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MIDDLE GRAPH OF SEMIRING VALUED GRAPHS

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Abstract. In this paper, we define middle graph of semiring valued graph $M(G^S)$ and study the regularity of $M(G^S)$ where G^S is the semiring valued graph (or simply S-valued graph).

Keywords: Middle graph, S-valued graph, regularity.

AMS Subject Classification: 05C22, 05C25, 05C38.

1. Introduction

The middle graph $M(G)$ of a graph G is an intersection graph $\Omega(F)$ on the vertex set $V(G)$ of any graph G. Let $E(G)$ be an edge set of G and $F = V'(G) \cup E(G)$, where $V'(G)$ indicates the family of all one vertex subsets of the set $V(G)$. This concept was introduced by T. Hamada and I. Yoshimura [1] and studied by V.R. Kulli and H.P. Patil.

Jonathan S. Golan, was the first person who introduced the notion of S-valued graphs where he defined a function $g : V \times V \to S$ such that $g(v_1, v_2) \neq \emptyset$. Here V is the vertex set of a graph G and S is a semiring. Golan consider the S-valued graph by assinging values to the edges only. Further, M.Rajkumar, S. Jeyalakshmi and M. Chandramouleeswaran precisely studied the graphs whose vertices and edges are assigned values from the semiring. However, they assign values to every vertex of G and the edges of G are assigned values according to the minimum value of vertices incident with the edges.

This motivated us to study the middle graph of graphs whose vertices and edges are assigned values from the semiring S. However, we assign values to every vertex of a middle graph of S-valued graph as same as in S-valued graph whenever vertex lies in G. Otherwise we assign the value of the vertex to be the value of its corresponding edge. We also assign the value for edges in $M(G)$ in relation to values of vertices incident with that edges.

2. Basic definitions

In this section, we recall some basic definitions from the theory of semirings and S-valued graphs. [2].

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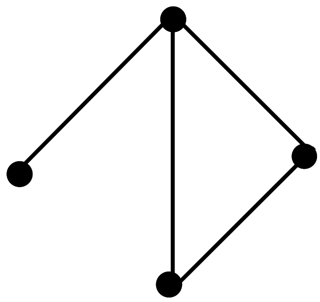
Definition 2.1. [1] *Let $G = (V, E)$ be a graph with $E \neq \emptyset$. The middle graph of a graph G, denoted by $M(G)$, is the graph whose vertex set is V_M and edge set is E_M, where $V_M = V \cup \{e_i^j = [v_i, v_j] : (v_i, v_j) \in E\}$ and $E_M = \{(e, f) : e$ and f are adjacent$\}$.*
Note: Adjacent in the sense that the corresponding edges are adjacent in G (in case of both vertices are edges). Otherwise, one is a vertex and the other is an edge incident with it.

Example 2.1. *Middle graph of the graph G.*



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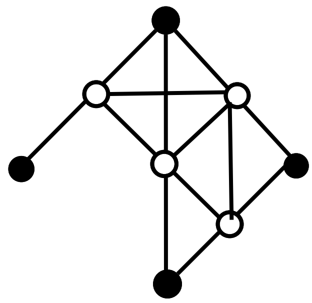


$M(G)$

Definition 2.2. *A semiring $(S, +, \cdot)$ is an algebraic system with a non-empty set S together with two binary operation $+$ and . such that*
1. $(S, +)$ is a commutative monoid.
2. $(S, \cdot)$ is a semigroup.
3. For all $a, b, c \in S, a \cdot (b + c) = a \cdot b + a \cdot c$ and $(a + b) \cdot c = a \cdot c + b \cdot c$.
4. $0 \cdot x = x \cdot 0 = 0$, $\forall x \in S$.

