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# INTERVAL-VALUED INTUITIONISTIC FUZZY SOFT GRAPHS WITH APPLICATION

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ABSTRACT. The concept of interval-valued intuitionistic fuzzy soft sets and fuzzy graphs structure together constitute a new structure called an interval-valued intuitionistic fuzzy soft graph. The definitions of interval-valued intuitionistic fuzzy soft subgraph and strong interval-valued intuitionistic fuzzy soft graph are introduced with suitable examples. The degree of the good influence of a parameter in a fuzzy network and there is no influence by an interval number in the same system. Similarly, the effectiveness and non-effectiveness of the other fuzzy system on other parameters is measured by the concept of soft graphs in this article. Also, several different types of operations, including Cartesian product, strong product and composition on interval-valued intuitionistic fuzzy soft graphs are presented. Some related properties of these operations are investigated. Finally, we give a real-life application of interval-valued intuitionistic fuzzy soft graphs on social media and find out the most affected person in social media.

Keyword: Interval-valued intuitionistic fuzzy soft set, interval-valued intuitionistic fuzzy soft graph, Operations on interval-valued intuitionistic fuzzy soft graphs.

AMS Subject Classification: 05C72.

## 1. Introduction

It is seen in the maximum time that graph theory is found as an essential part of connectivity in some fields of geometry, algebra, topology, number theory, computer science, operation research as well as optimization. Fuzzy graph theory is finding an increasing number of applications in real-time modelling systems where the level of information inherent in the system varies with different levels of precision. The notion of the fuzzy graph was first initiated by Rosenfeld [44]. Mordeson and Nair [31] defined the concept of the

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complement of the fuzzy graph and studied some operations on fuzzy graphs. Vertex covering problem on fuzzy graph studied by Bhattacharya and pal [9]. In [36, 37, 38, 39, 40] Islam et al. studied Wiener index, hyper-Wiener index, first Zagreb index, F-index and hyper-connectivity index for fuzzy graph. Moderson and Peng [30] introduce various operations on fuzzy graphs.

Many important applications in different fields are handled with fuzzy soft sets. To deal with vague and imprecise parameters, classical soft sets are not appropriate. Maji et al. [24] represented the generalization of the standard soft sets in the form of initialization of the notion of fuzzy soft sets. Some characteristics of the said notion are investigated by Maji et al. [24, 25, 26]. Also, several researchers have used the idea on various mathematical and algebraic structures [2, 5, 14, 15, 20, 21].

In 1983, Atanassov [6] introduced the concept of intuitionistic fuzzy set as a generalisation of fuzzy sets and Maji et al. [27] introduced the concept of intuitionistic fuzzy soft set and presented some operations on intuitionistic fuzzy soft sets. Akram and Shahzadi [4] introduced the concept of intuitionistic fuzzy soft graphs combining the notion of intuitionistic fuzzy soft sets with graphs. Also, Shyla and Varkey [51, 54] gave the concept of the intuitionistic fuzzy soft graph and discussed some properties of this graph.

One extension of fuzzy set [57] is an interval-valued fuzzy set, initiated by Zadeh [58] in 1975, in which the measure of membership grades are intervals instead of the numbers. Interval-valued fuzzy sets are more flexible than traditional fuzzy sets of some description of uncertainty. Some researchers thoroughly studied the interval-valued fuzzy sets. Many works have been done on interval-valued fuzzy sets as well as on interval-valued fuzzy graphs [1, 7, 22, 23, 32, 35, 41, 56]. Later, the notion of interval-valued fuzzy soft sets was initiated by Yang [56], which is a more realistic model of uncertainty compared to a fuzzy set. In 2015, Tripathy and Sooraj [52] gave on interval-valued fuzzy soft sets and their application in group decision making, Jiang et al. [21] provided interval-valued intuitionistic fuzzy soft sets and their properties. Some new operations on interval-valued intuitionistic fuzzy soft sets were initiated by Wang and Tang [55]. Tripathy and Panigrahi [53] gave interval-valued intuitionistic fuzzy parameterized soft set theory and its application in decision-making. Motivated enough above these works and to the best of our knowledge, there is no work available on interval-valued intuitionistic fuzzy soft graphs. For this reason, based on the basic concepts of intuitionistic fuzzy graphs, interval-valued fuzzy graphs and fuzzy soft theory; we develop interval-valued intuitionistic fuzzy soft graphs.

For other interesting work on fuzzy graphs see [10, 11, 12, 13, 16, 17, 18, 19, 33, 34, 45, 46, 47, 42, 43, 48, 49, 50, 28].

In this paper, the concept of interval-valued intuitionistic fuzzy soft sets and graph structure are combined, which induced a kind of fuzzy graph called interval-valued intuitionistic fuzzy soft graph. The definitions of interval-valued intuitionistic fuzzy soft subgraph and strong interval-valued intuitionistic fuzzy soft graph are introduced with suitable examples. Also, several different types of operations, including Cartesian product, strong product and composition on interval-valued intuitionistic fuzzy soft graphs, are presented. Some related properties of these operations are investigated. Finally, we give a real-life application of interval-valued intuitionistic fuzzy soft graphs on social media and find out the most affected person in social media.

**Motivation:** Many people are connected on a social media. Some people have good influence in his/her life by social media and there is a little bit no influence by the same social media. For other people, the influence may be reversed. It is tough to measure good influence as a point. So one can represent it by an interval. The degree of the good

Table 1. The list of abbreviation

| Abbreviation | Meaning | Abbreviation | Meaning |
|---|---|---|---|
| FS | Fuzzy set | IFS | Intuitionistic fuzzy set |
| SS | Soft set | IVIF | Interval valued intuitionistic fuzzy |
| FG | Fuzzy graph | IFG | Intuitionistic fuzzy graph |
| MS | Membership | IVFS | Interval valued fuzzy set |
| NMS | Non membership | | |

influence of a person by social media is [0.2,0.4] and there is no influence by the same social media on the person is considered as [0.1,0.2]. Similarly, the effectiveness and non-effectiveness of the other social media on other people is measured. To find out the most affected person from a particular group of persons who are connected by social media, we have used the model of interval valued intuitionistic fuzzy soft graph. For this combined concept, anyone can analysis the range of characteristic together with membership and non-membership fact in a certain way. For this reason, we are interested to work with this concept. Also, it is very interesting to develop and analysis such combined graphs with examples and related theorems. These definitions and theorems are definitely improve the existing concepts of fuzzy soft graphs and more reliable for solving any complicated real-life problem.

## 2. Preliminaries

Some essential notions recall here which are needed for the article.

**Definition 2.1.** [57] *Taking $T$ as a universal set, then a FS $A$ over $T$ is meant as $A = \{(u, \mu_A(u)) : u \in T\}$, where $\mu_A : T \to I$. Here, $\mu_A(u)$ is the measure of MS of $u$ in $A$ and $I = [0, 1]$.*

In 1965, the concept of FSs and fuzzy relations was initiated by Zadeh and later, Rosenfeld took fuzzy relations on FSs and improved the concept of FGs in 1975.

**Definition 2.2.** [44] *Let $\acute{G} = (\acute{V}, \sigma, \mu)$ be a FG, where $\acute{V}$ is non-empty set (called set of vertices), jointly with two functions $\sigma : \acute{V} \to I$ and $\mu : \acute{V} \times \acute{V} \to I$ such that for all $u, v \in \acute{V}$, $\mu(uv) \leq \min\{\sigma(u), \sigma(v)\}$, $\mu$ is a symmetrical fuzzy relation on $\sigma$ and $I = [0, 1]$.*

In 1986, Atanassov [6] included a new type of uncertainty in FS, which is a measure of NMS function and gave a new definition name of IFS.

**Definition 2.3.** [6] *Taking $T$ as a non-empty set, an IFS $\tilde{A}$ over $T$ is defined by $\tilde{A} = \{(u, \mu_{\tilde{A}}(u), \nu_{\tilde{A}}(u)) : u \in T\}$, where $\mu_{\tilde{A}}(u) \in I$ is the measure of MS of $u$ in $\tilde{A}$ and $\nu_{\tilde{A}}(t) \in I$ is the measure of NMS of $u$ in $\tilde{A}$ with the condition $0 \leq \mu_{\tilde{A}}(u) + \nu_{\tilde{A}}(u) \leq 1$.*

Here, $S_{\tilde{A}}(u) = 1 - (\mu_{\tilde{A}}(u) + \nu_{\tilde{A}}(u))$ is the measure of suspicion of $u$ in $\tilde{A}$, which excludes the measure of MS and NMS.

**Definition 2.4.** [3] *An IFG is denoted by $\acute{G} = (\acute{V}, \mu, \gamma)$ where $\mu = (\mu_1, \mu_2), \gamma = (\gamma_1, \gamma_2)$ and (a) $\mu_1 : \acute{V} \to I$ and $\mu_2 : \acute{V} \to I$ denote the measure of MS and NMS of the vertex $u \in \acute{V}$ respectively and $0 \leq \mu_1(u) + \mu_2(u) \leq 1$ for every $u \in \acute{V}$. (b) $\gamma_1 : \acute{V} \times \acute{V} \to I$ and $\gamma_2 : \acute{V} \times \acute{V} \to I$, where $\gamma_1(uv)$ and $\gamma_2(uv)$ denote the measure of MS and NMS value of the edge $uv$ respectively such that $\gamma_1(uv) \leq \min\{\mu_1(u), \mu_1(v)\}$ and $\gamma_2(uv) \geq \max\{\mu_2(u), \mu_2(v)\}$, $0 \leq \gamma_1(uv) + \gamma_2(uv) \leq 1$ for every $uv$.*

In 1999, Molodtsov [29] proposed SS theory, generalization of FS theory to deal with uncertainty in a parametric manner.

**Definition 2.5.** [29] *A pair $(\phi, Q)$ is called a SS over $X$, where $X$ and $Q$ are a universal set and set of parameters respectively and $\phi$ is a mapping of $Q$ into a power set of $X$.*

In another way, a SS over $X$ is a family of a parametric subset of $X$. For $e \in Q$, $\phi(e)$ can be taken as the set of $e$-elements of the SS $(\phi, Q)$.

**Definition 2.6.** [26] *Suppose $(\phi, Q)$, $(\psi, R)$ be two SSs on a universe $X$. Then we tell that $(\phi, Q)$ is a soft subset of $(\psi, R)$, if it satisfies (i) $Q \subseteq R$, and (ii) $\phi(e)$ and $\psi(e)$ are identical approximations, for all $e \in Q$.*

In an interval-valued fuzzy set, the measure of membership grades are intervals instead of a number. The MS degree of each element on an IVFS is defined on a closed sub-interval $I$. Let $L(I)$ be the set of all closed sub-intervals of $I$.

