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AN INTRODUCTION TO PYTHAGOREAN FUZZY HYPERIDEALS IN HYPERSEMIGROUPS

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Abstract. As the generalization of intuitionistic fuzzy set, Pythagorean fuzzy set was introduced. It is a pair of membership and non-membership grade where the sum of the squares of membership and non-membership grade should be less than or equal to 1. Pythagorean fuzzy set get more attention to deal with uncertainity. In this paper we apply Pythagorean fuzzy set in ideal theory of hypersemigroups. We introduce Pythagorean fuzzy left(right) hyperideals in hypersemigroups. We define $t-$level cut of Pythagorean fuzzy hyperideal is in hypersemigroup. Also we introduce Pythagorean fuzzy interior hyperideals in hypersemigroups and explain it with detailed example. Some theorems and results are also studied. Relation between Pythagorean fuzzy right(left) hyperideal, Pythagorean fuzzy subsemihypergroup and Pythagorean fuzzy interior hyperideal is given.

Keywords: Pythagorean fuzzy set, Pythagorean fuzzy hyperideal, Pythagorean fuzzy interior hyperideal.

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1. Introduction

Fuzzy set theory was proposed by Zadeh[11]. Several researchers have done remarkable works on the generalization of fuzzy sets. Atanassov[2] introduced a new generalization, intuitionistic fuzzy set in 1986. He defined some new operations on intuitionistic fuzzy sets[3]. Yager[9] generalized the concept of intuitionistic fuzzy set and propounded the theory of Pythagorean fuzzy set. In recent days the Pythagorean fuzzy set got more attention among the researchers. It plays an important role to tackle the uncertainities. Yager[10], Zhang[12] have applied the concept of Pythagorean fuzzy set in decision making problem. Kumar et al.[7] approached transportation decision making problems using Pythagorean fuzzy set.

Marty[8] extended the algebraic structures to algebraic hyperstructures. Hussain et

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al.[6] applied the concept of rough set in Pythagorean fuzzy ideals in semigroups. Chinram et al.[4] extended the idea of [6] to ternary semigroups. Akram[1] established the properties of fuzzy lie algebras. Davvaz[5] studied fuzzy hyperideals and intuitionistic fuzzy hyperideals in hypersemigroups. In this paper we introduce Pythagorean fuzzy left(right) hyperideals, Pythagorean fuzzy subsemihypergroup and Pythagorean fuzzy interior hyperideal. We also study some theorems and results. We define both the concepts and explain with clear examples.

2. Preliminaries

In this section we recall the definitions of ideals in hyperstructures such as hyperideal, subsemihypergroup and bi hyperideal.

Definition 2.1. [5] *Let $\mathscr{G}$ be a non-empty universe set and $\mathcal{F}(\mathscr{G})$ is the collection of all subsets of $\mathscr{G}$. The hyperoperation $\bullet$ on $\mathscr{G}$ is defined by*
$\bullet : \mathscr{G} \times \mathscr{G} \to \mathcal{F}(\mathscr{G})$
The set $\mathscr{G}$ with the hyperoperation $\bullet$ is called hypergroupoid(say $\mathscr{G}^\bullet$). The image of $(g_1, g_2) \in \mathscr{G} \times \mathscr{G}$ is denoted by $g_1 \bullet g_2$.

Let $\mathscr{G}_1$ and $\mathscr{G}_2$ be the subsets of $\mathcal{F}(\mathscr{G})$ then the hyperoperation($\star$) between $\mathscr{G}_1$ and $\mathscr{G}_2$ is defined by

$$\mathscr{G}_1 \star \mathscr{G}_2 = \bigcup_{(g_1,g_2)\in\mathscr{G}_1\times\mathscr{G}_2} g_1 \bullet g_2 \tag{1}$$

where
$\star : \mathcal{F}(\mathscr{G}) \times \mathcal{F}(\mathscr{G}) \to \mathcal{F}(\mathscr{G})$.

Definition 2.2. [5] *A hypergroupoid ($\mathscr{G}^\bullet$) is called a hypersemigroup(say $\mathscr{G}^\star$) if $(\{r\} \star \{s\}) \star \{t\} = \{r\} \star (\{s\} \star \{t\})$ for all $r, s, t \in \mathscr{G}$.*

Example 2.1. *Let us consider a universe set $\mathscr{G}$ as $\mathscr{G} = \{r, s, t\}$. Define a hyperoperation $\bullet$ on $\mathscr{G}$ is defined by $r \bullet s = \{r, s\}$ $\forall$ $r, s \in \mathscr{G}$ which is a hypergroupoid. Then we have*

$$\begin{aligned} r \bullet (s \bullet t) &= \{r\} \star \{s, t\} \\ &= (r \bullet s) \cup (r \bullet t) \\ &= \{r, s\} \cup \{r, t\} \\ &= \{r, s, t\} \end{aligned}$$

And also we have $(r \bullet s) \bullet t = \{r, s, t\}$ Hence $r \bullet (s \bullet t) = (r \bullet s) \bullet t$ holds for all $r, s, t \in \mathscr{G}$. Therefore $\mathscr{G}$ is a hypersemigroup on the hyper-operation $\star$.

Definition 2.3. *A non-empty set A of a hypersemigroup $\mathscr{G}^\star$ is said to be a left(right) hyperideal if $\mathscr{G}^\star \star A \subseteq A(A \star \mathscr{G}^\star \subseteq A)$. A hyperideal is both a left and right hyperideal.*

A non-empty set A of a hypersemigroup $\mathscr{G}^\star$ is said to be a subsemihypergroup if $A \star A \subseteq A$.

A subsemihypergroup A of a hypersemigroup $\mathscr{G}^\star$ is said to be an interior hyperideal if $\mathscr{G}^\star \star A \star \mathscr{G}^\star \subseteq A$.