Example 2.2. $(M_{2\times2}(\mathbb{R}^+), +, \cdot)$ - *Set of all 2×2 matrices whose entires are positive real numbers forms semiring under matrix addition and matrix multiplication which is not a ring.*

Definition 2.3. *Let $(S, +, \cdot)$ be a semiring. $\preceq$ is said to be a Canonical preorder if for $a, b \in S, a \preceq b$ if and only if there exist $c \in S$ such that $a + c = b$.*

Example 2.3. *Let us take a semiring $\mathbb{N} \cup \{0\}$. $1 \preceq 2$ because there exists $1 \in \mathbb{N} \cup \{0\}$ such that $1 + 1 = 2$. But $1 \npreceq 0$.*

Example 2.4. *Let $(S, +, \cdot)$ be a semiring with binary operations '+'and '.'defined by the following Cayley tables.*

+	0	a	b	c
0	0	a	b	c
a	a	a	b	c
b	b	b	b	c
c	c	c	c	b

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	a	a	a
c	0	a	a	a

Clearly, $0 \preceq 0, 0 \preceq a, 0 \preceq b, 0 \preceq c, a \preceq a, a \preceq b, a \preceq c, b \preceq b, b \preceq c, c \preceq c, c \preceq b$.

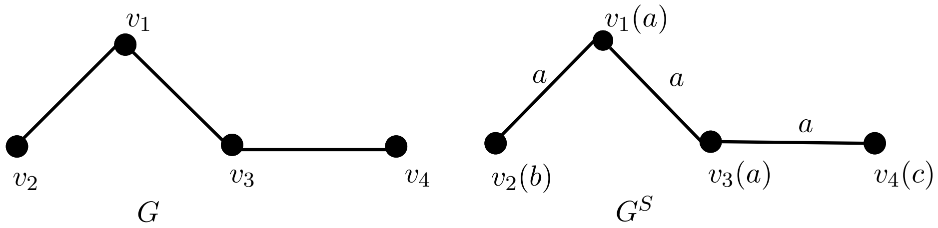
Definition 2.4. *Let $G = (V,E)$ be given graph with both $V,E \neq \emptyset$. For any semiring $(S,+,\cdot)$, a semiring-valued graph (or a S- valued graph), G^S, is defined to be the graph $G^S = (V, E, \sigma, \psi)$, where $\sigma : V \to S$ and $\psi : E \to S$ is defined to be*

$$\psi(x, y) = \begin{cases} min\{\sigma(x), \sigma(y)\} & \text{if } \sigma(x) \preceq \sigma(y) \text{ or } \sigma(y) \preceq \sigma(x) \\ 0 & \text{otherwise} \end{cases}$$

Remark 2.1. *The vertices and edges of G^S are the vertices and edges as in its underlying graph G. Since every semiring posses a canonical pre-order, σ, ψ are well defined. In general, both vertices and edges of a S-valued graph have values in the semiring S, called S-values. We call σ, a S-vertex set and ψ, a S-edge set of S-valued graph G^S.*

Example 2.5. *Consider the semiring with the canonical pre-order $\preceq$ given in Example 2.4. Let $G = (V, E)$ be the graph with $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{(v_1, v_2), (v_1, v_3), (v_3, v_4)\}$. Corresponding to the graph G, we define the S-graph G^S as follows:*

Define $\sigma : V \to S$ and $\psi : E \to S$ by $\sigma(v_1) = \sigma(v_3) = a; \sigma(v_2) = b; \sigma(v_4) = c$ and $\psi(v_1, v_2) = \psi(v_1, v_3) = \psi(v_3, v_4) = a$.



Definition 2.5. *Let $G^S = (V, E, \sigma, \psi)$ be S-valued graph. A S-walk $v_0 - v_n$ in G^S is an alternating sequence of vertices and edges $(v_0, \sigma(v_0))$, $(e_0^1, \psi(e_0^1)), (v_1, \sigma(v_1)), (e_1^2, \psi(e_1^2))$, $\cdots, (v_{n-1}, \sigma(v_{n-1})), (e_{n-1}^n, \psi(e_{n-1}^n)), (v_n, \sigma(v_n))$, beginning and ending with vertices v_0 and v_n respectively, such that v_{i-1} and v_i are end vertices of the edge e_{i-1}^i, $1 \leq i \leq n$.*

If $(v_0, \sigma(v_0)) = (v_n, \sigma(v_n))$, then the $v_0 - v_n$ S-walk is said to be a closed S-walk.

A $v_0 - v_n$ S-walk is a S-trial if any two edges in it are distinct. That is, in a S-trial, vertices may be repeated but all the edges are different.