**Definition 2.7.** [58] *An IVFS $\tilde{A}$, on the universe $T(\neq \phi)$, is a set such that $\tilde{A} = \{(t, \mu_{\tilde{A}}(t) = [\mu^-_{\tilde{A}}(t), \mu^+_{\tilde{A}}(t)]) : t \in T\}$, where the function $\mu_{\tilde{A}} : T \to L(I)$ is called the MS function and $\mu^-_{\tilde{A}}(t)$ and $\mu^+_{\tilde{A}}(t)$ are the lower and upper MS values of $t$ to $T$ where $0 \leq \mu^-_{\tilde{A}}(t) \leq \mu^+_{\tilde{A}}(t) \leq 1$.*

Atanassov and Gargov [7] first proposed the IVIF set. It is characterized by an interval-valued MS degree and an interval-valued NMS degree.

**Definition 2.8.** [8, 7] *An IVIF set on a universe $T$ is an object of the form $\tilde{A} = \{(t, \mu_{\tilde{A}}(t), \gamma_{\tilde{A}}(t)) : t \in T\}$, where $\mu_{\tilde{A}}(t) : T \to L(I)$ and $\gamma_{\tilde{A}}(t) : T \to L(I)$ satisfy the condition: for all $t \in T$, $\sup \mu_{\tilde{A}}(t) + \sup \gamma_{\tilde{A}}(t) \leq 1$.*

A combination of IVIF set with a soft set, called IVIF soft set, was given by Jiang et al. [21] in 2010. Let $IVIF^T$ be the IVIF power set of $T$.

**Definition 2.9.** [21] *Let $T$ be an initial universe, $Q$ be a set of parameters and $\acute{A} \subset Q$. Let us define a mapping $\phi$ from $\acute{A}$ to $IVIF^T$ as : $\phi : \acute{A} \to IVIF^T$. Then $(\phi, \acute{A})$ is called IVIF soft set over $T$ and is defined by $\{(t_i, [\mu^L_{\acute{A}}(t_i), \mu^U_{\acute{A}}(t_i)], [\gamma^L_{\acute{A}}(t_i), \gamma^U_{\acute{A}}(t_i)]) : \forall t_i \in T, \forall \alpha_i \in \acute{A}\}$, where $\mu^L_{\acute{A}} : T \to I$, $\mu^U_{\acute{A}} : T \to I$, $\gamma^L_{\acute{A}} : T \to I$ and $\gamma^U_{\acute{A}} : T \to I$ are such that $0 \leq \mu^U_{\acute{A}}(t) + \gamma^U_{\acute{A}}(t) \leq 1$.*

## 3. Interval valued intuitionistic fuzzy soft graphs

In this section, we introduce IVIF soft graph, IVIF soft subgraph and strong IVIF soft graph.

**Definition 3.1.** *Let $\Gamma^* = (\acute{V}, \acute{E})$ be a crisp graph. An IVIF soft graph with underlying set $\acute{V}$ is denoted by $\widetilde{\Gamma} = (\Gamma^*, \phi, \psi, \acute{A})$ such that*

*(i) $\acute{A}$ be a non-empty parameter set, (ii) $(\phi, \acute{A})$ be an IVIF soft set on $\acute{V}$,*
*(iii) $(\psi, \acute{A})$ be an IVIF soft set on $\acute{E}$, (iv) $(\phi(e), \psi(e))$ be an IVIF graph for all $e \in \acute{A}$. That is*

$$\mu^L_{\psi(e)}(xy) \leq (\mu^L_{\phi(e)}(x) \wedge \mu^L_{\phi(e)}(y)), \mu^U_{\psi(e)}(xy) \leq (\mu^U_{\phi(e)}(x) \wedge \mu^U_{\phi(e)}(y))$$
$$\gamma^L_{\psi(e)}(xy) \geq (\gamma^L_{\phi(e)}(x) \vee \gamma^L_{\phi(e)}(y)), \gamma^U_{\psi(e)}(xy) \geq (\gamma^U_{\phi(e)}(x) \vee \gamma^U_{\phi(e)}(y))$$

*for all $xy \in \acute{V} \times \acute{V}$ and $\mu^L_{\psi(e)}(xy) = \mu^U_{\psi(e)}(xy) = 0$, $\gamma^L_{\psi(e)}(xy) = \gamma^U_{\psi(e)}(xy) = 0$ for all $xy \in \acute{V} \times \acute{V} \setminus \acute{E}$. Note that $(\phi, \acute{A})$ is called an IVIF soft vertex and $(\psi, \acute{A})$ is called an IVIF* *soft edge. Here,* $\mu^L_{\phi(e)}(x)$ *and* $\mu^L_{\psi(e)}(xy)$ *denote the degree of lower MS of the vertex* $x$ *and edge* $xy$ *respectively corresponding to parameter* $e$, $\mu^U_{\phi(e)}(x)$ *and* $\mu^U_{\psi(e)}(xy)$ *denote the degree of upper MS of the vertex* $x$ *and edge* $xy$ *respectively corresponding to parameter* $e$, $\gamma^L_{\phi(e)}(x)$ *and* $\gamma^L_{\psi(e)}(xy)$ *denote the degree of lower NMS of the vertex* $x$ *and edge* $xy$ *respectively corresponding to parameter* $e$, $\gamma^U_{\phi(e)}(x)$ *and* $\gamma^U_{\psi(e)}(xy)$ *denote the degree of upper NMS of the vertex* $x$ *and edge* $xy$ *respectively corresponding to parameter* $e$*. Here,* $\phi(e)$ *and* $\psi(e)$ *represent the IVIF soft vertex and edge set respectively and* $M(e) = (\phi(e), \psi(e))$ *represents the IVIF graph corresponding to parameter* $e$*.*

Throughout this paper, we indicate $\Gamma^* = (\acute{V}, \acute{E})$ a crisp graph, $M(e) = (\phi(e), \psi(e))$ an IVIF graph and $\widetilde{\Gamma} = (\Gamma^*, \phi, \psi, \acute{A}) = ((\phi, \acute{A}), (\psi, \acute{A}))$ an IVIF soft graph. An IVIF soft graph is a parameterized family of IVIF graphs of $\Gamma^*$.

Let us consider an example of IVIF soft graph as follows :

**Example 1.** *Take* $\Gamma^* = (\acute{V}, \acute{E})$ *such that* $\acute{V} = \{t_1, t_2, t_3, t_4, t_5\}$ *and* $\acute{E} = \{t_1t_2, t_2t_3, t_3t_4, t_4t_5, t_5t_1\}$*. Consider* $\acute{A} = \{e_1, e_2\}$ *be a parameter set and* $(\phi, \acute{A})$ *be an IVIF soft set over* $\acute{V}$ *defined by*

$\phi(e_1) = \{(t_1, [0.2, 0.4], [0.3, 0.5]), (t_2, [0.1, 0.3], [0.4, 0.6]), (t_3, [0.2, 0.5], [0.1, 0.4])$
$(t_4, [0.4, 0.5], [0.2, 0.5]), (t_5, [0.2, 0.3], [0.3, 0.4])\}$

$\phi(e_2) = \{(t_1, [0.1, 0.4], [0.2, 0.6]), (t_3, [0.2, 0.3], [0.3, 0.5]), (t_5, [0.3, 0.5], [0.2, 0.4])\}$

*Now let* $(\psi, \acute{A})$ *be an IVIF soft set over* $\acute{E}$ *defined by*

$\psi(e_1) = \{(t_1t_2, [0.1, 0.2], [0.5, 0.7]), (t_2t_3, [0.1, 0.3], [0.4, 0.6]), (t_3t_4, [0.2, 0.4], [0.3, 0.5]),$
$(t_4t_5, [0.1, 0.2], [0.4, 0.6]), (t_5t_1, [0.2, 0.3], [0.4, 0.5])\}$

$\psi(e_2) = \{(t_1t_3, [0.1, 0.2], [0.3, 0.7]), (t_3t_5, [0.2, 0.3], [0.4, 0.6]), (t_5t_1, [0.1, 0.4], [0.3, 0.6])\}$

*It is clearly seen that* $M(e_1) = (\phi(e_1), \psi(e_1))$ *and* $M(e_2) = (\phi(e_2), \psi(e_2))$ *are IVIF graphs corresponding to the parameters* $e_1$ *and* $e_2$ *respectively, as displayed in Figure 1.*

*Hence* $\widetilde{\Gamma} = (\Gamma^*, \phi, \psi, \acute{A})$ *is an IVIF soft graph.*

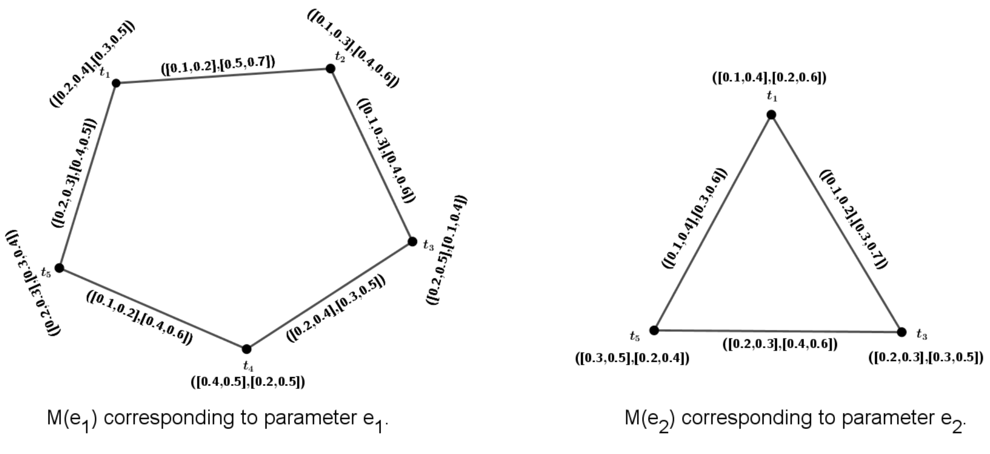



FIGURE 1. Interval valued intuitionistic fuzzy soft graph $\widetilde{\Gamma}$.

