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THE HUB NUMBER OF A FUZZY GRAPH

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Abstract. In this paper, we introduced the concepts of hub number in fuzzy graph and is denoted by $h(G)$. The hub number of fuzzy graph G is the minimum fuzzy cardinality among all minimal fuzzy hub sets . We determine the hub number $h(G)$ for several classes of fuzzy graph and obtain Nordhaus-Gaddum type results for this parameter. Further, some bounds of $h(G)$ are investigated. Also the relations between $h(G)$ and other known parameters in fuzzy graphs are investigated.

Keywords: Graphs, Fuzzy graph, Hub number in graph and Hub number in fuzzy graph.

AMS Subject Classification: 03E72, 05C69, 05C72.

1. Introduction

A graph is a mathematical representation of a network and it describes the relationship between vertices and edges. Graph theory is used to represent real-life phenomena, but sometimes graphs are not able to properly represent many phenomena because uncertainty of different attributes of the systems exists naturally. Many real-world phenomena provided motivation to define the fuzzy graphs. Kauffman [2] introduced fuzzy graphs using Zadeh's fuzzy relation [19]. Fuzzy-graph theory is growing rapidly, with numerous applications in many domains, including networking, communication, data mining, clustering, image capturing, image segmentation, planning, and scheduling.

The origin of graph theory dates back to Euler's solution of the puzzle of Koigsberg's bridges in 1736 [1]. Graph theory has numerous applications to problems in systems analysis, operations research, transportation, and economics. However, in many cases, some aspects of a graph theoretic problem may be uncertain. For example, the vehicle travel time or vehicle capacity on a road network may not be known exactly. In such cases, it is natural to deal with the uncertainty using fuzzy set theory. The fuzzy graph theory as a

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generalization of Euler's graph theory was first introduced by Rosenfeld [6] in 1975. Up to the present, fuzzy graphs have been studied by some researchers [12, 3, 7, 8, 9, 10, 11]. For example, Poulik and Chorai [13] introduced the detour g-interior nodes and detour g-boundary nodes in bipolar fuzzy graph with applications.
Also, Poulik and Chorai [15] introduced the certain indices of graphs under bipolar fuzzy environment with applications.

The note on Bipolar fuzzy graphs with applications introduced by Poulik and Chorai in 2020 [14].
In 2020, Poulik et al. [16] introduced the Pragmatic results in Taiwan education system based $IVFG$ and $IVNG$.
The determination of journeys order based on graph's Wiener absolute index with bipolar fuzzy information introduced by Poulik and Chorai in 2021 [18].
Poulik and Chorai 2020 [17] introduced the empirical results on operations of bipolar fuzzy graphs. Consider the fuzzy graphs that represent transportation networks that is vertices can be taken to be locations or destination and an edge exists between two vertices precisely when there is an "easy passage" between the corresponding locations for example, a city's network of streets, with vertices representing intersection or other points of intersect and edges road segments we are connected with a certain kind of connectivity specifically we want a set such that any traffic between disparate points in our network passes solely through vertices in this street. The hub number was introduced by Matthew, W., in 2006 [4]. This motivated us to introduce the concepts of hub number of fuzzy graphs.