3. Pythagorean fuzzy hyperideals of hypersemigroups

Definition 3.1. *Let $\mathscr{G}^\star$ be a hypersemigroup. A Pythagorean fuzzy set $\mathcal{P} = (\mathcal{P}_1, \mathcal{P}_2)$ is defined by*

$$\mathcal{P} = \{\frac{x}{(\mu_{\mathcal{P}_1}(x), \mu_{\mathcal{P}_2}(x))} : x \in \mathscr{G}^\star\} \text{ such that } 0 \leq \mu_{\mathcal{P}_1}(x)^2 + \mu_{\mathcal{P}_2}(x)^2 \leq 1.$$

where $\mu_{\mathcal{P}_1}(x)$ is the membership function from the universe set $\mathscr{G}^\star$ to the closed interval $[0,1]$ and $\mu_{\mathcal{P}_2}(x)$ is the non-membership function from the universe set $\mathscr{G}^\star$ to the closed interval $[0,1]$.

Definition 3.2. *If $\mathcal{P}$ and $\mathcal{Q}$ are two Pythagorean fuzzy sets of $\mathscr{G}^\star$ then the following operations are defined as:*

(i) $\mathcal{P} \cap \mathcal{Q} = \{\frac{x}{(\mu_{\mathcal{P}_1}(x) \wedge \mu_{\mathcal{Q}_1}(x), \mu_{\mathcal{P}_2}(x) \vee \mu_{\mathcal{Q}_2}(x))} : x \in \mathscr{G}^\star\}$

(ii) $\mathcal{P} \cup \mathcal{Q} = \{\frac{x}{(\mu_{\mathcal{P}_1}(x) \vee \mu_{\mathcal{Q}_1}(x), \mu_{\mathcal{P}_2}(x) \wedge \mu_{\mathcal{Q}_2}(x))} : x \in \mathscr{G}^\star\}$

(iii) $\Box\mathcal{P} = (\mathcal{P}_1, \hat{\mathcal{P}}_1)$*, where* $\hat{\mathcal{P}}_1 = 1 - \mathcal{P}_1$.

Definition 3.3. *A Pythagorean fuzzy set $\mathcal{P}$ of $\mathscr{G}^\star$ is said to be a Pythagorean fuzzy left hyperideal if for $x \in \mathscr{G}^\star$ we have*

(i) $\mu_{\mathcal{P}_1}(b) \leq \inf_{x \in a \star b} \mu_{\mathcal{P}_1}(x)$

(ii) $\mu_{\mathcal{P}_2}(b) \geq \sup_{x \in a \star b} \mu_{\mathcal{P}_2}(x)$ *for all* $a, b \in \mathscr{G}^\star$.

Definition 3.4. *A Pythagorean fuzzy set $\mathcal{P}$ of $\mathscr{G}^\star$ is said to be a Pythagorean fuzzy right hyperideal if for $x \in \mathscr{G}^\star$ we have*

(i) $\mu_{\mathcal{P}_1}(a) \leq \inf_{x \in a \star b} \mu_{\mathcal{P}_1}(x)$

(ii) $\mu_{\mathcal{P}_2}(a) \geq \sup_{x \in a \star b} \mu_{\mathcal{P}_2}(x)$ *for all* $a, b \in \mathscr{G}^\star$.

Definition 3.5. *A Pythagorean fuzzy set $\mathcal{P}$ is said to be a Pythagorean fuzzy hyperideal of $\mathscr{G}^\star$ if $\mathcal{P}$ is both a Pythagorean fuzzy left hyperideal and Pythagorean fuzzy right hyperideal of $\mathscr{G}^\star$.*

Example 3.1. *Let us consider a hypersemigroup $\mathscr{G}^\star = \{a_1, a_2, a_3, a_4\}$ with the hyperoperation ($\bullet$):*

$\bullet$	a_1	a_2	a_3	a_4
a_1	$\{a_1\}$	$\{a_1\}$	$\{a_1\}$	$\{a_1\}$
a_2	$\{a_1\}$	$\{a_1\}$	$\{a_1\}$	$\{a_1\}$
a_3	$\{a_1\}$	$\{a_1\}$	$\{a_1, a_2\}$	$\{a_1, a_2\}$
a_4	$\{a_1\}$	$\{a_1\}$	$\{a_1, a_2\}$	$\{a_1\}$

Define a Pythagorean fuzzy set $\mathcal{P}$ as

$\mathcal{P} = \{\frac{a_1}{(0.5, 0.6)}, \frac{a_2}{(0.5, 0.7)}, \frac{a_3}{(0.3, 1)}, \frac{a_4}{(0.2, 0.9)}\}$. *We have*

$\mu_{\mathcal{P}_1}(a_1) = 0.5 = \inf_{a_1 \in a_1 \star a_2} \mu_{\mathcal{P}_1}(a_1)\ \forall\ a_2 \in \mathscr{G}^\star$,

$\mu_{\mathcal{P}_1}(a_2) = 0.5 = \inf_{a_1 \in a_2 \star a_3} \mu_{\mathcal{P}_1}(a_1)\ \forall\ a_3 \in \mathscr{G}^\star$,

$\mu_{\mathcal{P}_1}(a_3) = 0.3 < 0.5 = \inf_{a_1, a_2 \in a_3 \star a_4} \{\mu_{\mathcal{P}_1}(a_1), \mu_{\mathcal{P}_1}(a_2)\}\ \forall\ a_4 \in \mathscr{G}^\star$,

$\mu_{\mathcal{P}_1}(a_4) = 0.5 = \inf_{a_1, a_2 \in a_4 \star a_3} \{\mu_{\mathcal{P}_1}(a_1), \mu_{\mathcal{P}_1}(a_2)\}\ \forall\ a_2 \in \mathscr{G}^\star$.

Therefore $\mathcal{P}_1$ is a Pythagorean fuzzy right hyperideal. Now,

$\mu_{\mathcal{P}_2}(a_1) = 0.6 = \sup_{a_1 \in a_1 \star a_2} \mu_{\mathcal{P}_2}(a_1) \ \forall \ a_2 \in \mathscr{G}^\star,$
$\mu_{\mathcal{P}_2}(a_2) = 0.7 > 0.6 = \sup_{a_1 \in a_2 \star a_3} \mu_{\mathcal{P}_2}(a_1) \ \forall \ a_3 \in \mathscr{G}^\star,$
$\mu_{\mathcal{P}_2}(a_3) = 1 > 0.7 = \sup_{a_1, a_2 \in a_3 \star a_4} \{\mu_{\mathcal{P}_2}(a_1), \mu_{\mathcal{P}_2}(a_1)\} \ \forall \ a_4 \in \mathscr{G}^\star,$
$\mu_{\mathcal{P}_2}(a_4) = 0.9 > 0.7 = \sup_{a_1, a_2 \in a_4 \star a_3} \{\mu_{\mathcal{P}_2}(a_1), \mu_{\mathcal{P}_2}(a_1)\} \ \forall \ a_2 \in \mathscr{G}^\star.$
Therefore $\mathcal{P}_2$ is a Pythagorean fuzzy right hyperideal. Hence $\mathcal{P}$ is a Pythagorean fuzzy right hyperideal of $\mathscr{G}^\star$. In the same way we can show that $\mathcal{P}$ is a Pythagorean fuzzy left hyperideal of $\mathscr{G}^\star$.