**Definition 3.2.** *Let* $\widetilde{\Gamma_1} = (\Gamma^*, \phi_1, \psi_1, \acute{A})$ *and* $\widetilde{\Gamma_2} = (\Gamma^*, \phi_2, \psi_2, \acute{B})$ *be two IVIF soft graphs of* $\Gamma^*$*. Then* $\widetilde{\Gamma_1}$ *is said IVIF soft subgraph of* $\widetilde{\Gamma_2}$ *if (i)* $\acute{A} \subseteq \acute{B}$ *and (ii)* $M_1(e) = (\phi_1(e), \psi_1(e))$ *is a partial IVIF subgraph of* $M_2(e) = (\phi_2(e), \psi_2(e))$ *for all* $e \in \acute{A}$*.*

**Example 2.** *Take the IVIF soft graph $\widetilde{\Gamma} = (\Gamma^*, \phi, \psi, \acute{A})$ as taken in Example 1. Let $\acute{B} = \{e_1, e_2\}$ be a parameter set, $(\phi_1, \acute{B})$ be an IVIF soft set over $\acute{V}$ and $(\psi_1, \acute{B})$ be an IVIF soft set on $\acute{E}$ defined by* $\phi_1(e_1) = \{(t_2, [0.1, 0.2], [0.4, 0.5]), (t_3, [0.2, 0.3], [0.2, 0.5]), (t_4, [0.1, 0.4], [0.3, 0.6]), (t_5, [0.2, 0.5], [0.1, 0.5])\}$, $\psi_1(e_1) = \{(t_2t_3, [0.1, 0.2], [0.4, 0.6]), (t_3t_4, [0.1, 0.2], [0.4, 0.6]), (t_4t_5, [0.1, 0.4], [0.3, 0.6])\}$, $\phi_1(e_2) = \{(t_1, [0.3, 0.6], [0.1, 0.4]), (t_3, [0.2, 0.4], [0.3, 0.6])\}$,$\psi_1(e_2) = \{(t_1t_3, [0.1, 0.3], [0.3, 0.7])\}$

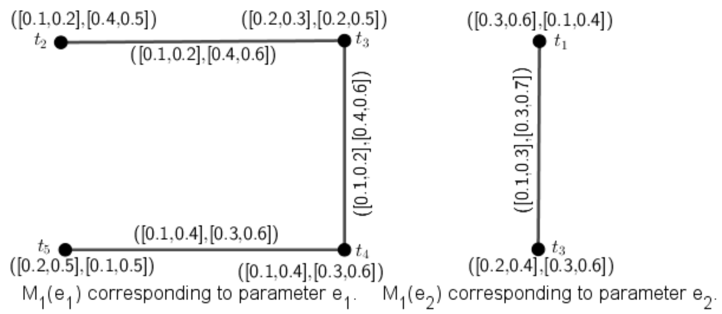



FIGURE 2. Interval valued intuitionistic fuzzy soft graph $\widetilde{\Gamma_1}$.

*It is clearly seen that $M_1(e_1) = (\phi_1(e_1), \psi_1(e_1))$ and $M_1(e_2) = (\phi_1(e_2), \psi_1(e_2))$ are IVIF graphs corresponding to the parameters $e_1$ and $e_2$, respectively as shown in Figure 2. Also, $\widetilde{\Gamma_1} = (\Gamma^*, \phi_1, \psi_1, \acute{B})$ is an IVIF soft graph. Hence $\widetilde{\Gamma_1}$ is an IVIF soft subgraph of $\widetilde{\Gamma}$.*

**Theorem 3.1.** *Let $\widetilde{\Gamma_1} = (\Gamma^*, \phi_1, \psi_1, \acute{A})$ and $\widetilde{\Gamma_2} = (\Gamma^*, \phi_2, \psi_2, \acute{B})$ be two IVIF soft graphs of $\Gamma^*$. Then $\widetilde{\Gamma_1}$ is an IVIF soft subgraph of $\widetilde{\Gamma_2}$ if and only if $\phi_1(a) \subseteq \phi_2(a)$ and $\psi_1(a) \subseteq \psi_2(a)$ for all $a \in \acute{A}$.*

**Proof:** Suppose that $\widetilde{\Gamma_1}$ is an IVIF soft subgraph of $\widetilde{\Gamma_2}$. Then $\acute{A} \subseteq \acute{B}$ and $M_1(a)$ is an IVIF subgraph of $M_2(a)$ for all $a \in \acute{A}$.

Conversely, suppose that $\phi_1(a) \subseteq \phi_2(a)$ and $\psi_1(a) \subseteq \psi_2(a)$ for all $a \in \acute{A}$. Since $\widetilde{\Gamma_1}$ is an IVIF soft subgraph of $\Gamma^*$, $M_1(a)$ is an IVIF subgraph of $\Gamma^*$ for all $a \in \acute{A}$. Since $\widetilde{\Gamma_2}$ is an IVIF soft subgraph of $\Gamma^*$, $M_2(a)$ is an IVIF subgraph of $\Gamma^*$ for all $a \in \acute{B}$. Thus, $M_1(a)$ is a partial IVIF subgraph of $M_2(a)$ for all $a \in \acute{A}$. Hence, $\widetilde{\Gamma_1}$ is an IVIF soft subgraph of $\widetilde{\Gamma_2}$.

**Definition 3.3.** *A IVIF soft graph $\widetilde{\Gamma} = (\Gamma^*, \phi, \psi, \acute{A})$ is called strong IVIF soft graph if*

$$\mu^L_{\psi(e)}(xy) = (\mu^L_{\phi(e)}(x) \wedge \mu^L_{\phi(e)}(y)), \mu^U_{\psi(e)}(xy) = (\mu^U_{\phi(e)}(x) \wedge \mu^U_{\phi(e)}(y))$$
$$\gamma^L_{\psi(e)}(xy) = (\gamma^L_{\phi(e)}(x) \vee \gamma^L_{\phi(e)}(y)), \gamma^U_{\psi(e)}(xy) = (\gamma^U_{\phi(e)}(x) \vee \gamma^U_{\phi(e)}(y))$$

*for all $e \in \acute{A}$ and $xy \in \acute{E}$.*

**Example 3.** *Let $\Gamma^* = (\acute{V}, \acute{E})$ such that $\acute{V} = \{t_1, t_2, t_3, t_4\}$ and $\acute{E} = \{t_1t_2, t_2t_3, t_3t_4, t_4t_1\}$. Also, let $\acute{A} = \{e_1, e_2\}$ be a parameter set and $(\phi, \acute{A})$ be an IVIF soft set over $\acute{V}$ defined by* $\phi(e_1) = \{(t_1, [0.2, 0.4], [0.1, 0.3]), (t_2, [0.1, 0.5], [0.2, 0.4]), (t_3, [0.3, 0.4], [0.4, 0.6]), (t_4, [0.1, 0.5], [0.3, 0.5])\}$ ,$\phi(e_2) = \{(t_1, [0.1, 0.5], [0.4, 0.5]), (t_2, [0.2, 0.3], [0.3, 0.4]), (t_3, [0.4, 0.5], [0.2, 0.4]), (t_4, [0.1, 0.4], [0.2, 0.4])\}$ *Now, let $(\psi, \acute{A})$ be an IVIF soft set over $\acute{E}$ defined by* $\psi(e_1) = \{(t_1t_2, [0.1, 0.4], [0.2, 0.4]), (t_2t_3, [0.1, 0.4], [0.4, 0.6]), (t_3t_4, [0.1, 0.4], [0.4, 0.6]), (t_4t_1, [0.1, 0.4], [0.3, 0.5])\}$, $\psi(e_2) = \{(t_1t_2, [0.1, 0.3], [0.4, 0.5]), (t_2t_3, [0.2, 0.3], [0.3, 0.4]), (t_3t_4, [0.1, 0.4], [0.2, 0.4]), (t_4t_1, [0.1, 0.4], [0.4, 0.5])\}$

*Obviously, $M(e_1) = (\phi(e_1), \psi(e_1))$ and $M(e_2) = (\phi(e_2), \psi(e_2))$ are IVIF graphs. Here, the edge MS values corresponding to parameter $e_1$ are given by*

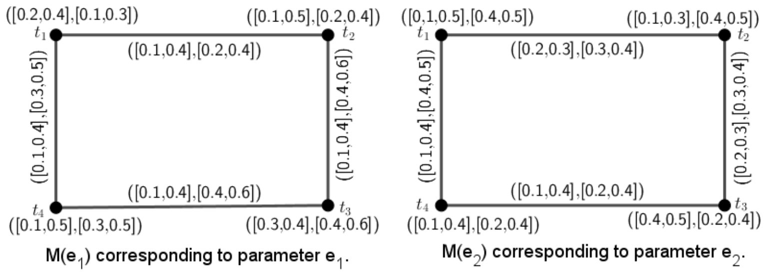



FIGURE 3. Strong interval valued intuitionistic fuzzy soft graph $\widetilde{\Gamma}$.

$\mu^L_{\psi(e_1)}(t_1t_2) = (\mu^L_{\phi(e_1)}(t_1) \wedge \mu^L_{\phi(e_1)}(t_2)) = (0.2) \wedge (0.1) = 0.1, \mu^U_{\psi(e_1)}(t_1t_2) = 0.4$
$\gamma^L_{\psi(e_1)}(t_1t_2) = (\gamma^L_{\phi(e_1)}(t_1) \vee \gamma^L_{\phi(e_1)}(t_2)) = (0.1) \vee (0.2) = 0.2, \gamma^U_{\psi(e_1)}(t_1t_2) = 0.4,$
*similarly,* $\mu^L_{\psi(e_1)}(t_2t_3) = 0.1, \mu^U_{\psi(e_1)}(t_2t_3) = 0.4, \gamma^L_{\psi(e_1)}(t_2t_3) = 0.4, \gamma^U_{\psi(e_1)}(t_2t_3) = 0.6;$
$\mu^L_{\psi(e_1)}(t_3t_4) = 0.1, \mu^U_{\psi(e_1)}(t_3t_4) = 0.4, \gamma^L_{\psi(e_1)}(t_3t_4) = 0.4, \gamma^U_{\psi(e_1)}(t_3t_4) = 0.6;$ *and*
$\mu^L_{\psi(e_1)}(t_4t_1) = 0.1, \mu^U_{\psi(e_1)}(t_4t_1) = 0.4, \gamma^L_{\psi(e_1)}(t_4t_1) = 0.3, \gamma^U_{\psi(e_1)}(t_4t_1) = 0.5.$
*Again, the edge MS values corresponding to parameter* $e_2$ *similarly are given by*
$\mu^L_{\psi(e_2)}(t_1t_2) = 0.1, \mu^U_{\psi(e_2)}(t_1t_2) = 0.3, \gamma^L_{\psi(e_2)}(t_1t_2) = 0.4, \gamma^U_{\psi(e_2)}(t_1t_2) = 0.5;$
$\mu^L_{\psi(e_2)}(t_2t_3) = 0.2, \mu^U_{\psi(e_2)}(t_2t_3) = 0.3, \gamma^L_{\psi(e_2)}(t_2t_3) = 0.3, \gamma^U_{\psi(e_2)}(t_2t_3) = 0.4;$
$\mu^L_{\psi(e_2)}(t_3t_4) = 0.1, \mu^U_{\psi(e_2)}(t_3t_4) = 0.4, \gamma^L_{\psi(e_2)}(t_3t_4) = 0.2, \gamma^U_{\psi(e_2)}(t_3t_4) = 0.4;$
$\mu^L_{\psi(e_2)}(t_4t_1) = 0.1, \mu^U_{\psi(e_2)}(t_4t_1) = 0.4, \gamma^L_{\psi(e_2)}(t_4t_1) = 0.4, \gamma^U_{\psi(e_2)}(t_4t_1) = 0.5.$
*By routine computations, evidently* $\widetilde{\Gamma} = (\Gamma^*, \phi, \psi, \acute{A})$ *is a strong IVIF soft graph of* $\Gamma^*$ *as shown in Figure 3.*