2. DEFINITIONS

In this section, we review briefly some definitions in graphs, fuzzy graphs, hub number in graphs and domination number in fuzzy graphs.
The vertex sets and the edges set of G are denoted by $V(G)$ and $E(G)$ respectively, with order $p = |V|$ and size $q = |E|$.
A graph is a pair (V, E), where V is a set and E is a relation on V. The elements of V are thought of as vertices of the graph and the elements of E are thought of as the edges.[3]
Suppose that $H \subseteq V(G)$ and let $x, y \in V(G)$. An H-path between x and y is a path where all intermediate vertices are from H. (This includes the degenerate cases where the path consists of the single edge xy or a single vertex x if $x = y$, call such an H-path trivial). A set $H \subseteq V(G)$ is a hub set of G if it has the property that, for any $x, y \in V(G) - H$, there is an H-path in G between x and y. The minimum cardinality of a hub set in G is called a hub number of G, and is denoted by $h(G)$ [4].
A path P of a graph $G = (V, E)$ is a graph which all vertices are distinct.
A graph $G = (V, E)$ is called star graph if $V(G) = \{u\} \cup \{v_i\}$, where $v_i = V - \{u\}$ and denoted by $K_{1,m}$.[3]
A stare graph $G = (V, E)$ is called a wheel graph if $V(G) = \{u\} \cup \{V_i\}$, where $V_i = V - \{u\}$ and all vertex are connected.[3]
A graph $G = (V, E)$ is called connected if each pair of vertices in G belong to a path; otherwise, G is not connected.
A fuzzy graph $G = (\mu, \rho)$ is a set V with two function $\mu, V \to [0, 1]$ and $\rho, E \to [0, 1]$ such that $\rho(\{u, v\}) \leq \mu(u) \wedge \mu(v)$ for all $u, v \in V$. We write $\rho(\{u, v\})$ for $\rho(u, v)$.[3]
The order p and size q of a fuzzy graph $G = (\mu, \rho)$ are defined to be $p = \sum_{u \in V} \mu(u)$ and $q = \sum_{(u,v) \in E} \rho(u, v)$.
A path P in a fuzzy graph $G = (\mu, \rho)$ is a sequence of distinct vertices $v_0, v_1, v_2, ..., v_n$ (except possibly v_0 and v_n) such that $\mu(v_i) > 0,\ \rho(v_{i-1},\ v_i) > 0,\ 0 \leq i \leq 1$. Here $n \geq 1$

is called the length of the path P. We say a path P is a cycle if $v_0 = v_n$ and $n \geq 3$.
A fuzzy graph $G = (\mu, \rho)$ is a star if and only if $G^* = (\mu^*, \rho^*)$ is a star and is denoted by $K_{\mu(u),\ \mu(v_i)}$, where $V = u \cup v_i$, $i = 1, 2, ..., n-1$.
A fuzzy graph $G = (\mu, \rho)$ is a wheel if and only if $G^* = (\mu^*, \rho^*)$ is a wheel and is denoted by $W_{\mu(u),\ \mu(v_i)}$, where $V = u \cup v_i$, where $V_i = V - \{u\}, u$ is a root.
A fuzzy graph $G = (V, \mu)$ is called complete fuzzy graph if $\rho(u, v) = \mu(u) \wedge \mu(v)$ for all $u, v \in V$.
A fuzzy graph G is said to be bipartite fuzzy graph if the vertex set V can be partitioned in to two nonempty sets V_1 and V_2 such that $\rho(u, v) = 0$ if $u, v \in V_1$ or $u, v \in V_2$.
We say that a bipartite fuzzy graph is complete bipartite fuzzy graph if $\rho(u, v) = \mu(u) \wedge \mu(v)$ for all $u \in V_1, v \in V_2$.
Let $G = (\mu, \rho)$ be a fuzzy graph. Then the degree of vertex $v \in V(G)$ is defined as $d(v) = \sum_{u \neq v} \rho(u, v)$. The maximum degree of G is $\Delta(G) = \vee\{d(v), v \in V\}$, and the minimum degree of G is $\delta(G) = \wedge\{d(v), v \in V\}$.
Let $G = (\mu, \rho)$ be a fuzzy graph and let $v \in V(G)$. Then $N(v) = \{u \in V, \rho(u, v) = \mu(u) \wedge \mu(v)\}$ is called the open neighborhood set of v and $N[v] = N(v) \cup \{v\}$ is called the closed neighborhood set of v.
Let $G = (\mu, \rho)$ be a fuzzy graph and $v \in V(G)$. Then the neighborhood degree of v is defined as $d_N(v) = \sum \mu(u), u \in N(v)$.
The minimum neighborhood degree of a fuzzy graph G is $\delta_N(G) = min\{d_N(v), v \in V\}$ and the maximum neighborhood degree of G is $\Delta_N(G) = max\{d_N(v), v \in V\}$.
The effective degree of a vertex v in fuzzy graph is denoted by $d_E(V) = \sum_{u \neq v} \rho(u, v)$
The maximum degree taken of all effective degree is denoted by $\Delta_E(G) = max\{d_E(v), v \in V(G)\}$ [3].
The minimum degree of G taken of an effective degree is denoted by $\delta_E(G) = min\{d_E(v), v \in V\}$.
A subset D of V is called a dominating set of G if for every $v \in V - D$ there exists $u \in D$ such that u dominates v[5].
A dominating set D of a fuzzy graph G is called minimal dominating set if $D - \{v\}$ is not dominating set of G for all $v \in D$ [3].
The minimum fuzzy cardinality taken over all minimal dominating set in fuzzy graph G is called domination number of G and denoted by $\gamma(G)$.
A fuzzy graph $G = (V, \mu, \rho)$ is called connected fuzzy graph if each pair of vertices in G belong to a path; otherwise, G is not connected.
Let D is a $\gamma - set$ then D is connected dominating set if the fuzzy subgraph $< D >$ induced by D is connected.
The connected domination number of a fuzzy graph G is the minimum cardinality taken over all connected dominating set of G and is denoted by $\gamma_c(G)$.