Theorem 3.1. *If $\mathcal{P}$ and $\mathcal{Q}$ are two Pythagorean fuzzy left(right) hyperideals of $\mathscr{G}^\star$ then $\mathcal{P} \cap \mathcal{Q}$ is also a Pythagorean fuzzy left(right) hyperideals of $\mathscr{G}^\star$.*

Proof. Consider for $x \in \mathscr{G}^\star$

$$\begin{aligned} \mu_{\mathcal{P}_1}(a) \wedge \mu_{\mathcal{Q}_1}(a) &\leq \inf_{x \in a \star b} \mu_{\mathcal{P}_1}(x) \wedge \inf_{x \in a \star b} \mu_{\mathcal{Q}_1}(x) \\ &\leq \inf_{x \in a \star b} \{\mu_{\mathcal{P}_1}(x) \wedge \mu_{\mathcal{Q}_1}(x)\} \\ &\leq \inf_{x \in a \star b} \mu_{\mathcal{P}_1 \cap \mathcal{Q}_1}(x) \ \forall \ a, b \in \mathscr{G}^\star. \end{aligned}$$

Now,

$$\begin{aligned} \mu_{\mathcal{P}_2}(a) \vee \mu_{\mathcal{Q}_2}(a) &\geq \sup_{x \in a \star b} \mu_{\mathcal{P}_2}(x) \vee \sup_{x \in a \star b} \mu_{\mathcal{Q}_2}(x) \\ &\geq \sup_{x \in a \star b} \{\mu_{\mathcal{P}_2}(x) \vee \mu_{\mathcal{Q}_2}(x)\} \\ &\geq \sup_{x \in a \star b} \mu_{\mathcal{P}_2 \cup \mathcal{Q}_2}(x) \ \forall \ a, b \in \mathscr{G}^\star. \end{aligned}$$

Thus $\mathcal{P} \cap \mathcal{Q}$ is a Pythagorean fuzzy right hyperideal. Similarly we can prove for Pythagorean fuzzy left hyperideal. □

Theorem 3.2. *If $\mathcal{P}$ and $\mathcal{Q}$ are two Pythagorean fuzzy left(right) hyperideals of $\mathscr{G}^\star$ then $\mathcal{P} \cup \mathcal{Q}$ is also a Pythagorean fuzzy left(right) hyperideals of $\mathscr{G}^\star$.*

Proof. The proof is similar as in Theorem 3.1. □

Definition 3.6. *If $\mathcal{P} = (\mathcal{P}_1, \mathcal{P}_2)$ is a Pythagorean fuzzy set of $\mathscr{G}^\star$ then the image of $\mathcal{P}$ is defined by $IM(\mathcal{P}) = IM(\mathcal{P}_1) \cup IM(\mathcal{P}_2)$ where*
$IM(\mathcal{P}_1) = \{\mu_{\mathcal{P}_1}(x) : x \in \mathscr{G}^\star\}$ and $IM(\mathcal{P}_2) = \{\mu_{\mathcal{P}_2}(x) : x \in \mathscr{G}^\star\}$.

Definition 3.7. *Let $\mathcal{P} = (\mathcal{P}_1, \mathcal{P}_2)$ be a Pythagorean fuzzy set of $\mathscr{G}^\star$ and let $t \in IM(\mathcal{P})$ then the sets*
$\mathcal{P}_1^t = \{x \in \mathscr{G}^\star : \mu_{\mathcal{P}_1}(x) \geq t\}$ and
$\mathcal{P}_2^t = \{x \in \mathscr{G}^\star : \mu_{\mathcal{P}_2}(x) \leq t\}$
are called $t-$ level cut of $\mathcal{P}_1$ and $t-$ level cut of $\mathcal{P}_2$ respectively.

Theorem 3.3. *If $\mathcal{P} = (\mathcal{P}_1, \mathcal{P}_2)$ is a Pythagorean fuzzy hyperideal of $\mathscr{G}^\star$ and for $t \in IM(\mathcal{P})$ then $\mathcal{P}^t = (\mathcal{P}_1^t, \mathcal{P}_2^t)$ is a hyperideal of $\mathscr{G}^\star$.*

Proof. Let $a, b, x \in \mathscr{G}^\star$ such that $a \in \mathcal{P}_1^t$ then

$$\mu_{\mathcal{P}_1}(a) \geq t \tag{2}$$

Since $\mu_{\mathcal{P}_1}(a) \leq \inf_{x \in a \star b} \mu_{\mathcal{P}_1}(x)$
$\implies \inf_{x \in a \star b} \mu_{\mathcal{P}_1}(x) \geq t$ by (2)

$\Longrightarrow \mu_{\mathcal{P}_1}(x) \geq t$ for $x \in a \star b$
$\Longrightarrow a \star b \in \mathcal{P}_1^t$.
Thus $\mathcal{P}_1^t$ is a right hyperideal.
Now, let $a, b, x \in \mathscr{G}^\star$ such that $a \in \mathcal{P}_2^t$ then

$$\mu_{\mathcal{P}_2}(a) \leq t \tag{3}$$

$\Longrightarrow \mu_{\mathcal{P}_2}(a) \geq \sup_{x \in a \star b} \mu_{\mathcal{P}_2}(x)$
$\Longrightarrow \sup_{x \in a \star b} \mu_{\mathcal{P}_2}(x) \leq t$ by (3)
$\Longrightarrow \mu_{\mathcal{P}_2}(x) \leq t$ for $x \in a \star b$
$\Longrightarrow a \star b \in \mathcal{P}_2^t$.
Thus $\mathcal{P}_2^t$ is a right hyperideal. Similarly we can prove that $\mathcal{P}^t$ is a left hyperideal of $\mathscr{G}^\star$. □