## 4. OPERATION ON INTERVAL-VALUED INTUITIONISTIC FUZZY SOFT GRAPHS

**Definition 4.1.** *Take* $\widetilde{\Gamma_1} = (\Gamma^*_1, \phi_1, \psi_1, \acute{A})$ *and* $\widetilde{\Gamma_2} = (\Gamma^*_2, \phi_2, \psi_2, \acute{B})$ *be two IVIF soft graphs of the crisp graphs* $\Gamma^*_1 = (\acute{V}_1, \acute{E}_1)$ *and* $\Gamma^*_2 = (\acute{V}_2, \acute{E}_2)$ *respectively. The Cartesian product of* $\widetilde{\Gamma_1}$ *and* $\widetilde{\Gamma_2}$ *is indicated by* $\widetilde{\Gamma_1} \times \widetilde{\Gamma_2} = (\Gamma^*, \phi, \psi, \acute{A} \times \acute{B})$*, where* $\Gamma^* = (\acute{V}_1 \times \acute{V}_2, \acute{E}_1 \times \acute{E}_2)$ *and is defined by*
$(\mu^L_{\phi_1(e_1)} \times \mu^L_{\phi_2(e_2)})(t_1, t_2) = (\mu^L_{\phi_1(e_1)}(t_1) \wedge \mu^L_{\phi_2(e_2)}(t_2)), (\mu^U_{\phi_1(e_1)} \times \mu^U_{\phi_2(e_2)})(t_1, t_2) = (\mu^U_{\phi_1(e_1)}(t_1) \wedge \mu^U_{\phi_2(e_2)}(t_2))$
$(\gamma^L_{\phi_1(e_1)} \times \gamma^L_{\phi_2(e_2)})(t_1, t_2) = (\gamma^L_{\phi_1(e_1)}(t_1) \vee \gamma^L_{\phi_2(e_2)}(t_2)), (\gamma^U_{\phi_1(e_1)} \times \gamma^U_{\phi_2(e_2)})(t_1, t_2) = (\gamma^U_{\phi_1(e_1)}(t_1) \vee \gamma^U_{\phi_2(e_2)}(t_2))$*, for all* $(t_1, t_2) \in \acute{V}_1 \times \acute{V}_2$*,* $e_1 \in \acute{A}$ *and* $e_2 \in \acute{B}$*.*

$(\mu^L_{\psi_1(e_1)} \times \mu^L_{\psi_2(e_2)})((t, t_2)(t, u_2)) = (\mu^L_{\phi_1(e_1)}(t) \wedge \mu^L_{\psi_2(e_2)}(t_2u_2)), (\mu^U_{\psi_1(e_1)} \times \mu^U_{\psi_2(e_2)})((t, t_2)(t, u_2)) = (\mu^U_{\phi_1(e_1)}(t) \wedge \mu^U_{\psi_2(e_2)}(t_2u_2))$
$(\gamma^L_{\psi_1(e_1)} \times \gamma^L_{\psi_2(e_2)})((t, t_2)(t, u_2)) = (\gamma^L_{\phi_1(e_1)}(t) \vee \gamma^L_{\psi_2(e_2)}(t_2u_2)), (\gamma^U_{\psi_1(e_1)} \times \gamma^U_{\psi_2(e_2)})((t, t_2)(t, u_2)) = (\gamma^U_{\phi_1(e_1)}(t) \vee \gamma^U_{\psi_2(e_2)}(t_2u_2))$*, for all* $t \in \acute{V}_1$*,* $t_2u_2 \in \acute{E}_2$*,* $e_1 \in \acute{A}$ *and* $e_2 \in \acute{B}$*.*

$(\mu^L_{\psi_1(e_1)} \times \mu^L_{\psi_2(e_2)})((t_1, v)(u_1, v)) = (\mu^L_{\psi_1(e_1)}(t_1u_1) \wedge \mu^L_{\phi_2(e_2)}(v)), (\mu^U_{\psi_1(e_1)} \times \mu^U_{\psi_2(e_2)})((t_1, v)(u_1, v)) = (\mu^U_{\psi_1(e_1)}(t_1u_1) \wedge \mu^U_{\phi_2(e_2)}(v))$
$(\gamma^L_{\psi_1(e_1)} \times \gamma^L_{\psi_2(e_2)})((t_1, v)(u_1, v)) = (\gamma^L_{\psi_1(e_1)}(t_1u_1) \vee \gamma^L_{\phi_2(e_2)}(v)), (\gamma^U_{\psi_1(e_1)} \times \gamma^U_{\psi_2(e_2)})((t_1, v)(u_1, v)) = (\gamma^U_{\psi_1(e_1)}(t_1u_1) \vee \gamma^U_{\phi_2(e_2)}(v))$ *for all* $v \in \acute{V}_2$*,* $t_1u_1 \in \acute{E}_1$*,* $e_1 \in \acute{A}$ *and* $e_2 \in \acute{B}$*. Here* $(t_1, t_2)$ *and* $((t, t_2)(t, u_2))$*,* $((t_1, v)(u_1, v))$ *represent the vertex and edges respectively of* $\widetilde{\Gamma_1} \times \widetilde{\Gamma_2}$*.*

**Example 4.** *Consider two graphs* $\Gamma_1^* = (\acute{V}_1, \acute{E}_1)$ *and* $\Gamma_2^* = (\acute{V}_2, \acute{E}_2)$ *such that* $\acute{V}_1 = \{t_1, u_1, v_1, w_1\}$, $\acute{E}_1 = \{t_1u_1, v_1w_1\}$ *and* $\acute{V}_2 = \{t_2, u_2, v_2, w_2\}$, $\acute{E}_2 = \{t_2u_2, v_2w_2\}$. *Take* $\acute{A} = \{e_1\}$ *be a parameter set and take* $(\phi_1, \acute{A})$ *and* $(\psi_1, \acute{A})$ *be two IVIF soft sets over* $\acute{V}_1$ *and* $\acute{E}_1$*, respectively, defined by*
$\phi_1(e_1) = \{(t_1, [0.1, 0.4], [0.2, 0.3]), (u_1, [0.2, 0.4], [0.3, 0.5]), (v_1, [0.4, 0.6], [0.3, 0.4]),$
$(w_1, [0.3, 0.5], [0.3, 0.5])\}$
$\psi_1(e_1) = \{(t_1u_1, [0.1, 0.4], [0.3, 0.5]), (v_1w_1, [0.3, 0.5], [0.3, 0.5])\}$
*Now, take* $\acute{B} = \{e_2\}$ *be parameter set and take* $(\phi_2, \acute{B})$ *and* $(\psi_2, \acute{B})$ *be two IVIF soft sets over* $\acute{V}_2$ *and* $\acute{E}_2$*, respectively, defined by*

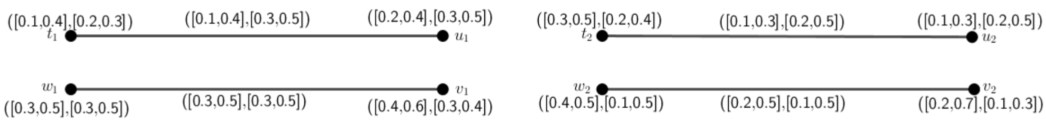



FIGURE 4. Interval valued intuitionistic fuzzy soft graphs $\widetilde{\Gamma_1}$ and $\widetilde{\Gamma_2}$.

$\phi_2(e_2) = \{(t_2, [0.3, 0.5], [0.2, 0.4]), (u_2, [0.1, 0.3], [0.2, 0.5]), (v_2, [0.2, 0.7], [0.1, 0.3]),$
$(w_2, [0.4, 0.5], [0.1, 0.5])\}$
$\psi_2(e_2) = \{(t_2u_2, [0.1, 0.3], [0.2, 0.5]), (v_2w_2, [0.2, 0.5], [0.1, 0.5])\}$

*It is easy to see that,* $M(e_1) = (\phi_1(e_1), \psi_1(e_1))$ *and* $M(e_2) = (\phi_2(e_2), \psi_2(e_2))$ *are IVIF graphs. Hence* $\widetilde{\Gamma_1} = (\Gamma_1^*, \phi_1, \psi_1, \acute{A})$ *and* $\widetilde{\Gamma_2} = (\Gamma_2^*, \phi_2, \psi_2, \acute{B})$ *are IVIF soft graphs of* $\Gamma_1^*$ *and* $\Gamma_2^*$*, respectively as shown in Figure 4. The Cartesian product of* $\widetilde{\Gamma_1}$ *and* $\widetilde{\Gamma_2}$ *is as shown in Figure 5a.*

**Theorem 4.1.** *If* $\widetilde{\Gamma_1}$ *and* $\widetilde{\Gamma_2}$ *are two IVIF soft graphs, then the Cartesian product* $\widetilde{\Gamma_1} \times \widetilde{\Gamma_2} = (\Gamma^*, \phi, \psi, \acute{A} \times \acute{B})$ *is also an IVIF soft graph.*

**Proof:** Let $\widetilde{\Gamma_1} = (\Gamma_1^*, \phi_1, \psi_1, \acute{A})$ and $\widetilde{\Gamma_2} = (\Gamma_2^*, \phi_2, \psi_2, \acute{B})$ be two IVIF soft graphs of simple graphs $\Gamma_1^* = (\acute{V}_1, \acute{E}_1)$ and $\Gamma_2^* = (\acute{V}_2, \acute{E}_2)$ respectively. From Definition 4.1, for all $e_1 \in \acute{A}$ and $e_2 \in \acute{B}$, there are three cases.

**Case(i)** If $t_1 \in \acute{V}_1$ and $t_2 \in \acute{V}_2$, then

$(\mu^L_{\phi_1(e_1)} \times \mu^L_{\phi_2(e_2)})(t_1, t_2) = \min(\mu^L_{\phi_1(e_1)}(t_1), \mu^L_{\phi_2(e_2)}(t_2)) = \min\{(\mu^L_{\phi_1(e_1)} \times \mu^L_{\phi_2(e_2)})(t_1),$
$(\mu^L_{\phi_1(e_1)} \times \mu^L_{\phi_2(e_2)})(t_2)\}$

$(\mu^U_{\phi_1(e_1)} \times \mu^U_{\phi_2(e_2)})(t_1, t_2) = \min\{(\mu^U_{\phi_1(e_1)} \times \mu^U_{\phi_2(e_2)})(t_1), (\mu^U_{\phi_1(e_1)} \times \mu^U_{\phi_2(e_2)})(t_2)\}$

$(\gamma^L_{\phi_1(e_1)} \times \gamma^L_{\phi_2(e_2)})(t_1, t_2) = \max(\gamma^L_{\phi_1(e_1)}(t_1), \gamma^L_{\phi_2(e_2)}(t_2)) = \max\{(\gamma^L_{\phi_1(e_1)} \times \gamma^L_{\phi_2(e_2)})(t_1),$
$(\gamma^L_{\phi_1(e_1)} \times \gamma^L_{\phi_2(e_2)})(t_2)\}$

$(\gamma^U_{\phi_1(e_1)} \times \gamma^U_{\phi_2(e_2)})(t_1, t_2) = \max\{(\gamma^U_{\phi_1(e_1)} \times \gamma^U_{\phi_2(e_2)})(t_1), (\gamma^U_{\phi_1(e_1)} \times \gamma^U_{\phi_2(e_2)})(t_2)\}$

