INSERTION LOSS DESIGN of RF FILTERS with MIXED LUMPED - DISTRIBUTED REALIZATION

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INSERTION LOSS DESIGN of RF FILTERS with MIXED LUMPED - DISTRIBUTED REALIZATION

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Abstract

In this study, the purpose was to develop a user friendly software tool which enables to design microwave filters with lumped circuit elements, distributed transmission lines and mixed lumped-distributed circuit elements. For the design tool, modern insertion loss approach is preferred because of several advantages such as flexible filter specification and easy control of filter characteristic. The work is mainly concentrated on low pass Butterworth and Chebyshev type filter designs. For the lumped element and distributed element filter designs the available insertion loss methods are explained and implemented. Solution to filter problems with lumped elements alone is well established in literature. However at microwave frequencies use of lumped elements or distributed elements alone in the circuit realization has serious implementation problems. Thus mixed lumped and distributed filter design have several advantages and flexibilities in monolithic integrated circuit layouts. Unfortunately, designing filters with both lumped and distributed elements has not been solved analytically yet. Beacuse of this reason, in this work a new approximation scheme is proposed to construct filters with lumped and distributed elements. Our new proposed approximation for mixed lumped-distributed filter design and microwave filter design tool will provide new possibilities and flexibilities in designing microwave filters.

KARIŞIK TOPLU-DAĞINIK GERÇEKLEŞTİRMEYLE RADYO FREKANS (RF) FİLTRELERİNİN GİRİŞ KAYBI TASARIMI

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Anahtar Kelimeler: Giriş kaybı tasarımı,Radyo Frekans (RF) filtreleri, Toplu devre elemanlı filtre tasarımı, Dağınık devre elemanlı filtre tasarımı, Karışık Toplu-Dağınık devre elemanlı filtre tasarımı

Özet

Bu çalışmadaki amaç, toplu devre elemanları, dağınık devre elemanları ve karışık toplu ve dağınık devre elemanları içeren, mikrodalga filtreleri tasarlayan kolay kullanılır bir yazılım aracı geliştirmekti. Filtre davranışını kolay kontrol etme ve esnek filtre belirtimi gibi birçok avantajlardan dolayı tasarım aracı için , modern giriş kaybı yaklaşımı tercih edildi.Çalışmada alçak geçiren Butterworth ve Chebyshev tipi filtre tasarımlarında odaklanıldı.Toplu eleman ve dağınık eleman filtre tasarımları için giriş kaybı methodu açıklandı. Literatürde toplu elemanlardan oluşan filtre problemlerinin çözümü iyi belirlenmiştir. Fakat mikrodalga frekanslarda toplu elemanların veya dağınık elemanların tek başına devre gerçekleştirmelerinde kullanılması ciddi uygulama problemlerine neden olmaktadır. Bu nedenle karışık toplu ve dağınık filtre tasarımı birçok avantaj ve tümleşik devre düzenlerinde esnekliğe sahiptir. Maalesef karışık toplu ve dağınık elemanlardan oluşan filtre tasarımı analitik olarak henüz çözümlenememiştir. Bu sebeble, bu çalışmada karışık toplu ve dağınık elemanlardan oluşan filtreleri kurmak için yeni bir yaklaşım düzeni önerildi. Karışık toplu ve dağınık filtre tasarımı için önerdiğimiz yeni yaklaşım ve mikrodalga filtre tasarım aracı, mikrodalga filtre tasarımında yeni olanaklar ve esneklikler sağlayacaktır.

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CHAPTER 1

INTRODUCTION

Radio frequency (RF) filter is a two-port network used to control the frequency response of a high frequency system. In the literature there exist different approaches of filter design for high frequency applications. Most commonly used design methods are known as the image parameter method and the insertion loss method. Image parameter method consists of cascade of two-port filter sections to provide desired cutoff frequencies. Image filter design method is simple, however it must be iterated many times to achieve desired values. Second method is the insertion loss based design. This method uses network synthesis technique to design filter for specified frequency response. The design is simplified by low-pass filter prototypes. Transformation can be then applied to convert the other types of filters such as high-pass, band-pass and stop-band.

In this thesis, we studied insertion loss based filter design approach because of its advantages over the image parameter approach. In the work, three practically important types of filter designs are investigated. These are lumped element design, distributed element design and mixed lumped-distributed element design. For the lumped element and distributed element filter designs the available insertion loss methods are explained and implemented. For the mixed lumped-distributed filters a new transformation based approximation technique is developed.