Example 3.2. *Consider Example 3.1. We have a Pythagorean fuzzy set $\mathcal{P}$ as*
$\mathcal{P} = \{\frac{a_1}{(0.5, 0.6)}, \frac{a_2}{(0.5, 0.7)}, \frac{a_3}{(0.3, 1)}, \frac{a_4}{(0.2, 0.9)}\}$.
The image of $\mathcal{P}$ is $IM(\mathcal{P}) = \{0.2, 0.3, 0.5, 0.6, 0.7, 0.9, 1\}$. The $t-$level set of $\mathcal{P}$ is defined in Table 1:

TABLE 1. $t-$level set of $\mathcal{P}$ $(\mathcal{P}^t)$

$\mathcal{P}^t/t$	0.2	0.3	0.5	0.6	0.7	0.9	1
$\mathcal{P}_1^t$	$\mathscr{G}^\star$	$\{a, b, c\}$	$\{a, b\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\mathcal{P}_2^t$	$\emptyset$	$\emptyset$	$\emptyset$	$\{a\}$	$\{a, b\}$	$\{a, b, c\}$	$\mathscr{G}^\star$

Since by Example 3.1 we know that $\mathcal{P}$ is a Pythagorean fuzzy hyperideal. By routine calculation we can say that the sets $\{a\}, \{a, b\}$ and $\{a, b, c\}$ are hyperideals.

Theorem 3.4. *If $\mathcal{P} = (\mathcal{P}_1, \mathcal{P}_2)$ of $\mathscr{G}^\star$ is a Pythagorean hyperideal then so $\Box\mathcal{P} = (\mathcal{P}_1, \hat{\mathcal{P}}_1)$ where $\hat{\mathcal{P}}_1 = 1 - \mathcal{P}_1$.*

Proof. Since $\mathcal{P}_1$ is a Pythagorean fuzzy hyperideal. Now to show that $\hat{\mathcal{P}}_1$ is a Pythagorean fuzzy hyperideal. Consider for $x, a, b \in \mathscr{G}^\star$,

$$\begin{aligned} \mu_{\hat{\mathcal{P}}_1}(a) &= 1 - \mu_{\mathcal{P}_1}(a) \\ &\geq 1 - \sup_{x \in a \star b} \mu_{\mathcal{P}_1}(x) \\ &\geq \sup_{x \in a \star b} (1 - \mu_{\mathcal{P}_1}(x)) \\ &\geq \sup_{x \in a \star b} \mu_{\hat{\mathcal{P}}_1}(x) \end{aligned}$$

Thus $\Box\mathcal{P}$ is a Pythagorean fuzzy hyperideal of $\mathscr{G}^\star$. □

4. Pythagorean fuzzy interior hyperideal

Definition 4.1. *A Pythagorean fuzzy set $\mathcal{P}$ of $\mathscr{G}^\star$ is said to be Pythagorean fuzzy sub-semihypergroup if for $x \in \mathscr{G}^\star$ we have*
(i) $\mu_{\mathcal{P}_1}(a) \wedge \mu_{\mathcal{P}_1}(b) \leq \inf_{x \in a \star b} \mu_{\mathcal{P}_1}(x)$
(ii) $\mu_{\mathcal{P}_2}(a) \vee \mu_{\mathcal{P}_2}(b) \geq \sup_{x \in a \star b} \mu_{\mathcal{P}_2}(x)$ for all $a, b \in \mathscr{G}^\star$.

Example 4.1. *Let us consider a hypersemigroup $\mathscr{G}^\star = \{c_1, c_2, c_3, c_4\}$ with the hyperoperation $(\bullet)$:*

$\bullet$	c_1	c_2	c_3	c_4
c_1	$\{c_1\}$	$\{c_1\}$	$\{c_1\}$	$\{c_1\}$
c_2	$\{c_1\}$	$\{c_1, c_2\}$	$\{c_1, c_3\}$	$\{c_1\}$
c_3	$\{c_1\}$	$\{c_1\}$	$c_1\}$	$\{c_1\}$
c_4	$\{c_1\}$	$\{c_1, c_4\}$	$\{c_1\}$	$\{c_1\}$

Define a Pythagorean fuzzy set $\mathcal{P}$ as
$\mathcal{P} = \{\frac{c_1}{(1, 0.3)}, \frac{c_2}{(0.2, 1)}, \frac{c_3}{(0.2, 0.9)}, \frac{c_4}{(0.7, 0.6)}\}$. *We have*
$\mu_{\mathcal{P}_1}(c_2) \wedge \mu_{\mathcal{P}_1}(c_4) = 0.2 < 1 = \inf_{c_1 \in c_2 \star c_4} \mu_{\mathcal{P}_1}(c_1) \ \forall \ c_2, c_4 \in \mathscr{G}^\star$,
$\mu_{\mathcal{P}_2}(c_3) \vee \mu_{\mathcal{P}_2}(c_4) = 0.9 > 0.3 = \sup_{c_1 \in c_3 \star c_4} \mu_{\mathcal{P}_2}(c_1) \ \forall \ c_3, c_4 \in \mathscr{G}^\star$,
Similarly (i) and (ii) of Definition 4.1 holds for all $c_1, c_2, c_3, c_4 \in \mathscr{G}^\star$. Hence $\mathcal{P}$ is a Pythagorean fuzzy subsemihypergroup.