**Case(ii)** If $t_1 \in \acute{V}_1$ and $t_2u_2 \in \acute{E}_2$, then

$$\begin{aligned}(\mu^L_{\psi_1(e_1)} \times \mu^L_{\psi_2(e_2)})((t, t_2)(t, u_2)) &= \min(\mu^L_{\phi_1(e_1)}(t), \mu^L_{\psi_2(e_2)}(t_2u_2)) \\ &\leq \min\{(\mu^L_{\phi_1(e_1)}(t)), \min(\mu^L_{\phi_2(e_2)}(t_2), \mu^L_{\phi_2(e_2)}(u_2))\} \\ &= \min\{\min(\mu^L_{\phi_1(e_1)}(t), \mu^L_{\phi_2(e_2)}(t_2)), \min(\mu^L_{\phi_1(e_1)}(t), \\ &\quad \mu^L_{\phi_2(e_2)}(u_2))\} \\ &= \min\{(\mu^L_{\phi_1(e_1)} \times \mu^L_{\phi_2(e_2)})(t, t_2), (\mu^L_{\phi_1(e_1)} \times \mu^L_{\phi_2(e_2)})(t, u_2)\}\end{aligned}$$

$$(\mu^U_{\psi_1(e_1)} \times \mu^U_{\psi_2(e_2)})((t, t_2)(t, u_2)) \leq \min\{(\mu^U_{\phi_1(e_1)} \times \mu^U_{\phi_2(e_2)})(t, t_2), (\mu^U_{\phi_1(e_1)} \times \mu^U_{\phi_2(e_2)})(t, u_2)\}$$

$$\begin{aligned}(\gamma^L_{\psi_1(e_1)} \times \gamma^L_{\psi_2(e_2)})((t,t_2)(t,u_2)) &= \max(\gamma^L_{\phi_1(e_1)}(t), \gamma^L_{\psi_2(e_2)}(t_2u_2)) \\ &\geq \max\{(\gamma^L_{\phi_1(e_1)}(t)), \max(\gamma^L_{\phi_2(e_2)}(t_2), \gamma^L_{\phi_2(e_2)}(u_2))\} \\ &= \max\{\max(\gamma^L_{\phi_1(e_1)}(t), \gamma^L_{\phi_2(e_2)}(t_2)), \max(\gamma^L_{\phi_1(e_1)}(t), \\ &\quad \gamma^L_{\phi_2(e_2)}(u_2))\} \\ &= \max\{(\gamma^L_{\phi_1(e_1)} \times \gamma^L_{\phi_2(e_2)})(t,t_2), (\gamma^L_{\phi_1(e_1)} \times \gamma^L_{\phi_2(e_2)})(t,u_2)\}\end{aligned}$$

$$(\gamma^U_{\psi_1(e_1)} \times \gamma^U_{\psi_2(e_2)})((t,t_2)(t,u_2)) \geq \max\{(\gamma^U_{\phi_1(e_1)} \times \gamma^U_{\phi_2(e_2)})(t,t_2), (\gamma^U_{\phi_1(e_1)} \times \gamma^U_{\phi_2(e_2)})(t,u_2)\}$$

**Case(iii)** If $v \in \acute{V}_2$ and $t_1u_1 \in \acute{E}_1$, then in the similar way we can prove that

$(\mu^L_{\psi_1(e_1)} \times \mu^L_{\psi_2(e_2)})((t_1,v)(u_1,v)) = \min\{(\mu^L_{\phi_1(e_1)} \times \mu^L_{\phi_2(e_2)})(t_1,v), (\mu^L_{\phi_1(e_1)} \times \mu^L_{\phi_2(e_2)})(u_1,v)\}$,
$(\mu^U_{\psi_1(e_1)} \times \mu^U_{\psi_2(e_2)})((t_1,v)(u_1,v)) \leq \min\{(\mu^U_{\phi_1(e_1)} \times \mu^U_{\phi_2(e_2)})(t_1,v), (\mu^U_{\phi_1(e_1)} \times \mu^U_{\phi_2(e_2)})(u_1,v)\}$,
$(\gamma^L_{\psi_1(e_1)} \times \gamma^L_{\psi_2(e_2)})((t_1,v)(u_1,v)) = \max\{(\gamma^L_{\phi_1(e_1)} \times \gamma^L_{\phi_2(e_2)})(t_1,v), (\gamma^L_{\phi_1(e_1)} \times \gamma^L_{\phi_2(e_2)})(u_1,v)\}$,
$(\gamma^U_{\psi_1(e_1)} \times \gamma^U_{\psi_2(e_2)})((t_1,v)(u_1,v)) \geq \max\{(\gamma^U_{\phi_1(e_1)} \times \gamma^U_{\phi_2(e_2)})(t_1,v), (\gamma^U_{\phi_1(e_1)} \times \gamma^U_{\phi_2(e_2)})(u_1,v)\}$

Therefore, $\widetilde{\Gamma_1} \times \widetilde{\Gamma_2}$ is an IVIF soft graph.

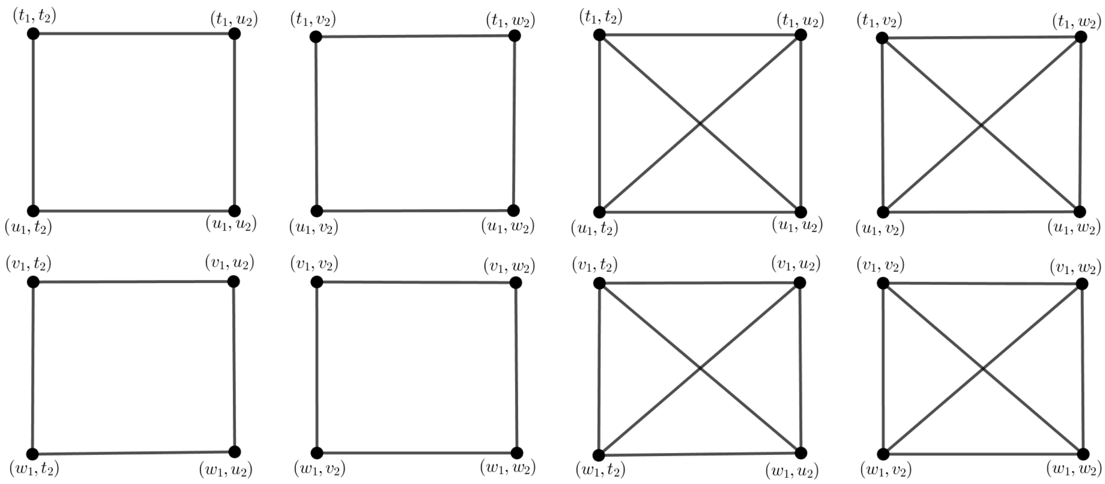
**Definition 4.2.** *Let $\widetilde{\Gamma_1} = (\Gamma^*_1, \phi_1, \psi_1, \acute{A})$ and $\widetilde{\Gamma_2} = (\Gamma^*_2, \phi_2, \psi_2, \acute{B})$ be two IVIF soft graphs of crisp graphs $\Gamma^*_1 = (\acute{V}_1, \acute{E}_1)$ and $\Gamma^*_2 = (\acute{V}_2, \acute{E}_2)$ respectively. The strong product of $\widetilde{\Gamma_1}$ and $\widetilde{\Gamma_2}$ is indicated by $\widetilde{\Gamma_1} \bigotimes \widetilde{\Gamma_2} = (\Gamma^*, \phi, \psi, \acute{A} \times \acute{B})$ and is defined by*
$(\mu^L_{\phi_1(e_1)} \otimes \mu^L_{\phi_2(e_2)})(t_1,t_2) = (\mu^L_{\phi_1(e_1)}(t_1) \wedge \mu^L_{\phi_2(e_2)}(t_2)), (\mu^U_{\phi_1(e_1)} \otimes \mu^U_{\phi_2(e_2)})(t_1,t_2) = (\mu^U_{\phi_1(e_1)}(t_1) \wedge \mu^U_{\phi_2(e_2)}(t_2))$
$(\gamma^L_{\phi_1(e_1)} \otimes \gamma^L_{\phi_2(e_2)})(t_1,t_2) = (\gamma^L_{\phi_1(e_1)}(t_1) \vee \gamma^L_{\phi_2(e_2)}(t_2)), (\gamma^U_{\phi_1(e_1)} \otimes \gamma^U_{\phi_2(e_2)})(t_1,t_2) = (\gamma^U_{\phi_1(e_1)}(t_1) \vee \gamma^U_{\phi_2(e_2)}(t_2))$ *for all* $(t_1,t_2) \in \acute{V}_1 \times \acute{V}_2$, $e_1 \in \acute{A}$ *and* $e_2 \in \acute{B}$.

$(\mu^L_{\psi_1(e_1)} \otimes \mu^L_{\psi_2(e_2)})((t,t_2)(t,u_2)) = (\mu^L_{\phi_1(e_1)}(t) \wedge \mu^L_{\psi_2(e_2)}(t_2u_2))$,
$(\mu^U_{\psi_1(e_1)} \otimes \mu^U_{\psi_2(e_2)})((t,t_2)(t,u_2)) = (\mu^U_{\phi_1(e_1)}(t) \wedge \mu^U_{\psi_2(e_2)}(t_2u_2))$
$(\gamma^L_{\psi_1(e_1)} \otimes \gamma^L_{\psi_2(e_2)})((t,t_2)(t,u_2)) = (\gamma^L_{\phi_1(e_1)}(t) \vee \gamma^L_{\psi_2(e_2)}(t_2u_2))$,
$(\gamma^U_{\psi_1(e_1)} \otimes \gamma^U_{\psi_2(e_2)})((t,t_2)(t,u_2)) = (\gamma^U_{\phi_1(e_1)}(t) \vee \gamma^U_{\psi_2(e_2)}(t_2u_2))$ *for all* $t \in \acute{V}_1$, $t_2u_2 \in \acute{E}_2$, $e_1 \in \acute{A}$ *and* $e_2 \in \acute{B}$.

$(\mu^L_{\psi_1(e_1)} \otimes \mu^L_{\psi_2(e_2)})((t_1,v)(u_1,v)) = (\mu^L_{\psi_1(e_1)}(t_1u_1) \wedge \mu^L_{\phi_2(e_2)}(v))$,
$(\mu^U_{\psi_1(e_1)} \otimes \mu^U_{\psi_2(e_2)})((t_1,v)(u_1,v)) = (\mu^U_{\psi_1(e_1)}(t_1u_1) \wedge \mu^U_{\phi_2(e_2)}(v))$
$(\gamma^L_{\psi_1(e_1)} \otimes \gamma^L_{\psi_2(e_2)})((t_1,v)(u_1,v)) = (\gamma^L_{\psi_1(e_1)}(t_1u_1) \vee \gamma^L_{\phi_2(e_2)}(v))$,
$(\gamma^U_{\psi_1(e_1)} \otimes \gamma^U_{\psi_2(e_2)})((t_1,v)(u_1,v)) = (\gamma^U_{\psi_1(e_1)}(t_1u_1) \vee \gamma^U_{\phi_2(e_2)}(v))$ *for all* $v \in \acute{V}_2$, $t_1u_1 \in \acute{E}_1$, $e_1 \in \acute{A}$ *and* $e_2 \in \acute{B}$.