A $v_0 - v_n$ S-walk is a closed S-trial (or a S-tour) if $(v_0, \sigma(v_0)) = (v_n, \sigma(v_n))$ and any two edges in it are distinct.

A $v_0 - v_n$ S-path in G^S is a S-trial in which all the vertices are distinct.

A S-closed path, that is $(v_0, \sigma(v_0)) = (v_n, \sigma(v_n))$ in a S-path, is called a S-cycle.

Definition 2.6. [2] *If $\sigma(v) = a$, $\forall v \in V$ and some $a \in S$, then the corresponding S-valued graph G^S is called vertex regular S-valued graph. If $\psi(v_i, v_j) = a$, $\forall (v_i, v_j) \in E$ and some $a \in S$, then the corresponding S- valued graph G^S is called edge regular S-valued graph. An S-valued graph G^S is said to be S-regular if it is both a vertex regular and an edge regular S-valued graph.*

Lemma 2.1. [2] *If G^S is vertex regular S-valued graph, then G^S is edge regular S-valued graph.*

3. Middle graph of S-valued graphs $M(G^S)$

In this section, we define the middle graph of S-valued graph and discuss some of its properties.

Definition 3.1. *Let $G = (V, E)$ be a graph, $G^S = (V, E, \sigma, \psi)$ be a semiring valued graph and $M(G) = (V_M, E_M)$ be a middle graph of G. Define $M(G^S) = (V_M, E_M, \sigma_M, \psi_M)$ where $\sigma_M : V_M \to S$ and $\psi_M : E_M \to S$ are defined by*

$$\sigma_M(v) = \begin{cases} \sigma(v) & \text{if } v \in V \\ \psi(v_i, v_j) & \text{if } v = e_i^j \in V_M \setminus V \end{cases}$$

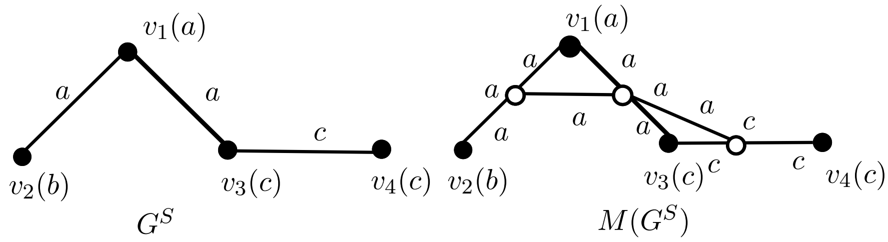
$$\psi_M(e, f) = \begin{cases} min\{\sigma_M(e), \sigma_M(f)\} & \text{if } \sigma_M(e) \preceq \sigma_M(f) \text{ or } \sigma_M(f) \preceq \sigma_M(e) \\ 0 & \text{otherwise} \end{cases}$$

Remark 3.1. *(1) The vertices and edges of $M(G^S)$ are the vertices and edges as in its underlying middle graph $M(G)$. Since every semiring posses a canonical pre-order, σ_M, ψ_M are well defined. In general, both vertices and edges of a S-valued graph have values in the semiring S, called S-values. We call σ_M, a S-vertex set and ψ_M, a S-edge set of S-valued graph $M(G^S)$.*

(2) The number of vertices of the middle graph of S-valued graph G is twice the number of vertices of G. i.e., $|V_M|_S = 2|V|$.

(3) The number of edjes of the middle graph of S-valued graph G is twice the number of edjes of G. i.e., $|E_M|_S = 2|E|$.

Example 3.1. *Consider a semiring as in Example 2.4. The middle graph $M(G^S)$ of the S-valued graph G^S is given by:*



Theorem 3.1. *The middle graph of S-cycle has atleast two S-cycles.*

Proof. Consider the cycle $C_n = v_1, (v_1, v_2), v_2, \ldots, v_n, (v_1, v_n), v_1$. Then we can construct the cycles in $M(C_n)$ as follows:
$A = v_1, (v_1, e_1^2), e_1^2, (e_1^2, v_2), v_2, (v_2, e_2^3), \cdots, (e_{n-1}^n, v_n), v_n, (v_n, e_1^n), e_1^n, (e_1^n, v_1), v_1$ and
$B = e_1^2, (e_1^2, e_2^3), e_2^3, (e_2^3, e_3^4), e_3^4, \cdots, e_{n-1}^n, (e_{n-1}^n, e_1^n), e_1^n, (e_1^n, e_1^2), e_1^2$.