Theorem 4.1. *Let $\mathcal{P} = (\mathcal{P}_1.\mathcal{P}_2)$ be a Pythagorean fuzzy set of $\mathscr{G}^\star$. For $t \in IM(\mathcal{P})$, the $t-$level cut of $\mathcal{P}(\mathcal{P}^t)$ is a subsemihypergroup then $\mathcal{P}$ is a Pythagorean fuzzy subsemihypergroup of $\mathscr{G}^\star$.*

Proof. Let $\mathcal{P}_1^t$ is a subsemihypergroup of $\mathscr{G}^\star$ then for $a, b \in \mathcal{P}_1^t$ we have

$$a \star b \in \mathcal{P}_1^t \tag{4}$$

Suppose if $\mu_{\mathcal{P}_1}(a) \wedge \mu_{\mathcal{P}_1}(b) \nleq \inf_{x \in a \star b} \mu_{\mathcal{P}_1}(x)$ then we have
$\mu_{\mathcal{P}_1}(a) \wedge \mu_{\mathcal{P}_1}(b) > \inf_{x \in a \star b} \mu_{\mathcal{P}_1}(x)$.
Then there exists some $t_\alpha \in IM(\mathcal{P})$ such that
$\mu_{\mathcal{P}_1}(a) \wedge \mu_{\mathcal{P}_1}(b) > t_\alpha > \inf_{x \in a \star b} \mu_{\mathcal{P}_1}(x)$. This implies that
$\mu_{\mathcal{P}_1}(a) \wedge \mu_{\mathcal{P}_1}(b) > t_\alpha$ and $\inf_{x \in a \star b} \mu_{\mathcal{P}_1}(x) < t_\alpha$.
$\Longrightarrow \mu_{\mathcal{P}_1}(a) \wedge \mu_{\mathcal{P}_1}(b) > t_\alpha$ and $\mu_{\mathcal{P}_1}(x) < t_\alpha$ for $x \in a \star b$
$\Longrightarrow a \star b \notin \mathcal{P}_1^t$ and either $a \in \mathcal{P}_1^t$ or $b \in \mathcal{P}_1^t$.
Which is a contradiction to Equation (4). Hence $\mu_{\mathcal{P}_1}(a) \wedge \mu_{\mathcal{P}_1}(b) \leq \inf_{x \in a \star b} \mu_{\mathcal{P}_1}(x)$.

Now, Since $\mathcal{P}_2^t$ is a subsemihypergroup then for $a, b \in \mathcal{P}_2^t$ we have

$$a \star b \in \mathcal{P}_2^t \tag{5}$$

Suppose if $\mu_{\mathcal{P}_2}(a) \vee \mu_{\mathcal{P}_2}(b) \ngeq \sup_{x \in a \star b} \mu_{\mathcal{P}_2}(x)$ then we have
$\mu_{\mathcal{P}_2}(a) \vee \mu_{\mathcal{P}_2}(b) < \sup_{x \in a \star b} \mu_{\mathcal{P}_2}(x)$.
Then there exists some $t_\beta \in IM(\mathcal{P})$ such that
$\mu_{\mathcal{P}_2}(a) \vee \mu_{\mathcal{P}_2}(b) < t_\beta < \sup_{x \in a \star b} \mu_{\mathcal{P}_2}(x)$. This implies that
$\mu_{\mathcal{P}_2}(a) \vee \mu_{\mathcal{P}_2}(b) < t_\beta$ and $\sup_{x \in a \star b} \mu_{\mathcal{P}_2}(x) > t_\beta$
$\Longrightarrow \mu_{\mathcal{P}_2}(a) \vee \mu_{\mathcal{P}_2}(b) < t_\beta$ and $\mu_{\mathcal{P}_2}(x) > t_\beta$ for $x \in a \star b$
$\Longrightarrow a \star b \notin \mathcal{P}_2^t$ and either $a \in \mathcal{P}_2^t$ or $b \in \mathcal{P}_2^t$.
Which is a contradiction to Equation (5). Hence $\mu_{\mathcal{P}_2}(a) \vee \mu_{\mathcal{P}_2}(b) \leq \inf_{x \in a \star b} \mu_{\mathcal{P}_2}(x)$.

□

Definition 4.2. *A Pythagorean fuzzy subsemihypergroup $\mathcal{P}$ is said to be Pythagorean fuzzy interior hyperideal of $\mathscr{G}^\star$ if for $x, z \in \mathscr{G}^\star$ we have*
(i) $\mu_{\mathcal{P}_1}(z) \leq \inf\limits_{x \in a\star z\star b} \mu_{\mathcal{P}_1}(x)$
(ii) $\mu_{\mathcal{P}_2}(z) \geq \sup\limits_{x \in a\star z\star b} \mu_{\mathcal{P}_2}(x)$ *for all $a, b \in \mathscr{G}^\star$.*

Theorem 4.2. *Let $\mathcal{P} = (\mathcal{P}_1.\mathcal{P}_2)$ be a Pythagorean fuzzy set of $\mathscr{G}^\star$. For $t \in IM(\mathcal{P})$, the $t-$level cut of $\mathcal{P}$ is an interior hyperideal of $\mathscr{G}^\star$ then $\mathcal{P}$ is a Pythagorean fuzzy interior hyperideal of $\mathscr{G}^\star$.*

Proof. Let $\mathcal{P}$ be a Pythagorean fuzzy set of $\mathscr{G}^\star$. From Theorem 4.1 $\mathcal{P}$ is a Pythagorean fuzzy subsemihypergroup.
Since $\mathcal{P}_1^t$ is an interior ideal then for $a, b \in \mathscr{G}^\star$ and $z \in \mathcal{P}_1^t$ we have

$$a \star z \star b \in \mathcal{P}_1^t \tag{6}$$

Suppose if $\mathcal{P}_1$ is not a Pythagorean fuzzy interior hyperideal, we have
$\mu_{\mathcal{P}_1}(z) \nleq \inf\limits_{x \in a\star z\star b} \mu_{\mathcal{P}_1}(x)$

$\Longrightarrow \mu_{\mathcal{P}_1}(z) > \inf\limits_{x \in a\star z\star b} \mu_{\mathcal{P}_1}(x)$.
Then there exists some $t_\alpha \in IM(\mathcal{P})$ such that
$\mu_{\mathcal{P}_1}(z) > t_\alpha > \inf\limits_{x \in a\star z\star b} \mu_{\mathcal{P}_1}(x)$.
This implies that $\mu_{\mathcal{P}_1}(z) > t_\alpha$ and $\inf\limits_{x \in a\star z\star b} \mu_{\mathcal{P}_1}(x) < t_\alpha$.
i.e., $\mu_{\mathcal{P}_1}(z) > t_\alpha$ and $\mu_{\mathcal{P}_1}(x) < t_\alpha$ for $x \in a \star z \star b$
$\Longrightarrow a \star z \star b \notin \mathcal{P}_1^t$ and $z \in \mathcal{P}_1^t$.
Which is a contradiction to Equation (6). Hence $\mu_{\mathcal{P}_1}(z) \leq \inf\limits_{x \in a\star z\star b} \mu_{\mathcal{P}_1}(x)$.
Now, Since $\mathcal{P}_2^t$ is an interior ideal then for $a, b \in \mathscr{G}^\star$ and $z \in \mathcal{P}_2^t$ we have