$(\mu^L_{\psi_1(e_1)} \otimes \mu^L_{\psi_2(e_2)})((t_1,v_1)(u_1,v_2)) = (\mu^L_{\psi_1(e_1)}(t_1u_1) \wedge \mu^L_{\psi_2(e_2)}(v_1v_2))$,
$(\mu^U_{\psi_1(e_1)} \otimes \mu^U_{\psi_2(e_2)})((t_1,v_1)(u_1,v_2)) = (\mu^U_{\psi_1(e_1)}(t_1u_1) \wedge \mu^U_{\psi_2(e_2)}(v_1v_2))$
$(\gamma^L_{\psi_1(e_1)} \otimes \gamma^L_{\psi_2(e_2)})((t_1,v_1)(u_1,v_2)) = (\gamma^L_{\psi_1(e_1)}(t_1u_1) \vee \gamma^L_{\psi_2(e_2)}(v_1v_2)), (\gamma^U_{\psi_1(e_1)} \otimes \gamma^U_{\psi_2(e_2)})((t_1,v_1)(u_1,v_2)) = (\gamma^U_{\psi_1(e_1)}(t_1u_1) \vee \gamma^U_{\psi_2(e_2)}(v_1v_2))$ *for all* $t_1u_1 \in \acute{E}_1$, $v_1v_2 \in \acute{E}_2$, $e_1 \in \acute{A}$ *and* $e_2 \in \acute{B}$.

**Example 5.** *Consider two IVIF soft graphs $\widetilde{\Gamma_1} = (\Gamma^*_1, \phi_1, \psi_1, \acute{A})$ and $\widetilde{\Gamma_2} = (\Gamma^*_2, \phi_2, \psi_2, \acute{B})$ have already been shown in Example 4. The strong product of $\widetilde{\Gamma_1}$ and $\widetilde{\Gamma_2}$ is as displayed in Figure 5b.*




(A) Cartesian product of interval valued intuitionistic fuzzy soft graphs $\widetilde{\Gamma_1}$ and $\widetilde{\Gamma_2}$.

(B) Strong product of interval valued intuitionistic fuzzy soft graphs $\widetilde{\Gamma_1}$ and $\widetilde{\Gamma_2}$.

FIGURE 5

**Theorem 4.2.** *If $\widetilde{\Gamma_1}$ and $\widetilde{\Gamma_2}$ are two IVIF soft graphs, then so is $\widetilde{\Gamma_1} \otimes \widetilde{\Gamma_2}$.*

**Proof:** The proof is similar to Theorem 4.1.

**Definition 4.3.** *Let $\widetilde{\Gamma_1} = (\Gamma_1^*, \phi_1, \psi_1, \acute{A})$ and $\widetilde{\Gamma_2} = (\Gamma_2^*, \phi_2, \psi_2, \acute{B})$ be two IVIF soft graphs of crisp graphs $\Gamma_1^* = (\acute{V}_1, \acute{E}_1)$ and $\Gamma_2^* = (\acute{V}_2, \acute{E}_2)$ respectively. The composition of $\widetilde{\Gamma_1}$ and $\widetilde{\Gamma_2}$ is indicated by $\widetilde{\Gamma_1} \circ \widetilde{\Gamma_2} = (\Gamma^*, \phi, \psi, \acute{A} \times \acute{B})$ and is defined by*

$(\mu^L_{\phi_1(e_1)} \circ \mu^L_{\phi_2(e_2)})(t_1, t_2) = (\mu^L_{\phi_1(e_1)}(t_1) \wedge \mu^L_{\phi_2(e_2)}(t_2)), (\mu^U_{\phi_1(e_1)} \circ \mu^U_{\phi_2(e_2)})(t_1, t_2) = (\mu^U_{\phi_1(e_1)}(t_1) \wedge \mu^U_{\phi_2(e_2)}(t_2))$
$(\gamma^L_{\phi_1(e_1)} \circ \gamma^L_{\phi_2(e_2)})(t_1, t_2) = (\gamma^L_{\phi_1(e_1)}(t_1) \vee \gamma^L_{\phi_2(e_2)}(t_2)), (\gamma^U_{\phi_1(e_1)} \circ \gamma^U_{\phi_2(e_2)})(t_1, t_2) = (\gamma^U_{\phi_1(e_1)}(t_1) \vee \gamma^U_{\phi_2(e_2)}(t_2))$ *for all* $(t_1, t_2) \in \acute{V}_1 \times \acute{V}_2$, $e_1 \in \acute{A}$ *and* $e_2 \in \acute{B}$.

$(\mu^L_{\psi_1(e_1)} \circ \mu^L_{\psi_2(e_2)})((t, t_2)(t, u_2)) = (\mu^L_{\phi_1(e_1)}(t) \wedge \mu^L_{\psi_2(e_2)}(t_2u_2))$,
$(\mu^U_{\psi_1(e_1)} \circ \mu^U_{\psi_2(e_2)})((t, t_2)(t, u_2)) = (\mu^U_{\phi_1(e_1)}(t) \wedge \mu^U_{\psi_2(e_2)}(t_2u_2))$
$(\gamma^L_{\psi_1(e_1)} \circ \gamma^L_{\psi_2(e_2)})((t, t_2)(t, u_2)) = (\gamma^L_{\phi_1(e_1)}(t) \vee \gamma^L_{\psi_2(e_2)}(t_2u_2)), (\gamma^U_{\psi_1(e_1)} \circ \gamma^U_{\psi_2(e_2)})((t, t_2)(t, u_2)) = (\gamma^U_{\phi_1(e_1)}(t) \vee \gamma^U_{\psi_2(e_2)}(t_2u_2))$ *for all* $t \in \acute{V}_1$, $t_2u_2 \in \acute{E}_2$, $e_1 \in \acute{A}$ *and* $e_2 \in \acute{B}$.

$(\mu^L_{\psi_1(e_1)} \circ \mu^L_{\psi_2(e_2)})((t_1, v)(u_1, v)) = (\mu^L_{\psi_1(e_1)}(t_1u_1) \wedge \mu^L_{\phi_2(e_2)}(v))$,
$(\mu^U_{\psi_1(e_1)} \circ \mu^U_{\psi_2(e_2)})((t_1, v)(u_1, v)) = (\mu^U_{\psi_1(e_1)}(t_1u_1) \wedge \mu^U_{\phi_2(e_2)}(v))$
$(\gamma^L_{\psi_1(e_1)} \circ \gamma^L_{\psi_2(e_2)})((t_1, v)(u_1, v)) = (\gamma^L_{\psi_1(e_1)}(t_1u_1) \vee \gamma^L_{\phi_2(e_2)}(v)), (\gamma^U_{\psi_1(e_1)} \circ \gamma^U_{\psi_2(e_2)})((t_1, v)(u_1, v)) = (\gamma^U_{\psi_1(e_1)}(t_1u_1) \vee \gamma^U_{\phi_2(e_2)}(v))$ *for all* $v \in \acute{V}_2$, $t_1u_1 \in \acute{E}_1$, $e_1 \in \acute{A}$ *and* $e_2 \in \acute{B}$.

$(\mu^L_{\psi_1(e_1)} \circ \mu^L_{\psi_2(e_2)})((t_1, v_1)(u_1, v_2)) = (\mu^L_{\psi_1(e_1)}(t_1u_1) \wedge \mu^L_{\psi_2(e_2)}(v_1v_2))$,
$(\mu^U_{\psi_1(e_1)} \circ \mu^U_{\psi_2(e_2)})((t_1, v_1)(u_1, v_2)) = (\mu^U_{\psi_1(e_1)}(t_1u_1) \wedge \mu^U_{\psi_2(e_2)}(v_1v_2))$
$(\gamma^L_{\psi_1(e_1)} \circ \gamma^L_{\psi_2(e_2)})((t_1, v_1)(u_1, v_2)) = (\gamma^L_{\psi_1(e_1)}(t_1u_1) \vee \gamma^L_{\psi_2(e_2)}(v_1v_2)), (\gamma^U_{\psi_1(e_1)} \circ \gamma^U_{\psi_2(e_2)})((t_1, v_1)(u_1, v_2)) = (\gamma^U_{\psi_1(e_1)}(t_1u_1) \vee \gamma^U_{\psi_2(e_2)}(v_1v_2))$ *for all* $t_1u_1 \in \acute{E}_1$, $v_1v_2 \in \acute{E}_2$, $e_1 \in \acute{A}$ *and* $e_2 \in \acute{B}$.

$(\mu^L_{\psi_1(e_1)} \circ \mu^L_{\psi_2(e_2)})((t_1, v_1)(u_1, v_2)) = (\mu^L_{\psi_1(e_1)}(t_1u_1) \wedge \mu^L_{\phi_2(e_2)}(v_1) \wedge \mu^L_{\phi_2(e_2)}(v_2)), (\mu^U_{\psi_1(e_1)} \circ \mu^U_{\psi_2(e_2)})$

$((t_1, v_1)(u_1, v_2)) = (\mu^U_{\psi_1(e_1)}(t_1u_1) \wedge \mu^U_{\phi_2(e_2)}(v_1) \wedge \mu^U_{\phi_2(e_2)}(v_2))$
$(\gamma^L_{\psi_1(e_1)} \circ \gamma^L_{\psi_2(e_2)})((t_1, v_1)(u_1, v_2)) = (\gamma^L_{\psi_1(e_1)}(t_1u_1) \vee \gamma^L_{\phi_2(e_2)}(v_1) \vee \gamma^L_{\phi_2(e_2)}(v_2)), (\gamma^U_{\psi_1(e_1)} \circ \gamma^U_{\psi_2(e_2)})$
$((t_1, v_1)(u_1, v_2)) = (\gamma^U_{\psi_1(e_1)}(t_1u_1) \vee \gamma^U_{\phi_2(e_2)}(v_1) \vee \gamma^U_{\phi_2(e_2)}(v_2))$ *for all* $t_1u_1 \in \acute{E}_1$, $v_1, v_2 \in \acute{V}_2$ *such that* $v_1 \neq v_2$ *and for all* $e_1 \in \acute{A}$ *and* $e_2 \in \acute{B}$.