It is easy to observe that the S-cycle A^S corresponding to the cycle A is a C_{2n}^S, S-cycle and the S-cycle B^S corresponding to the cycle B is a C_n^S, S-cycle. Therefore $M(C_n^S)$ has atleast two S-cycles. □

Theorem 3.2. *The middle graph of S-path has atleast two S-path.*

Proof. Consider the path $P_n = v_1, (v_1, v_2), v_2, \cdots, (v_{n-1}, v_n), v_n$. Then we can construct the paths in $M(P_n)$ as follows:
$Q = v_1, (v_1, e_1^2), e_1^2, (e_1^2, v_2), v_2, (v_2, e_2^3), \cdots, (e_{n-1}^n, v_n), v_n$ and
$R = e_1^2, (e_1^2, e_2^3), e_2^3, (e_2^3, e_3^4), e_3^4, \cdots, (e_{n-2}^{n-1}, e_{n-1}^n), e_{n-1}^n$

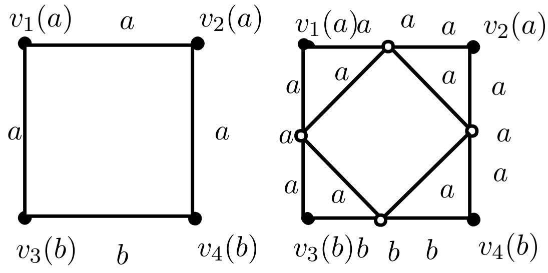
Let us take the S-paths Q^S and R^S corresponding to the paths Q and R. It is easy to observe that Q^S is a P_{2n-1}^S, S-path and R^S is a P_{n-1}^S, S-path. Therefore $M(P_n^S)$ has atleast two S-paths. □

Definition 3.2. *Let $M(G^S) = (V_M, E_M, \sigma_M, \psi_M)$ be the middle graph of S-valued graph $G^S = (V, E, \sigma, \psi)$. A S-valued graph $M(H^S) = (P_M, L_M, \tau_M, \gamma_M)$ is said to be S-subgraph of $M(G^S)$ if $H = (P, L)$ is a subgraph of G with $\tau_M \subset \sigma_M$ and $\gamma_M \subset \psi_M$.*

That is, $\tau_M \subset \sigma_M \Rightarrow \tau_M(v) \preceq \sigma_M(v)$, $\forall v \in P_M$ and $\gamma_M \subset \psi_M \Rightarrow \gamma_M(v_i, v_j) \preceq \psi_M(v_i, v_j)$, $\forall (v_i, v_j) \in L_M$.

Definition 3.3. *Let $M(G^S) = (V_M, E_M, \sigma_M, \psi_M)$ be a middle graph of S-valued graph. A S-valued graph $M(H^S) = (P_M, L_M, \tau_M, \gamma_M)$ is said to be S-subgraph of $M(G^S)$ induced by P_M if H is a subgraph of G with $P_M \subset V_M, L_M \subset E_M, \tau_M(v) = \sigma_M(v)$, $\forall v \in P_M$ and $\gamma_M(v_i, v_j) = \psi_M(v_i, v_j)$, $\forall (v_i, v_j) \in L_M$.*

Example 3.2. *Consider the S-valued graph G^S and its corresponding its middle graph $M(G^S)$.*



G^S $\quad$ $M(G^S)$

Consider the semiring with canonical pre-order $\preceq$ as given in Example 2.4. Let $M(G) = (V_M, E_M)$, where $V_M = \{v_1, v_2, v_3, v_4, e_1^2, e_2^3, e_3^4, e_1^4\}$ and $E_M = \{(v_1, e_1^2), (e_1^2, v_2), (v_2, e_2^3), (e_2^3, v_3), (v_3, e_3^4), (e_3^4, v_4), (v_4, e_1^4), (e_1^4, v_1), (e_1^2, e_2^3), (e_2^3, e_3^4), (e_1^2, e_1^4)\}$.

