of $I^X$ such that $f^{-1}(\lambda)$ is fuzzy open in P $\in \wp$ for any function f : (P, S) $\rightarrow$ (X, T). Suppose $\lambda \in \Im$ and $\lambda$) is a fuzzy path component of $x_t$ in the new fuzzy topology. Suppose f : (Z, R) $\rightarrow$ (X, T) is a function and f($\gamma$) $\leq \lambda$, where $\gamma$ is a fuzzy path component of $z_t \in \mathcal{FPCP}$(Z). As $f^{-1}(\lambda)$ is fuzzy open, there is a fuzzy connected neighbourhood $\mu$ of $z_t$ in (Z, R) satisfying f($\mu$) $\leq \lambda$. As f($\mu$) is fuzzy path connected, f($\mu$) $\leq \lambda$ and $\lambda \leq \Im$. □

## 4. Application: The fuzzy Hamilton cycle by using rail connectivity network

In this section, As an application of Theorem 3.1, we have to find the fuzzy Hamiltonian cycle by using the algorithm given in the preliminary.

From the map train routes between the cities Salem, Vellore, Viluppuram, cuddalore, chidambaram, Thanjavur and Tiruchirapalli are selected. Figure 4 represents the rail connectivity network in which the vertices denotes the cities Salem(S), Vellore(Ve), Viluppuram(Vi), cuddalore(Cu), chidambaram(Ch), Thanjavur(Th) and Tiruchirapalli(Ti). The membership value of the edge weight is the ratio of the distance between two cities to the total distance of all the routes in the selected network and the membership value of every vertex is the sum of all destinations from it.


Fuzzy graph of selected cities

$$
A(G) = \begin{array}{c c}
 & \begin{array}{ccccccc} S & Ve & Vi & Cu & Ch & Th & Ti \end{array} \\
\begin{array}{c} S \\ Ve \\ Vi \\ Cu \\ Ch \\ Th \\ Ti \end{array} &
\begin{bmatrix}
0 & 0.13 & 0 & 0.13 & 0 & 0 & 0.10 \\
0.11 & 0 & 0.09 & 0.13 & 0 & 0 & 0 \\
0 & 0.09 & 0 & 0.03 & 0.05 & 0 & 0 \\
0.13 & 0.13 & 0.03 & 0 & 0.02 & 0.09 & 0.13 \\
0 & 0 & 0.05 & 0.02 & 0 & 0.07 & 0 \\
0 & 0 & 0 & 0.09 & 0.07 & 0 & 0.08 \\
0.10 & 0 & 0 & 0.13 & 0 & 0.08 & 0
\end{bmatrix}
\end{array}
$$

**Iteration 1**: From Algorithm 2.1, the minimum non zero entry appear in the fourth row(Cu) and the fifth column(Ch) is 0.02. That is from 'Cu' it reaches 'Ch'(Cu-Ch). Now in the row 'Ch' the next minimum value is 0.05 which corresponds to 'Vi' Column. That is, (Cu-Ch-Vi). Continuing in this way, we obtained a fuzzy path is (Cu-Ch-Vi-Cu), which is rejected as it is a fuzzy circuit.

**Iteration 2**: Start with the other non zero minimum entry 0.02, we obtained a fuzzy path (Ch-Cu-Vi-Ch), again it is rejected.

**Iteration 3**: Now start with the other minimum non zero entry 0.03 represented by the row 'Cu' and the column 'Vi'. By algorithm, the fuzzy path obtained is (Cu-Vi-Ch-Th-Ti-S-Ve) which is a fuzzy Hamiltonian path. At this stage, the row 'Ve' is left unmarked. From the row 'Ve', select the non zero entry corresponding to the column 'Cu', to get a fuzzy Hamiltonian cycle (Cu-Vi-Ch-Th-Ti-S-Ve-Cu).
Thus the perfect shortest path to cover all the selected cities is Cu-Vi-Ch-Th-Ti-S-Ve-Cu.

## 5. Conclusion

In this paper, we introduced fuzzy Peano space and studied some of their interesting Properties. By Using the property of fuzzy path connectedness we can find the fuzzy Hamiltonian cycle using adjacency matrix of fuzzy graph. When the number of cities are higher, it becomes difficult to find the Shortest Path between the cities. But by Using this method we can easily find the Shortest path. Thus the fuzzy Hamiltonian Cycle is used in rail connectivity network to find the perfect shortest path.

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**K. Sugapriya** is a research scholar in mathematics in Sri Sarada College for Women(Autonomous), Salem, Tamilnadu, India. She is doing research in fuzzy path connected spaces.

**Dr. B. Amudhambigai** is working as an assistant professor in mathematics at Sri Sarada College for Women(Autonomous), Salem, Tamilnadu, India. She completed her M.Sc. and M.Phil. in mathematics. She did her Ph.D in the area of Fuzzy Topology. Her research area includes fuzzy Cryptography and Mathematical Modelling.