$$a \star z \star b \in \mathcal{P}_2^t \tag{7}$$

Suppose if $\mu_{\mathcal{P}_2}(z) \ngeq \sup\limits_{x \in a\star z\star b} \mu_{\mathcal{P}_2}(x)$ then we have
$\mu_{\mathcal{P}_2}(z) < \sup\limits_{x \in a\star z\star b} \mu_{\mathcal{P}_2}(x)$.
Then there exists some $t_\beta \in IM(\mathcal{P})$ such that
$\mu_{\mathcal{P}_2}(z) < t_\beta < \sup\limits_{x \in a\star z\star b} \mu_{\mathcal{P}_2}(x)$.
Which implies that $\mu_{\mathcal{P}_2}(z) < t_\beta$ and $\sup\limits_{x \in a\star z\star b} \mu_{\mathcal{P}_2}(x) > t_\beta$.
$\Longrightarrow \mu_{\mathcal{P}_2}(z) < t_\beta$ and $\mu_{\mathcal{P}_2}(x) > t_\beta$ for $x \in a \star z \star b$
$\Longrightarrow a \star z \star b \notin \mathcal{P}_2^t$ and $z \in \mathcal{P}_2^t$.
Which is a contradiction to Equation (7). Hence $\mu_{\mathcal{P}_2}(z) \geq \sup\limits_{x \in a\star z\star b} \mu_{\mathcal{P}_2}(x)$.
Therefore $\mathcal{P}$ is a Pythagorean fuzzy interior hyperideal of $\mathscr{G}^\star$. □

Example 4.2. *Let us check that the following Pythagorean set is a Pythagorean fuzzy interior hyperideal of $\mathscr{G}^\star$ with the hyperoperation $(\bullet)$ by using Theorem 4.2.*
Let $\mathscr{G}^\star = \{f_1, f_2, f_3, f_4\}$ be the universe set with the hyperoperation $(\bullet)$.

$\bullet$	f_1	f_2	f_3	f_4
f_1	$\{f_1\}$	$\{f_1, f_2\}$	$\{f_1, f_3\}$	$\{f_1\}$
f_2	$\{f_1\}$	$\{f_1, f_2\}$	$\{f_1, f_3\}$	$\{f_1\}$
f_3	$\{f_1\}$	$\{f_1, f_2\}$	$\{f_1, f_3\}$	$\{f_1\}$
f_4	$\{f_1\}$	$\{f_1, f_2\}$	$\{f_1, f_3\}$	$\{f_1\}$

The Pythagorean fuzzy set $\mathcal{P}$ *is given by*
$\mathcal{P} = \{\frac{f_1}{(0.7, 0.4)}, \frac{f_2}{(0.7, 0.4)}, \frac{f_3}{(0.7, 0.4)}, \frac{f_4}{(0.6, 0.8)}\}$
The $t-$*level cut of* $\mathcal{P}$ *is given in Table 2.*

TABLE 2. $t-$ level cut of $\mathcal{P}$ ($\mathcal{P}^t$)

$\mathcal{P}^t/t$	0.4	0.6	0.7	0.8
$\mathcal{P}_1$	$\mathscr{G}^\star$	$\mathscr{G}^\star$	$\{f_1, f_2, f_3\}$	$\emptyset$
$\mathcal{P}_2$	$\{f_1, f_2, f_3\}$	$\{f_1, f_2, f_3\}$	$\{f_1, f_2, f_3\}$	$\mathscr{G}^\star$

It is enough to show that the sets $\mathscr{G}^\star$, $\{f_1, f_2, f_3\}$ *and* $\emptyset$ *are interior hyperideals.* $\mathscr{G}^\star$ *and* $\emptyset$ *are trivial. Let* $F = \{f_1, f_2, f_3\}$ *. Since*
$f_1 \star f_1 = f_2 \star f_1 = f_3 \star f_1 = f_1 \subseteq F$,
$f_1 \star f_2 = f_2 \star f_2 = f_3 \star f_2 = \{f_1, f_2\} \subseteq F$,
$f_1 \star f_3 = f_2 \star f_3 = f_3 \star f_3 = \{f_1, f_3\} \subseteq F$.
Therefore F *is a subsemihypergroup.*
$f_4 \star f_1 \star f_4 = f_4 \star f_2 \star f_4 = f_4 \star f_3 \star f_4 = f_1$. *Hence* F *is an interior hyperideal. From the Theorem 4.2 we can say that* $\mathcal{P}$ *is a Pythagorean fuzzy interior hyperideal of* $\mathscr{G}^\star$.

5. Coincidence of Pythagorean fuzzy hyperideal and Pythagorean fuzzy interior hyperideal

Lemma 5.1. *Every Pythagorean fuzzy hyperideal of* $\mathscr{G}^\star$ *is a Pythagorean fuzzy subsemihypergroup.*

Proof. Let $\mathcal{P} = (\mathcal{P}_1, \mathcal{P}_2)$ be a Pythagorean fuzzy hyperideal. For $a, b, x \in \mathscr{G}^\star$ we have

$$\mu_{\mathcal{P}_1}(a) \wedge \mu_{\mathcal{P}_1}(b) \leq \inf_{x \in a \star b} \mu_{\mathcal{P}_1}(x) \leq \inf_{x \in a \star b} \mu_{\mathcal{P}_1}(x). \tag{8}$$

and

$$\mu_{\mathcal{P}_2}(a) \vee \mu_{\mathcal{P}_2}(b) \geq \sup_{x \in a \star b} \mu_{\mathcal{P}_2}(x) \geq \sup_{x \in a \star b} \mu_{\mathcal{P}_2}(x). \tag{9}$$

From Equations (8) and (9) we have $\mathcal{P}$ is a Pythagorean fuzzy subsemihypergroup. Therefore every Pythagorean fuzzy hyperideal is a Pythagorean fuzzy subsemihypergroup. □

The converse part of the above Lemma is not true. It has been proved by the following example.