Solution to filter problems with lumped elements alone is well established in literature. However at microwave and millimeter-wave frequencies use of lumped elements alone in the circuit realization has serious implementation problem because of the difficulties in the physical interconnection of components and parasitic effects. Therefore distributed structures composed of transmission lines can be used. Unfortunately, using distributed structures results in undesired harmonics in the performance of the system. Moreover, there exist further difficulties in the physical realization of lumped and distributed network elements during the implementation of microwave filter circuits. Thus lumped and distributed (mixed) filter design have several advantages and flexibilities in monolithic integrated circuit layouts. Designing filters with both lumped and distributed elements has not been solved analytically yet. An analytic treatment of this design requires characterization of the mixed element structures using multivariable functions. In this context, there have been valuable contributions for the characterization of some mixed element topologies. However there is still not available a complete theory for the approximation and synthesis problems for the mixed element networks.

The basic approach in obtaining filter networks with both lumped and distributed elements is to construct is realizable two-variable network functions from single-variable network functions. In this thesis, we gather the insertion loss design technique with an appropriate application of two-variable transformation and network replacement techniques.

We propose a new approximation scheme to construct filters with lumped and distributed elements. The proposed design approach for the mixed element lumped-distributed filters is presented with illustrative design examples and an integrated design tool on MATLAB platform is developed. The developed microwave filter design tool offers options for designing lumped, distributed and mixed element filters.

In chapter 2 of the thesis, some fundamental theoretical concepts of filters are reviewed in a brief manner. Insertion loss based filter design for Butterworth and Chebyshev filters with only lumped and distributed elements are studied. Applications of these filter design techniques are illustrated with design examples using the developed microwave filter design tool.

In chapter 3, construction of mixed lumped distributed filters is discussed. Here a new design approach is proposed and elaborated. In this part, proposed approaches for the generation of two-variable transfer function, two-variable ladder network and transformation of two-variable ladder into lumped and distributed filter are discussed. Using our approximation for two-variable low pass ladder with unit element design, we produce tabularized new filter circuit element values for mixed element Butterworth and Chebyshev type ladders.

In chapter 4, the MATLAB based filter design tool 'Microwave Filter Designer' is presented and explained with examples .

In chapter 5 concluding remarks are summarized.

CHAPTER 2

RF FILTER DESIGN

Passive RF filter is a two-port passive network used to control the frequency response at certain point in a microwave system by providing transmission at frequencies within the pass band of the filter and attenuation in the stop band of the filter. There exist an extensive work on the filter design in the literature. The issue is well elaborated and several design approaches are available. Among these approaches the image parameter approach and the modern insertion loss based CAD techniques are commonly used for RF and microwave filter design applications.

The image parameter method consist of a cascade of a simpler two-port filter sections to provide the desired cutoff a frequency response over the operating range. The procedure of image filter design is simple, and it must be iterated many times to achieve the desired results. Insertion loss method uses network synthesis techniques to design filters with a completely specified frequency response. The design is simplified by beginning low-pass filter prototypes that are normalized with respect to impedance and frequency. Transformations are then applied to convert the prototype designs to the desired frequency range and impedance level. Both the image parameter and insertion loss method of filter design provide lumped element circuits. Moreover for microwave applications designs must be modified to use distributed elements of transmission line sections. The Richard's transformation and Kuroda identities provide this step.

In this chapter filter design with only lumped or only distributed elements will be investigated along with the introduction of fundamental concepts. The insertion loss based design approaches for the lumped element and distributed element filters will be presented .

2.1 Fundamental Concepts on Filters

The function of a filter is to separate different frequency components of the input signal that passes through the filter network. Filters may be classified in a number of ways. For example, analog filters are used to process analog signals, which are a function of a continuous time variable. Digital filters, on the other hand, process digitized continuous waveforms. Analog passive filters may be classified as lumped element or distributed element devices. We may also classify filters as passive or active depending on the type of elements used in their construction. Five basic types of selective networks are commonly referred to in filter design. They include low-pass, high-pass, band-pass, all-pass and band-stop filters. The characteristics of the network are specified by a transfer function $H(p)$, where $p= j\omega$ represents the complex frequency defined for the Laplace transform. The transfer function is the ratio of output signal to input signal, voltage, or current:

$$\mathrm{H}(j\omega) = \frac{V_{out}}{V_{in}} = |H(j\omega)|\,\mathrm{e}^{\,j\phi(\omega)}$$
(2.1)

A filter network passes some of the input signal frequencies and stops others, and being a linear circuit, this function is performed without adding or generating new frequency components. The frequency band that passes, ideally without losses, defines the passband, and the band that stops the frequencies, ideally with infinite loss, is called the stopband. Figure 2.1.a shows this loss representation of the ideal low-pass filter with a pass band corner of ω_c. The frequency, ω_c, is called the cut-off frequency of the filter. An ideal low-pass filter is physically not realizable as this requires a circuit with an infinite number of elements due to an abrupt change from passband to stopband. In an actual filter transfer characteristic, the transition band is the frequency range that separates the passband and stopband where the loss make a transitions from a minimum to a maximum value.