**Example 6.** *Consider two IVIF soft graphs* $\widetilde{\Gamma_1} = (\Gamma_1^*, \phi_1, \psi_1, \acute{A})$ *and* $\widetilde{\Gamma_2} = (\Gamma_2^*, \phi_2, \psi_2, \acute{B})$ *have already been shown in Example 4. The composition of* $\widetilde{\Gamma_1}$ *and* $\widetilde{\Gamma_2}$ *is as shown in Figure 6.*

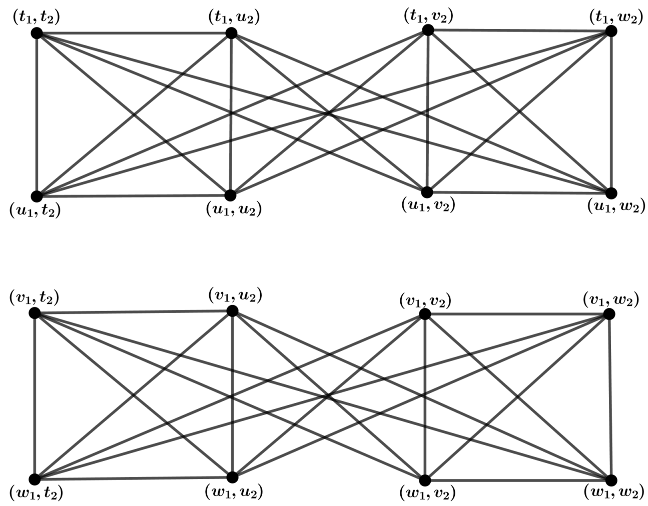



Figure 6. Composition of interval valued intuitionistic fuzzy soft graphs $\widetilde{\Gamma_1}$ and $\widetilde{\Gamma_2}$.

**Theorem 4.3.** *If* $\widetilde{\Gamma_1}$ *and* $\widetilde{\Gamma_2}$ *are two IVIF soft graphs, then so is* $\widetilde{\Gamma_1} \circ \widetilde{\Gamma_2}$.

**Proof:** The proof is similar to Theorem 4.1.

## 5. Application of interval-valued intuitionistic fuzzy soft graphs in social media

Nowadays, many people are connected by social media. The connected people are affected (positively or negatively) by social media like news, advertisement, gossip, fake news, videos, messages, etc. We consider some people who are connected by social media like WhatsApp, Facebook, Instagram, Twitter, etc. We aim to find out the most affected person by social media.

Let us consider each person as a vertex whose MS and NMS degree represent how much they are affected and not affected by social media in a particular time interval. The edges represent the connection between two persons. Each edge's MS and NMS degree represent how much affected and not affected on each other by social media in a particular time interval.

We consider a time interval as one week. In a week, many people are connected on a social media. Some people have a good influence in his/her life by social media and there is a little bit no influence by the same social media. For other people, the influence may be reversed. It is tough to measure good influence as a point. So one can represent it by an interval. The degree of the good influence of a person by social media is [0.2,0.4]

Table 2. Vertex membership values of IVIF soft graph

| $\phi$ | Ratan | Suchismita | Suparna |
|---|---|---|---|
| $e_1$ | ([0.1,0.5],[0.2,0.2]) | ([0.2,0.4],[0.1,0.2]) | ([0.4,0.5],[0.2,0.3]) |
| $e_2$ | ([0.2,0.3],[0.2,0.2]) | ([0.1,0.6],[0.1,0.1]) | ([0.4,0.4],[0.2,0.3]) |
| $e_3$ | ([0.1,0.2],[0.1,0.1]) | ([0.1,0.3],[0.1,0.2]) | ([0.2,0.2],[0.2,0.2]) |
| $\phi$ | Hirak | Nilkamal | Chandra |
| $e_1$ | ([0.2,0.4],[0.1,0.3]) | ([0.1,0.5],[0.3,0.4]) | ([0.4,0.5],[0.2,0.3]) |
| $e_2$ | ([0.3,0.6],[0.1,0.2]) | ([0.1,0.4],[0.1,0.2]) | ([0.4,0.6],[0.2,0.3]) |
| $e_3$ | ([0.1,0.2],[0.1,0.2]) | ([0.2,0.3],[0.1,0.1]) | ([0.3,0.3],[]0.1,0.3) |

and there is no influence by the same social media on the person is considered as [0.1,0.2]. Similarly, the effectiveness and non-effectiveness of the other social media on other people is measured.

For finding the most affected person in social media, we consider some people who are connected in the social media say Ratan, Suchismita, Suparna, Hirak, Nilkamal and Chandra. The vertex set for this members is consider as $\acute{V} = \{\text{Ratan } (x_1), \text{Suchismita } (x_2), \text{Suparna } (x_3), \text{Hirak } (x_4), \text{Nilkamal } (x_5), \text{Chandra } (x_6)\}$. The vertices represent persons and edges represent connection between two persons. Let the set of attributes be $\acute{A} = \{e_1 = WhatsApp, e_2 = Facebook, e_3 = Twitter\}$. An IVIF soft graph $\Gamma = \{M(e) = (\phi(e), \psi(e)) : e \in \acute{A}\}$ is given by Table 2 and Table 3. An IVIF graph $M(e_1) = (\phi(e_1), \psi(e_1))$ with respect to $e_1$ is displayed in Figure 7 and tabular representation of this graph are given in Table 2 and Table 3.

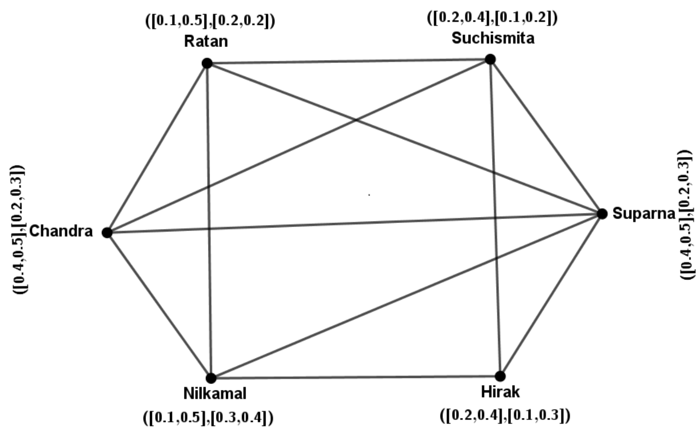



Figure 7. Interval valued intuitionistic fuzzy graph corresponding to attribute $e_1$.

The IVIF graphs $M(e_2) = (\phi(e_2), \psi(e_2))$ and $M(e_3) = (\phi(e_3), \psi(e_3))$ with respect to $e_2$ and $e_3$ are displayed in Figure 8. Tabular representations of these graphs are given in Table 2 and Table 3.

For finding the most affected person in social media, the core values are calculated by the following steps:

**Step 1:** The average of lower MS and upper MS values of each vertex (persons) with respect to $e_1$, $e_2$ and $e_3$ are calculated by the formula $S^A(x_i) = \frac{1}{2}[\sum_{j=1}^{3}(\mu^L_{\phi(e_j)}(x_i) + \mu^U_{\phi(e_j)}(x_i))]$.......(1), for each $i = 1, 2, \cdots, 6$, where $S^A(x_i)$ is average MS value of the vertex $x_i$ and $\mu^L_{\phi(e_j)}(x_i)$ and $\mu^U_{\phi(e_j)}(x_i)$ are the lower and upper MS values of the vertex

TABLE 3. Edge membership values of IVIF soft graph

| | | | |
|---|---|---|---|
| $\psi$ | $x_1x_2$ | $x_1x_3$ | $x_1x_5$ |
| $e_1$ | ([0.1,0.3],[0.2,0.3]) | ([0.1,0.4],[0.2,0.3]) | ([0.1,0.5],[0.3,0.4]) |
| $\psi$ | $x_1x_6$ | $x_2x_3$ | $x_2x_4$ |
| $e_1$ | ([0.1,0.5],[0.2,0.3]) | ([0.2,0.4],[0.2,0.3]) | ([0.2,0.4],[0.1,0.3]) |
| $\psi$ | $x_2x_6$ | $x_3x_5$ | $x_3x_4$ |
| $e_1$ | ([0.2,0.4],[0.2,0.3]) | ([0.1,0.5],[0.3,0.4]) | ([0.2,0.4],[0.2,0.3]) |
| $\psi$ | $x_3x_6$ | $x_4x_5$ | $x_5x_6$ |
| $e_1$ | ([0.3,0.5],[0.2,0.3]) | ([0.1,0.4],[0.3,0.4]) | ([0.1,0.5],[0.3,0.4]) |
| $\psi$ | $x_1x_2$ | $x_1x_4$ | $x_1x_6$ |
| $e_2$ | ([0.1,0.3],[0.2,0.2]) | ([0.2,0.3],[0.2,0.2]) | ([0.2,0.3],[0.2,0.3]) |
| $\psi$ | $x_2x_3$ | $x_2x_4$ | $x_2x_5$ |
| $e_2$ | ([0.1,0.4],[0.2,0.3]) | ([0.1,0.5],[0.1,0.2]) | ([0.1,0.4],[0.2,0.2]) |
| $\psi$ | $x_2x_6$ | $x_3x_4$ | $x_3x_6$ |
| $e_2$ | ([0.1,0.6],[0.2,0.3]) | ([0.3,0.4],[0.2,0.3]) | ([0.4,0.4],[0.3,0.3]) |
| $\psi$ | $x_4x_5$ | $x_4x_6$ | $x_5x_6$ |
| $e_2$ | ([0.1,0.4],[0.1,0.2]) | ([0.3,0.6],[0.2,0.3]) | ([0.1,0.4],[0.2,0.3]) |
| $\psi$ | $x_1x_2$ | $x_1x_3$ | $x_1x_4$ |
| $e_3$ | ([0.1,0.2],[0.1,0.2]) | ([0.1,0.2],[0.2,0.2]) | ([0.1,0.2],[0.1,0.2]) |
| $\psi$ | $x_1x_6$ | $x_2x_4$ | $x_2x_5$ |
| $e_3$ | ([0.1,0.2],[0.1,0.3]) | ([0.1,0.2],[0.1,0.2]) | ([0.1,0.3],[0.1,0.2]) |
| $\psi$ | $x_3x_4$ | $x_3x_5$ | $x_4x_5$ |
| $e_3$ | ([0.1,0.2],[0.2,0.2]) | ([0.2,0.2],[0.2,0.2]) | ([0.1,0.2],[0.1,0.2]) |
| $\psi$ | $x_4x_6$ | $x_5x_6$ | |
| $e_3$ | ([0.1,0.2],[0.1,0.3]) | ([0.2,0.3],[0.1,0.3]) | |

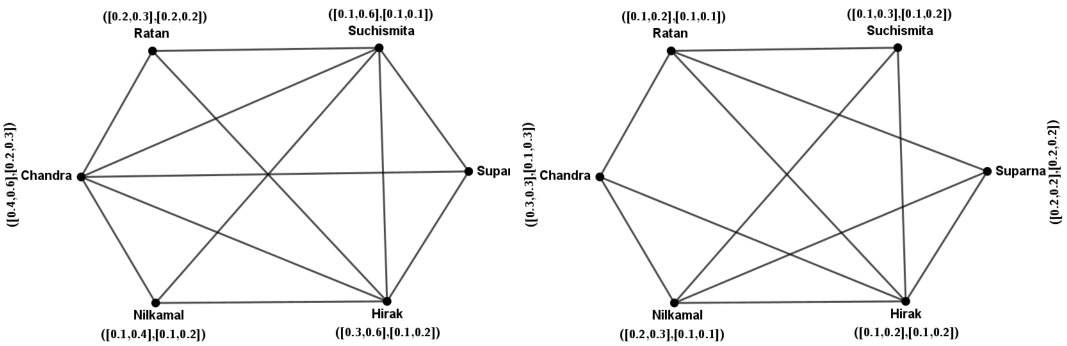



FIGURE 8. Interval valued intuitionistic fuzzy graph corresponding to attributes $e_2$ and $e_3$ respectively.