Example 5.1. *Let* $\mathscr{G}^\star = \{y_1, y_2, y_3, y_4, y_5, y_6, y_7\}$ *be a hypersemigroup with the hyperoperation* $\bullet$

$\bullet$	y_1	y_2	y_3	y_4	y_5	y_6	y_7
y_1	$\{y_1\}$	$\{y_1\}$	$\{y_1\}$	$\{y_1\}$	$\{y_1\}$	$\{y_1\}$	$\{y_1\}$
y_2	$\{y_1\}$	$\{y_2\}$	$\{y_2, y_3\}$	$\{y_4\}$	$\{y_4, y_5\}$	$\{y_6\}$	$\{y_6, y_7\}$
y_3	$\{y_1\}$	$\{y_3\}$	$\{y_3\}$	$\{y_5\}$	$\{y_5\}$	$\{y_7\}$	$\{y_5\}$
y_4	$\{y_1\}$	$\{y_4\}$	$\{y_4, y_5\}$	$\{y_4\}$	$\{y_4, y_5\}$	$\{y_4\}$	$\{y_4, y_5\}$
y_5	$\{y_1\}$	$\{y_5\}$	$\{y_5\}$	$\{y_5\}$	$\{y_5\}$	$\{y_5\}$	$\{y_5\}$
y_6	$\{y_1\}$	$\{y_6\}$	$\{y_6, y_7\}$	$\{y_4\}$	$\{y_4, y_5\}$	$\{y_6\}$	$\{y_6, y_7\}$
y_7	$\{y_1\}$	$\{y_7\}$	$\{y_7\}$	$\{y_5\}$	$\{y_5\}$	$\{y_7\}$	$\{y_7\}$

Define a Pythagorean fuzzy set $\mathcal{P}$ *as*
$\mathcal{P} = \{\frac{y_1}{(0.1, 1)}, \frac{y_2}{(0.1, 1)}, \frac{y_3}{(0.2, 0.7)}, \frac{y_4}{(0.5, 0.6)}, \frac{y_5}{(0.5, 0.6)}, \frac{y_6}{(0.9, 0.3)}, \frac{y_7}{(0.9, 0.3)}\}$.

$\mathcal{P}$ satisfies the conditions of (i) and (ii) in Definition 4.1. Hence we can say that $\mathcal{P}$ is a Pythagorean fuzzy subsemihypergroup. But $\mathcal{P}$ is not a Pythagorean fuzzy hyperideal of $\mathscr{G}^\star$. Since
$\mu_{\mathcal{P}_1}(y_4) = 0.5 \not\leq 0.1 = \inf\limits_{\mathscr{G}^\star \in y_1 \star y_4} \mu_{\mathcal{P}_1}(\mathscr{G}^\star)\ \forall\ y_1 \in \mathscr{G}^\star$
$\mathcal{P}_1$ is not a Pythagorean fuzzy left hyperideal.
$\mu_{\mathcal{P}_2}(y_3) = 0.7 \not\geq 0.1 = \sup\limits_{\mathscr{G}^\star \in y_1 \star y_3} \mu_{\mathcal{P}_2}(\mathscr{G}^\star)\ \forall\ y_1 \in \mathscr{G}^\star$
$\mathcal{P}_2$ is not a Pythagorean fuzzy left hyperideal.
$\mu_{\mathcal{P}_1}(y_4) = 0.5 \not\leq 0.1 = \inf\limits_{\{y_1, y_4, y_6\} \in y_4 \star y_1} \{\mu_{\mathcal{P}_1}(y_1), \mu_{\mathcal{P}_1}(y_4), \mu_{\mathcal{P}_1}(y_6)\}\ \forall\ y_1 \in \mathscr{G}^\star$
$\mathcal{P}_1$ is not a Pythagorean fuzzy right hyperideal.
$\mu_{\mathcal{P}_2}(y_6) = 0.3 \not\geq 1 = \sup\limits_{\{y_1, y_4, y_5, y_6, y_7\} \in y_6 \star y_1} \{\mu_{\mathcal{P}_2}(y_1), \mu_{\mathcal{P}_2}(y_4), \mu_{\mathcal{P}_2}(y_5), \mu_{\mathcal{P}_2}(y_6), \mu_{\mathcal{P}_2}(y_7)\}\ \forall\ y_1 \in \mathscr{G}^\star$
$\mathcal{P}_2$ is not a Pythagorean fuzzy right hyperideal.

Lemma 5.2. *Every Pythagorean fuzzy hyperideal of $\mathscr{G}^\star$ is a Pythagorean fuzzy interior hyperideal of $\mathscr{G}^\star$.*

Proof. Let $\mathcal{P} = (\mathcal{P}_1, \mathcal{P}_2)$ be a Pythagorean fuzzy hyperideal. For $a, b, x, z \in \mathscr{G}^\star$ we have

$$\begin{aligned} \inf_{x \in a \star z \star b} \mu_{\mathcal{P}_1}(x) &= \inf_{x \in (a \star z) \star b} \mu_{\mathcal{P}_1}(x) \\ &= \inf_{x \in (a \star z)} \mu_{\mathcal{P}_1}(x) \\ &\geq \mu_{\mathcal{P}_1}(z) \end{aligned}$$

Therefore $\mathcal{P}_1$ is a Pythagorean fuzzy interior hyperideal.
Now consider

$$\begin{aligned} \sup_{x \in a \star z \star b} \mu_{\mathcal{P}_2}(x) &= \sup_{x \in (a \star z) \star b} \mu_{\mathcal{P}_2}(x) \\ &= \sup_{x \in (a \star z)} \mu_{\mathcal{P}_2}(x) \\ &\leq \mu_{\mathcal{P}_2}(z) \end{aligned}$$

Therefore $\mathcal{P}_2$ is a Pythagorean fuzzy interior hyperideal.

□

The following example shows that every Pythagorean fuzzy interior hyperideal of $\mathscr{G}^\star$ need not be a Pythagorean fuzzy hyperideal of $\mathscr{G}^\star$.