3. The Hub Number of a Fuzzy Graph

The aim of this section is to introduce and study the concepts of hub number of a fuzzy graphs.

Definition 3.1. *Let $G = (V, \mu, \rho)$ be a fuzzy graph, a vertex subset D of $V(G)$ is said to be a fuzzy hub set of G if for any pair of vertices nonadjacent outside of D their is a fuzzy path between them, whose all intermediate vertices are in D.*

Definition 3.2. *A fuzzy hub set D of a fuzzy graph G is called minimal hub set if $D - \{v\}$ is not hub set of G for all $v \in D$.*

Definition 3.3. *The minimum fuzzy cardinality among all minimal fuzzy hub sets in a fuzzy graph G is called the hub number of G and is denoted by $h(G)$.*

Example 3.1. *Let $G = (V, \mu, \rho)$ be a fuzzy graph given in Figure(3.1), where $V = \{v_1, v_2, v_3, v_4, v_5\}, \mu(v_1) = 0.1,\ \mu(v_2) = 0.3,\ \mu(v_3) = 0.4,\ \mu(v_4) = 0.9,\ \mu(v_5) = 0.6$ and $\rho(u, v) = \mu(u) \wedge \mu(v)\ \forall\ (u, v) \in \rho*$.*

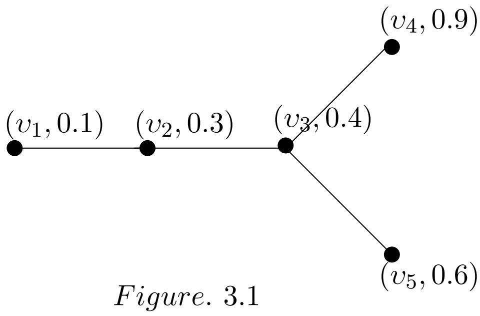

Figure. 3.1

We see that $h(G) = 0.7$.

In the following results we give $h(G)$ for some standard fuzzy graphs, we begin with the complete fuzzy graph K_μ.