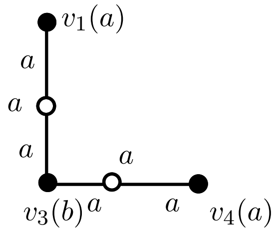
Consider the subgraph $M(H)$ of $M(G)$ such that $P_M = \{v_1, v_3, v_4, e_1^4, e_3^4\}$ and $L_M = \{(v_1, e_1^4), (e_1^4, v_4), (v_3, e_3^4), (e_3^4, v_4)\}$.

Define $\tau_M(v_1) = \tau_M(v_4) = a, \tau_M(v_3) = b$ and $\tau_M(e_1^4) = \tau_M(e_3^4) = a$. Therefore, $\tau_M(v) \preceq \sigma_M(v)$ for every $v \in P_M$. Hence $\tau_M \subset \sigma_M$.

Now define $\gamma_M : L \to S$ as follow $\gamma_M(v_1, e_1^4) = min\{\sigma_M(v_1), \sigma_M(e_1^4)\} = min\{a, a\}$.

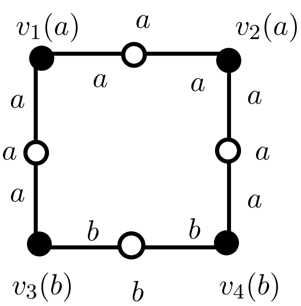
Similarly, $\gamma_M(e_1^4, v_4) = \gamma_M(v_3, e_3^4) = \gamma_M(e_3^4, v_4) = a$. Therefore $\gamma_M \preceq \psi_M$.

Thus the S-subgraph $M(H^S) = (P_M, L_M, \tau_M, \psi_M)$ is given by



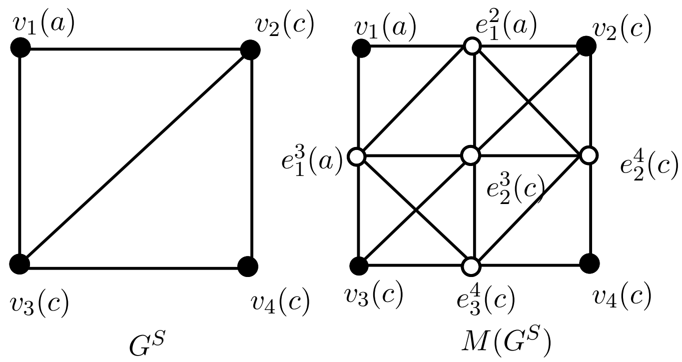
Definition 3.4. *Let $G^S = (V, E, \sigma, \psi)$ be a S-valued graph and $M(H^S) = (P_M, L_M, \tau_M, \gamma_M)$ be a S-subgraph of $M(G^S) = (V_M, E_M, \sigma_M, \psi_M)$. $M(H^S)$ is said to be a spanning S-subgraph of $M(G^S)$ if $P_M = V_M$, $L_M \subset E_M$, $\tau_M(v) = \sigma_M(v)$, $\forall v \in P_M$ and $\gamma_M(v_i, v_j) = \psi_M(v_i, v_j)$, $\forall (v_i, v_j) \in L_M$.*

Example 3.3. *Consider the middle graph $M(G^S)$ of S-valued graph as in Example 3.2. The spanning S-subgraph $M(H^S)$ of $M(G^S)$ is given by:*



Definition 3.5. *Let $G^S = (V, E, \sigma, \psi)$ be a S-valued graph where $(S, +, .)$ is a semiring with canonical preorder $\preceq$. For any $s \in S, M_s(G) = (\sigma_M^s, \psi_M^s)$ is a crisp graph with vertex set $\sigma_M^s = \{v \in V_M : s \preceq \sigma_M(v)\}$ and edge set $\psi_M^s = \{(v_i, v_j) \in E_M : s \preceq \psi_M(v_i, v_j)\}$.*