$x_i$ with respect to parameter $e_j$. From equation (1), we have determined the average effectiveness of each person due to using social media $e_1$, $e_2$ and $e_3$. For $i = 1$, $S^A(x_1) = \frac{1}{2}[\sum_{j=1}^{3}(\mu^L_{\phi(e_j)}(x_1) + \mu^U_{\phi(e_j)}(x_1))] = \frac{1}{2}[(\mu^L_{\phi(e_1)}(x_1) + \mu^U_{\phi(e_1)}(x_1)) + (\mu^L_{\phi(e_2)}(x_1) + \mu^U_{\phi(e_2)}(x_1)) + (\mu^L_{\phi(e_3)}(x_1) + \mu^U_{\phi(e_3)}(x_1))] = \frac{1}{2}[(0.1 + 0.5) + (0.2 + 0.3) + (0.1 + 0.2)] = 0.7$. Similarly, $S^A(x_2) = 0.85, S^A(x_3) = 1.05, S^A(x_4) = 0.9, S^A(x_5) = 0.8, S^A(x_6) = 1.25$. These values are shown in Table 4.

**Step 2:** We evaluate the average of lower NMS and upper NMS values of each vertices with respect to $e_1$, $e_2$ and $e_3$, using the formula $S^B(x_i) = \frac{1}{2}[\sum_{j=1}^{3}(\gamma^L_{\phi(e_j)}(x_i) +$

TABLE 4

| $S^A(x_1)$ | $S^A(x_2)$ | $S^A(x_3)$ | $S^A(x_4)$ | $S^A(x_5)$ | $S^A(x_6)$ |
|---|---|---|---|---|---|
| 0.7 | 0.85 | 1.05 | 0.9 | 0.8 | 1.25 |

TABLE 5

| $S^B(x_1)$ | $S^B(x_2)$ | $S^B(x_3)$ | $S^B(x_4)$ | $S^B(x_5)$ | $S^B(x_6)$ |
|---|---|---|---|---|---|
| 0.5 | 0.4 | 0.7 | 0.5 | 0.6 | 0.7 |

$\gamma^U_{\phi(e_j)}(x_i))]$.......(2) for each $i = 1, 2, 3, 4, 5, 6$ and the calculated values are given by the Table 5, where $S^B(x_i)$ is average NMS value of the vertex $x_i$ and $\gamma^L_{\phi(e_j)}(x_i)$ and $\gamma^U_{\phi(e_j)}(x_i)$ are the lower and upper NMS values of the vertex $x_i$ with respect to parameter $e_j$. From equation (2), we have determined the average non-effectiveness of each person due to using social media $e_1$, $e_2$ and $e_3$.

**Step 3:** Here, we evaluate the sum of average (lower and upper values) MS values of each vertices, affected by another vertices, with respect to the parameters $e_1$ (Figure 7), $e_2$ (Figure 8) and $e_3$ (Figure 8) using the formula $S^C(x_i) = \sum_{j=1}^{3}[\frac{1}{2}\{\sum_{l=1,l\neq i}^{6}(\mu^L_{\psi(e_j)}(x_ix_l) + \mu^U_{\psi(e_j)}(x_ix_l))\}]$.......(3) for each $i = 1, 2, 3, 4, 5, 6$. Any two vertices $x$ and $y$ are not connected by social media, then there is no edge between them. So, its MS and NMS values are 0, i.e, $\mu^L_{\psi(e_j)}(xy) = 0$, $\mu^U_{\psi(e_j)}(xy) = 0$, $\gamma^L_{\psi(e_j)}(xy) = 0$, $\gamma^U_{\psi(e_j)}(xy) = 0$. This sum of average MS value of each vertex $x_i$ is denoted by $S^C(x_i)$. Here $\mu^L_{\psi(e_j)}(x_ix_l)$ and $\mu^U_{\psi(e_j)}(x_ix_l)$ are the lower and upper MS values of the vertex $x_i$ affected by the vertex $x_l$ due to using social media $e_1$, $e_2$ and $e_3$. For $i = 1$,

$$
\begin{aligned}
S^C(x_1) &= \sum_{j=1}^{3}[\frac{1}{2}\{\sum_{l=2}^{6}(\mu^L_{\psi(e_j)}(x_1x_l) + \mu^U_{\psi(e_j)}(x_1x_l))\}] \\
&= \frac{1}{2}[\sum_{l=2}^{6}(\mu^L_{\psi(e_1)}(x_1x_l) + \mu^U_{\psi(e_1)}(x_1x_l))] + \frac{1}{2}[\sum_{l=2}^{6}(\mu^L_{\psi(e_2)}(x_1x_l) + \mu^U_{\psi(e_2)}(x_1x_l))] \\
&\quad + \frac{1}{2}[\sum_{l=2}^{6}(\mu^L_{\psi(e_3)}(x_1x_l) + \mu^U_{\psi(e_3)}(x_1x_l))] \\
&= \frac{1}{2}[\{(\mu^L_{\psi(e_1)}(x_1x_2) + \mu^U_{\psi(e_1)}(x_1x_2)) + (\mu^L_{\psi(e_1)}(x_1x_3) + \mu^U_{\psi(e_1)}(x_1x_3)) \\
&\quad + (\mu^L_{\psi(e_1)}(x_1x_5) + \mu^U_{\psi(e_1)}(x_1x_5)) + (\mu^L_{\psi(e_1)}(x_1x_6) + \mu^U_{\psi(e_1)}(x_1x_6))\} \\
&\quad + \{(\mu^L_{\psi(e_2)}(x_1x_2) + \mu^U_{\psi(e_2)}(x_1x_2) + (\mu^L_{\psi(e_2)}(x_1x_4) + \mu^U_{\psi(e_2)}(x_1x_4)) \\
&\quad + (\mu^L_{\psi(e_2)}(x_1x_6) + \mu^U_{\psi(e_2)}(x_1x_6))\} + \{(\mu^L_{\psi(e_3)}(x_1x_2) + \mu^U_{\psi(e_3)}(x_1x_2)) \\
&\quad + (\mu^L_{\psi(e_3)}(x_1x_3) + \mu^U_{\psi(e_3)}(x_1x_3)) + (\mu^L_{\psi(e_3)}(x_1x_4) + \mu^U_{\psi(e_3)}(x_1x_4)) \\
&\quad + (\mu^L_{\psi(e_3)}(x_1x_6) + \mu^U_{\psi(e_3)}(x_1x_6))\}] \\
&= \frac{1}{2}[\{(0.1 + 0.3) + (0.1 + 0.4) + (0.1 + 0.5) + (0.1 + 0.5)\} + \{(0.1 + 0.3)+ \\
&(0.2 + 0.3) + (0.2 + 0.3)\} + \{(0.1 + 0.2) + (0.1 + 0.2) + (0.1 + 0.2) + (0.1 + 0.2)\}] \\
&= 2.35.
\end{aligned}
$$

Table 6

| $S^C(x_1)$ | $S^C(x_2)$ | $S^C(x_3)$ | $S^C(x_4)$ | $S^C(x_5)$ | $S^C(x_6)$ |
|---|---|---|---|---|---|
| 2.35 | 2.65 | 3.05 | 3.2 | 2.7 | 3.55 |

Table 7

| $S^D(x_1)$ | $S^D(x_2)$ | $S^D(x_3)$ | $S^D(x_4)$ | $S^D(x_5)$ | $S^D(x_6)$ |
|---|---|---|---|---|---|
| 2.45 | 2.45 | 2.75 | 2.65 | 2.7 | 3.0 |

Table 8

| $F(x_1)$ | $F(x_2)$ | $F(x_3)$ | $F(x_4)$ | $F(x_5)$ | $F(x_6)$ |
|---|---|---|---|---|---|
| 0.1 | 0.65 | 0.65 | 0.95 | 0.2 | 1.1 |

Similarly, $S^C(x_2) = 2.65, S^C(x_3) = 3.05, S^C(x_4) = 3.2, S^C(x_5) = 2.7, S^C(x_6) = 3.55$. These core values are listed in Table 6.

**Step 4:** Sum of average (lower and upper values) NMS values of each vertices, affected by another vertices, with respect to the parameters $e_1$ (Figure 7), $e_2$ (Figure 8) and $e_3$ (Figure 8) are calculated by the formula $S^D(x_i) = \sum_{j=1}^{3}[\frac{1}{2}\{\sum_{l=1,l\neq i}^{6}(\gamma^L_{\psi(e_j)}(x_ix_l) + \gamma^U_{\psi(e_j)}(x_ix_l))\}]$.......(4) for each $i = 1, 2, 3, 4, 5, 6$. In Table 7, the score values are given. This sum of average NMS value of each vertex $x_i$ is denoted by $S^D(x_i)$. Here $\gamma^L_{\psi(e_j)}(x_ix_l)$ and $\gamma^U_{\psi(e_j)}(x_ix_l)$ are the lower and upper NMS values of the vertex $x_i$ affected by the vertex $x_l$ due to using social media $e_1$, $e_2$ and $e_3$.

**Step 5:** The final score value of the vertex $x_i$ is calculated by subtracting non-effectiveness score values from effectiveness score values using the equations (1), (2), (3) and (4), by the formula $F(x_i) = (S^A(x_i) + S^C(x_i)) - (S^B(x_i) + S^D(x_i))$ for each i=1,2,3,4,5,6. In Table 8, this score values are listed.

By Table 8, it is meant that the highest scoring value occurs by Chandra, which is 1.1. The second-highest scoring value is occurred by Hirak and the third is occurred by both Suchismita and Suparna. Hirak, Suchismita, Suparna, Nilkamal and Ratan score values are 0.95, 0.65, 0.65, 0.2 and 0.1, respectively. Hence the most affected person in social media is Chandra, second-most is Hirak, third-most are Suchismita and Suparna, fourth is Nilkamal and fifth is Ratan in a week.

## 6. Conclusion

In this article, new concepts of combining some graphs are introduced. Special types of IVIF soft graphs which is a combine representation of interval valued fuzzy graphs, intuitionistic fuzzy graphs and soft set theory in a new way, have been introduced and elaborately discussed throughout the paper. We have defined an IVIF soft graph and shown the new concepts about their properties as well as the proof of some theorems based on their aspects. The concept of IVIF soft graph is applied to a real-life application to find out the most affected person from a particular group of persons using social medias. In the application part of this paper, for this combine concept, any one can analysis the range of characteristic together with MS and NMS fact in a certain way. This analysis process is giving more efficient result with fuzziness than other existing fuzzy soft graphs. In future, we want to incorporate our research using the concept of soft set theory on bipolar fuzzy hyper-graphs and bipolar intuitionistic fuzzy graphs. As, bipolar sense includes positive as well as negative sense of any parameter; so introducing such new fuzzy soft graphs will be very interesting and helpful to solve many real-life problems with uncertainties and vagueness as well.

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