Theorem 3.1. *If $G = (V, \mu, \rho)$ is a complete fuzzy graph. Then $h(G) = 0$.*

Example 3.2. *Let $G = (V, \mu, \rho)$ be a complete fuzzy graph given in Figure(3.2), where $V = \{v_1, v_2, v_3, v_4\},\ \mu(v_1) = 0.3,\ \mu(v_2) = 0.1,\ \mu(v_3) = 0.5,\ \mu(v_4) = 0.4$ and $\rho(u, v) = \mu(u) \wedge \mu(v)\ \forall u, v \in V$.*

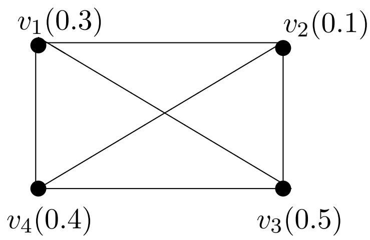

Figure. 3.2.

We see that $h(G) = 0$.

The following theorem gives $h(G)$ of the complete bipartite fuzzy graphs $K_{\mu_1,\ \mu_2}$.

Theorem 3.2. *If $G = K_{\mu_1, \mu_2}$ is a complete bipartite fuzzy graph. Then*

$$h(K_{\mu_1,\ \mu_2}) = \begin{cases} min\{\mu(v), v \in V_1\}, & if\ x, y \in V_2, \\ min\{\mu(u), u \in V_2\}, & if\ x, y \in V_1, \\ min\{\mu(u), u \in V_1\} + min\{\mu(v), v \in V_2\} & if\ \ x \in V_1,\ y \in V_2. \end{cases}$$

Proof. Let G be a complete bipartite fuzzy graph. Then we have three cases.
Case 1: If $x, y \in V_1$. Then their exits a path between x and y containing only one vertex from V_2. Hence, $h(K_{\mu_1,\mu_2}) = \min\{\mu(v), v \in V_2\}$.
Case 2: If $x, y \in V_2$. Then their exits a path between x and y containing only one vertex from V_1. Hence, $h(K_{\mu_1,\mu_2}) = \min\{\mu(u), u \in V_1\}$.
Case 3: If $x \in V_1, y \in V_2$. Then their exits a path between x and y containing a vertex of V_1 and vertex of V_2. Hence, $h(K_{\mu_1,\mu_2}) = min\{\mu(u); u \in V_1\} + min\{\mu(v), v \in V_2\}\ if\ x \in V_1, y \in V_2$. □

Theorem 3.3. *If G is any fuzzy graph. Then*
(i) $h(G) < p$;
(ii) $h(\bar{G}) < p$.

Proof. Proof follows from the definition. □

Theorem 3.4. *For any fuzzy graph G,*

$$h(G) + h(\bar{G}) < 2p.$$

Proof. By Theorem 3.3, we get the result. □

Theorem 3.5. *For any cycle fuzzy graph G,*

$$h(G) = min(\sum_{i=1}^{n-2} \mu(v_i)),\ n \geq 4.$$

Example 3.3. *Let G be a fuzzy graph given in the Figure 3.3, where* $V(G) = \{v_1, v_2, v_3, v_4, v_5\}$, $\mu(v_1) = 0.1, \mu(v_2) = 0.2, \mu(v_3) = 0.3, \mu(v_4) = 0.4,\ \mu(v_5) = 0.5$ *and* $\rho(u, v) = \mu(u) \wedge \mu(v)\ \forall\ (u, v) \in \rho*$.

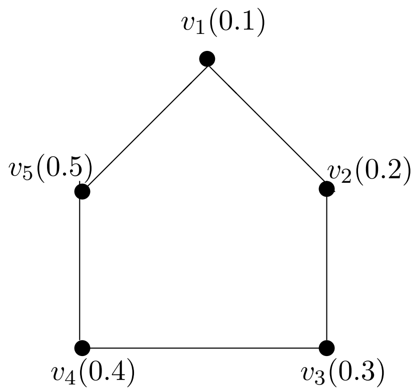

Figure. 3.3

We see that $h(G) = 0.6$.