Example 5.2. *Let $\mathscr{G}^\star = \{p_1, p_2, p_3, p_4\}$ be a hypersemigroup with the hyperoperation $\bullet$*

$\bullet$	p_1	p_2	p_3	p_4
p_1	$\{p_2, p_4\}$	$\{p_3, p_4\}$	$\{p_4\}$	$\{p_4\}$
p_2	$\{p_3, p_4\}$	$\{p_2\}$	$\{p_4\}$	$\{p_4\}$
p_3	$\{p_4\}$	$\{p_4\}$	$\{p_4\}$	$\{p_4\}$
p_4	$\{p_4\}$	$\{p_4\}$	$\{p_4\}$	$\{p_4\}$

The Pythagorean fuzzy set $\mathcal{P}$ is defined by
$\mathcal{P} = \{\frac{p_1}{(0.2, 1)}, \frac{p_2}{(0.5, 0.6)}, \frac{p_3}{(0.2, 1)}, \frac{p_4}{(0.9, 0.3)}\}$
The $t-$level cut of $\mathcal{P}$ is given by

TABLE 3. $t-$ level cut of $\mathcal{P}$ ($\mathcal{P}^t$)

$\mathcal{P}^t/t$	0.2	0.3	0.5	0.6	0.9	1
$\mathcal{P}_1$	$\mathscr{G}^\star$	$\{p_2, p_4\}$	$\{p_2, p_4\}$	$\{p_4\}$	$\{p_4\}$	$\emptyset$
$\mathcal{P}_2$	$\emptyset$	$\{p_4\}$	$\{p_4\}$	$\{p_2, p_4\}$	$\{p_2, p_4\}$	$\mathscr{G}^\star$

The set $\{p_4\}$ is a hyperideal but $\{p_2, p_4\}$ is not a hyperideal. Since for $p_1 \in \mathscr{G}^\star$ and $p_2 \in \{p_2, p_4\}$ then we have $p_1 \star p_2 = \{p_3, p_4\} \not\subseteq \{p_2, p_4\}$. Thus $\mathcal{P}$ is not a Pythagorean interior hyperideal of $\mathscr{G}^\star$

Definition 4.2 says that every Pythagorean fuzzy interior hyperideal is a Pythagorean fuzzy subsemihypergroup. But every Pythagorean fuzzy subsemihypergroup need not be a Pythagorean fuzzy interior hyperideal of $\mathscr{G}^\star$.

Example 5.3. *Let $\mathscr{G}^\star = \{k_1, k_2, k_3, k_4\}$ be a hypersemigroup with hyperoperation $\bullet$*

$\bullet$	k_1	k_2	k_3	k_4	k_5
k_1	$\{k_1\}$	$\{k_1, k_2, k_4\}$	$\{k_1\}$	$\{k_1, k_2, k_4\}$	$\{k_1, k_2, k_4\}$
k_2	$\{k_1\}$	$\{k_2\}$	$\{k_1\}$	$\{k_1, k_2, k_4\}$	$\{k_1, k_2, k_4\}$
k_3	$\{k_1\}$	$\{k_1, k_2\}$	$\{k_1, k_3\}$	$\{k_1, k_2, k_4\}$	$\mathscr{G}^\star$
k_4	$\{k_1\}$	$\{k_1, k_2\}$	$\{k_1\}$	$\{k_1, k_2, k_4\}$	$\{k_1, k_2, k_4\}$
k_5	$\{k_1\}$	$\{k_1, k_2\}$	$\{k_1, k_3\}$	$\{k_1, k_2, k_4\}$	$\mathscr{G}^\star$

The Pythagorean fuzzy set $\mathcal{P}$ is given by
$\mathcal{P} = \{\frac{k_1}{(0.7, 0.6)}, \frac{k_2}{(0.1, 1)}, \frac{k_3}{(0.7, 0.6)}, \frac{k_4}{(0.8, 0.5)}\}$
$\mathcal{P}$ is a Pythagorean fuzzy subsemihypergroup. But $\mathcal{P}$ is not a Pythagorean fuzzy interior hyperideal. Since
$\mu_{\mathcal{P}_1}(k_4) = 0.8 \not\leq \inf_{\{k_1,k_4\}\subseteq k_1\star k_4\star k_1} \{\mu_{\mathcal{P}_1}(k_1), \mu_{\mathcal{P}_1}(k_4)\}$.
Hence every Pythagorean fuzzy subsemigroup need not be a Pythagorean fuzzy interior hyperideal of $\mathscr{G}^\star$.

The following diagram explains the relation between Pythagorean fuzzy hyperideal, Pythagorean fuzzy subsemihypergroup and Pythagorean fuzzy interior hyperideal of $\mathscr{G}^\star$.

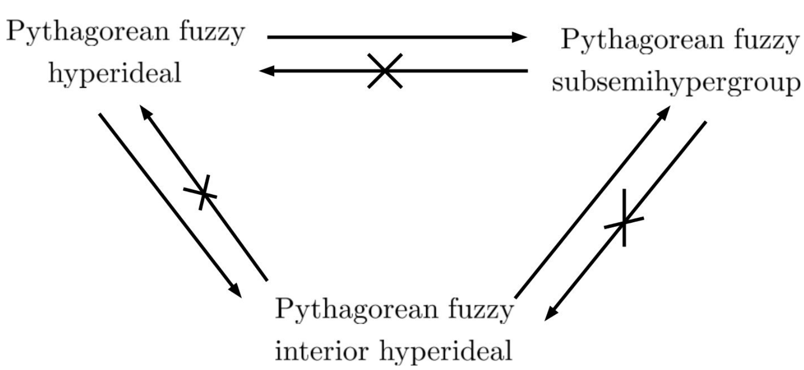

FIGURE 1. Diagrammatic representation

6. Conclusion

Pythagorean fuzzy set is one of the finest tool to deal with uncertainities. In this paper we applied Pythagorean fuzzy set in algebraic hyper-structures. We defined Pythagorean fuzzy hyperideal and explained with detailed example. We showed that the union and intersection of Pythagorean fuzzy hyperideals is also a Pythagorean fuzzy hyperideal. Then we introduce Pythagorean fuzzy subsemihypergroup and Pythagorean fuzzy interior hyperideal in hypersemigroup. We also investigated that every Pythagorean fuzzy hyperideal is a Pythagorean fuzzy interior hyperideal. In future work we apply the concept of pythagorean fuzzy set in ideal theory of ternary hypersemigroups.

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