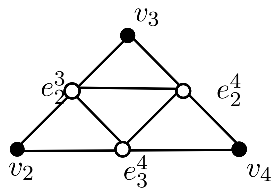
Example 3.4. *Let S be the semiring as in Example 2.4 and let $M(G^S)$ is given by:*



Suppose $s = b$. Then

$$\sigma_M^b = \{v \in V_M : b \preceq \sigma_M(v)\} = \{v_2, v_3, v_4, e_2^3, e_2^4, e_3^4\}$$
$$\psi_M^b = \{(e,f) \in E_M : b \preceq \psi_M(e,f)\}$$
$$= \{(v_2, e_2^3), (e_2^3, v_3), (v_3, e_3^4), (e_3^4, v_4), (e_2^4, v_4), (v_2, e_2^4), (e_2^4, e_2^3), (e_2^4, e_3^4), (e_2^3, e_3^4)\}$$

Therefore the required crisp graph $M_b(G)$ corresponding to $M(G^S)$ is



Theorem 3.3. *Let $G^S = (V, E, \sigma, \psi)$ be a S-valued graph. If $M(H^S)$ is a S-subgraph of $M(G^S)$, then $M_s(H)$ is a S-subgraph of $M_s(G)$, for any $s \in S$.*

Proof. Let $G^S = (V, E, \sigma, \psi)$ be a S-valued graph. Let $M(H^S) = (P_M, L_M, \tau_M, \gamma_M)$ be a S-subgraph of $M(G^S) = (V_M, E_M, \sigma_M, \psi_M)$ such that $P_M \subset V_M, L_M \subset E_M, \tau_M \subset \sigma_M$ and $\gamma_M \subset \psi_M$. i.e., $\tau_M(v) \preceq \sigma_M(v)$, for all $v \in P_M$, and $\gamma_M(v_i, v_j) \preceq \psi_M(v_i, v_j)$ for all $(v_i, v_j) \in L_M$. Let $s \in S$, $M_s(H) = (\tau_M^s, \gamma_M^s)$ and $M_s(G) = (\sigma_M^s, \psi_M^s)$, where
$\tau_M^s = \{v \in P_M : s \preceq \tau_M(v)\}$, $\gamma_M^s = \{(e,f) \in L_M : s \preceq \gamma_M(e,f)\}$
$\sigma_M^s = \{v \in V_M : s \preceq \sigma_M(v)\}$ and $\psi_M^s = \{(e,f) \in E : s \preceq \gamma_M(e,f)\}$.

Let $v \in \tau_M^s \Rightarrow s \preceq \tau_M(v) \preceq \sigma_M(v) \Rightarrow s \preceq \sigma_M(v) \Rightarrow v \in \sigma_M^s$. Therefore $\tau_M^s \subset \sigma_M^s$.

Let $(e,f) \in \gamma_M^s \Rightarrow s \preceq \gamma_M(e,f) \preceq \psi_M(e,f) \Rightarrow s \preceq \psi_M(e,f) \Rightarrow (e,f) \in \psi_M^s$. Therefore $\gamma_M^s \subset \psi_M^s$.

Hence $M_s(H)$ is a S-subgraph of $M_s(G)$. □

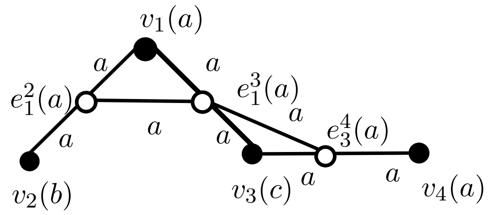
4. Regularity of $M(G^S)$

In this section, we study the regularity on $M(G^S)$.

Lemma 4.1. *If $M(G^S)$ is vertex regular S-valued graph, then $M(G^S)$ is edge regular S-valued graph.*

Proof. Since $M(G^S) = (V_M, E_M, \sigma_M, \psi_M)$ is vertex regular S-valued graph, $\sigma_M(v) = a$ for all $v \in V_M$ and for some $a \in S$. Let $(u,v) \in E_M$ be arbitrary. Then $\psi_M(u,v) = min\{\sigma_M(u), \sigma_M(v)\} = min\{a,a\} = a$. This proves $M(G^S)$ is an edge regular S-valued graph. □

Remark 4.1. *Converse of above lemma is not true. The following $M(G^S)$ is an edge regular S-valued graph but not vertex regular S-valued graph.*



Lemma 4.2. *$M(G^S)$ is vertex regular S-valued graph if and only if G^S is vertex regular S-valued graph.*

Proof. Assume $M(G^S)$ is vertex regular S-valued graph. Then $\sigma_M(v) = a$, $\forall v \in V_M$. In particular, $\sigma(v) = a$, $\forall v \in V$. Therefore G^S is vertex regular S-valued graph.