Theorem 3.6. *Let $G = (V, \mu, \rho)$ be any cycle fuzzy graph. Then*

$$h(G) \leq \gamma(G).$$

Proof. Let G be a cycle fuzzy graph and let S be a dominating set of G, since G is cycle. Then for any $u, v \in V(G) - S$ their is an $S - path$ between them. Thus we conclude. □

Theorem 3.7. *For any star fuzzy graph G,*

$$h(G) = \{\mu(u), u\ is\ a\ root\}.$$

Proof. Let G be any star fuzzy graph and $S = \{u, u\ is\ a\ root\}$. Since u is root. Then u is an intermediate vertices in every path joining any two vertices in $V(G) - S$. Then S is an $S - path$ set. Hence, $h(G) = \{\mu(u) : u\ is\ a\ root\}$. □

Theorem 3.8. *For any wheel fuzzy graph G,* $h(G) = \{\mu(u), u\ is\ a\ root\}$.

Example 3.4. *Let* $G = (V, \mu, \rho)$ *be a wheel fuzzy graph given in the Figure (3.4), where* $V = \{v_1, v_2, v_3, v_4, v_5\}$, $\mu(v_1) = 0.3$, $\mu(v_2) = 0.4$, $\mu(v_3) = 0.4$, $\mu(v_4) = 0.5$, $\mu(v_1) = 0.3$ *and* $\rho(u, v) = \mu(u) \wedge \mu(v)$, $\forall\ (u, v) \in \rho*$.

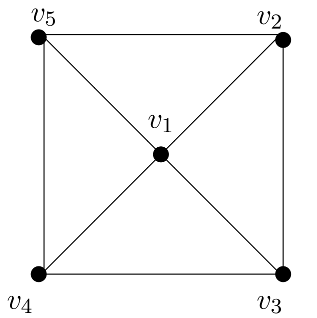

Figure. 3.4

We see that $S = \{v_1\}$ is a minimal hub set of G with $h(G) = 0.3$.

Definition 3.4. *A hub set D of a fuzzy graph* $G = (V, \mu, \rho)$ *is called connected hub set of G if the fuzzy subgraph* $< D >$ *induced by D is connected.*

Definition 3.5. *The minimum fuzzy cardinality taken over all connected hub sets in G is called connected hub number of G and is denoted by* $h_c(G)$.

Theorem 3.9. *For any connected fuzzy graph G,*

$$h_c(G) \leq \gamma_c(G).$$

Proof. Let G be a connected fuzzy graph and let S be a connected dominating set of G, since G is connected. Then for any two vertices outside S their is an $S - path$ between them. Therefor S is a connected hub set. Hence,

$$h_c(G) \leq \gamma_c(G).$$

□

Theorem 3.10. *For any connected fuzzy graph G,*

$$\gamma_c(G) \leq h(G) + 1.$$

Proof. Let G be a connected fuzzy graph, S be a hub set of G, and D be vertices subset of $V(G) - S$. Then for any two vertices $u, v \in D$ those u and v only form $S - path$ must therefor be trivial. Since we know that there are $S - pathes$ between all pairs of vertices in $V - S$, this implies that $G(D)$ is complete. Therefore $S \cup \{D\}$ must be a dominating set for any $v \in D$. □

Theorem 3.11. *Let G be a complete fuzzy graph. Then*

$$\triangle(G) = p - t, t = min\{\mu(v), v \in V(G)\} \ if \ and \ only \ if \ h(G) = t.$$

Proof. Let G be a complete fuzzy graph such that, $\triangle(G) = p-t, t = min\{\mu(v), v \in V(G)\}$. Suppose v is a vertex of G with $deg(v) = p - t$, since G is a complete fuzzy graph. Then every pair of vertices of $V - \{v\}$ are connected by a path whose intermediate vertex is v. Therefore $S = \{v\}$ is a hub set of G. Hence, $h(G) = t$. Conversely, suppose $h(G) = t$ and $S = \{v\}$ be a minimum hub set of G, since G is complete fuzzy graph. Then every hub set is a dominating set, it follows that v is dominates all other vertices in G. Since v has the minimum fuzzy cardinality. Then $\triangle(G) = p - t$. □

Theorem 3.12. *For any fuzzy graph G. Further, equality hold if G is complete.*
(i) $h(G) \leq p - \triangle_N(G)$;
(ii)$h(\bar{G}) < p - \triangle_N(\bar{G})$.