Conversely assume that $G^S = (V, E, \sigma, \psi)$ is a vertex regular S-valued graph, $\sigma(v) = a$ for all $v \in V$ and for some $a \in S$. By lemma 2.1, G^S is an edge regular S-valued graph and $\psi(v_i, v_j) = a$ for all $(v_i, v_j) \in E$. By definition 3.1, $\sigma_M(v) = a$ for all $v \in V_M$. Therefore $M(G^S)$ is vertex regular S-valued graph. □

Lemma 4.3. *$M(G^S)$ is edge regular S-valued graph if and only if G^S is edge regular S-valued graph.*

Proof. Assume $M(G^S)$ is edge regular S-valued graph. Then $\psi_M(e, f) = a$, $\forall (e, f) \in E_M$. By definition 3.1,

$$\psi_M(e,f) = min\{\sigma_M(e), \sigma_M(f)\} = \begin{cases} min\{\sigma(e), \psi(f)\} & \text{if } e \in V, f \in V_M \setminus V \\ min\{\psi(e), \psi(f)\} & \text{if } e, f \in V_M \setminus V \end{cases}$$

By the hypothesis, we have $min\{\sigma(e), \psi(f)\} = a$, $\forall e \in V, f \in V_M \setminus V$. Since e and f are adjacent in $M(G^S)$, f is an edge incident with e. So we have $\psi(f) \preceq \sigma(e)$. This implies $\psi(u, v) = a$, $\forall (u, v) \in E$. Hence G^S is an edge regular S-valued graph.

Conversely assume that G^S is edge regular S-valued graph. Then $\psi(u, v) = a$, $\forall (u, v) \in E$ and for some $a \in S$. Let $(e, f) \in E_M$, where $e, f \in V_M$.

Case(i) If $e \in V_M \setminus V, f \in V_M \setminus V$, then $\psi_M(e, f) = min\{\sigma_M(e), \sigma_M(f)\} = min\{\psi(e), \psi(f)\} = min\{a, a\} = a$.

Case(ii) Let $e \in V, f \in V_M \setminus V$. f is an edge incident with e, since e and f are adjacent in $M(G^S)$. So we have $\psi(f) \preceq \sigma(e)$. Consider $\psi_M(e, f) = min\{\sigma_M(e), \sigma_M(f)\} = min\{\sigma(e), \psi(f)\} = \psi(f) = a$.

In all cases, we have $\psi_M(e, f) = a$. Hence $M(G^S)$ is edge regular S-valued graph. □

Theorem 4.1. *$M(G^S)$ is S-regular if and only if G^S is S-regular.*

Proof. Assume $M(G^S)$ is S- regular. Then $\sigma_M(v) = a$ for all $v \in V_M$ and for some $a \in S$. That is, $\sigma(v) = a$ for all $v \in V$. Therefore G^S is vertex regular S-valued graph. By lemma 2.1, G^S is edge regular S-valued graph. Hence G^S is S-regular.

Conversely if G^S is S-regular graph, then $\sigma(v) = a$ for all $v \in V$ and by lemma 2.1, $\psi(v_i, v_j) = a$ for all $(v_i, v_j) \in E$ and for some $a \in S$. By definition 3.1,

$$\sigma_M(v) = \begin{cases} \sigma(v) & \text{for } v \in V \\ \psi(v_i, v_j) & \text{for } v = e_i^j \in V_M \setminus V \end{cases}$$

$\sigma_M(v) = a$ for all $v \in V_M$. This implies $M(G^S)$ is vertex regular S- graph. By lemma 4.1, $M(G^S)$ is an edge regular S- graph. Hence $M(G^S)$ is a S- regular graph. □

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