Proof. Let G be any fuzzy graph and let $v \in V(G)$ is a vertex such that $d_N(G) = \Delta_N(G)$. Then $V - N(v)$ is a hub set of G. Hence, $h(G) \leq |V - N(v)| = p - \triangle_N(G)$. If G is a complete fuzzy graph. Then $|V - N(v)| = |S|, S$ is a hub set contains only one vertex. Therefore, $h(G) = p - \triangle_N(G)$. Since $h(\bar{G}) < h(G)$. Then (ii) hold. □

Theorem 3.13. *For any fuzzy graph G,*

$$h(G) \leq p - \delta_N.$$

Proof. Let $v \in V$ with $d_N(v) = \delta_N$. Suppose S be a hub set of G. Clearly $v \in S$ and $S \subseteq V - N(v)$. Therefore, $h(G) = |S| \leq |V - N(v)| = p - |N(v)| \leq p - \delta_N$. □

Corollary 3.1. *For any fuzzy graph G,*

$$h(G) \leq p - \delta_E.$$

Proof. Clearly $\Delta_E \leq \Delta_N$, $\delta_E \leq \delta_N$ and by the above Theorem. Then

$$h(G) \leq p - \delta_E.$$

□

Theorem 3.14. *Let G be a fuzzy graph without isolated vertices. Then*

$$h(G) \leq \frac{2p}{\Delta(G) + 1}.$$

Proof. Let G be a fuzzy graph have no isolated vertices and let D be a hub set of G. Further, let $t = \sum \rho(e)$, where e is an edge in G which having one vertex in D and the other in $V - D$. Since $\Delta(G) \geq deg(v), \forall \ v \in D$, and each vertex in D has at least one neighbor in D, we have $t \geq (\Delta(G) + 1)|D| = (\triangle(G) + 1)h(G)$. Also, since each vertex in $V - D$ is adjacent to at least two vertices in D, we have $t \leq 2|V - D| = 2(p - h(G))$. Hence, $2(p - h(G)) \geq (\triangle(G) + 1)h(G)$, which reduces the bound of the Theorem. □

Theorem 3.15. *For any connected fuzzy graph bath G and $\bar{G}$*

$$h(G) + h(\bar{G}) < p + t, \ t = max\{\mu(v), \forall \ v \in V(G)\}.$$

Proof. From Theorem, $h(G) < p - \triangle(G)$ and $h(\bar{G}) < p - \Delta(\bar{G})$. We have

$$\begin{aligned} h(G) + h(\bar{G}) &< p - \triangle(G) + p - \triangle(\bar{G}) \\ &= 2p - (\triangle(G) + \triangle(\bar{G})) \\ &= 2p - (\triangle(G) + p - t - \delta(G)) \\ &= p - t - ((\triangle(G) + \delta(G)). \end{aligned}$$

Since $((\triangle(G) + \delta(G)) \geq 0$, we have

$$h(G) + h(\bar{G}) < p + t, \; t = max\{\mu(v), \forall \; v \in V(G)\}.$$

□

4. Conclusions

In this paper, we introduced the concept of the hub number in fuzzy graphs. We obtained the bounds and some properties for hub number of fuzzy graphs. Relationships between hub number of fuzzy graphs and some other parameters were established.

5. Future Work

For future studies, we propose some further research problems on the hub number in product fuzzy graph. It would interesting to extend the work of this research to solve external problems of certain product fuzzy graph and the hub number in bipolar fuzzy graphs.

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