As the selectivity defined by the transition band approaches the ideal steep characteristic, the more complex and costly the filter becomes. Similar considerations can be applied in the design of filters using phase linearity and/or group delay flatness. The concept of passband, stopband, and transition band permits specifications of five major types of filters whose transmission behaviors are as shown in Figure 2.1.

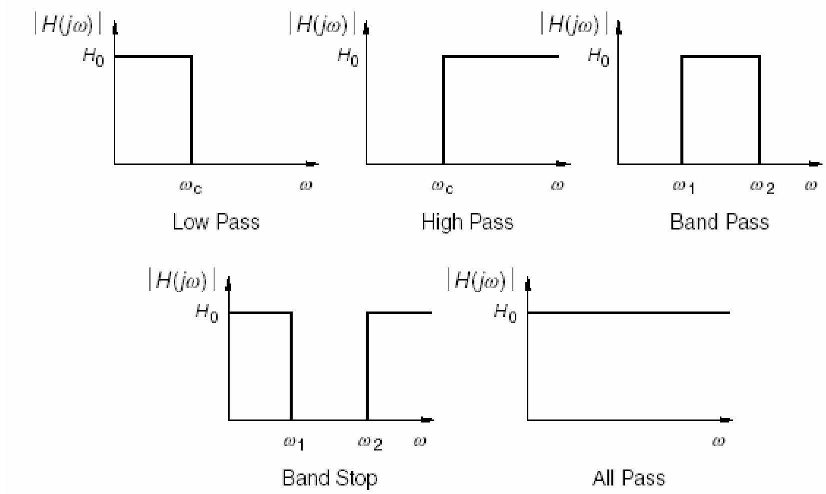


Figure 2.1 Transfer function characteristics for major filter types

The transfer function can easily be transformed from the time to the frequency domain when losses are small so that p= jω. The filter transfer function is the ratio of the output signal voltage to the input signal voltage (or current) and can be written as a ratio of two polynomials:

$$H(p) = \frac{P(p)}{Q(p)} = \frac{a_0 + a_1 p + a_2 p^2 + ...+ a_{m-1} p^m}{b_0 + b_1 p + b_2 p^2 + ...+ b_{n-1} p^n}$$
(2.2)

where polynomials P(p) and Q(p) in general are of order m and n. These polynomials are Hurwitz stable, which requires that the order of the numerator polynomial m be equal to or less than the denominator polynomial n, m ≤ n.

The order of polynomial Q(p) is the order of the filter as well. Polynomials P(p) and Q(p) can be factored and rewritten in the form

$$H(p) = \frac{(p-z_1)(p-z_2)(p-z_3)......(p-z_m)}{(p-k_1)(p-k_2)(p-k_3)......(p-k_n)}$$

(2.3)

The values $z_1, z_2, z_3, \ldots, z_m$, are called the zeros of the transfer function, or simply transmission zeros. The roots of Q(p), $k_1, k_2, k_3, \ldots, k_n$, are the poles of the transfer function. The poles and zeros can be real or complex, but complex poles and zeros must occur in conjugate pairs. The magnitude plot of voltage transfer function represents the loss or attenuation of the filter circuit, and in dB is given by

$$L_{dB} = 20 \log |H(p)|$$

(2.4)

Poles and zeros of realizable passive networks must follow certain rules:

- All poles of a transfer function occur in the left half p-plane. The left half p-plane includes the imaginary jω-axis.
- Complex poles and zeros occur in complex conjugate pairs. However, on the imaginary axis, poles and zeros may exist singly.[1]

2.2 Insertion Loss Based Filter Design

The insertion loss method allows a high degree of control over the passband and stopband amplitude and phase characteristics with a systematic way to synthesize a desired response. The necessary design can be evaluated to best meet the application requirements. If for example, a minimum insertion loss is most important, a binomial response could be used. Chebyshev response would satisfy a requirements for the sharpest cutoff. If it is possible to sacrifice the attenuation rate, a better phase response can be obtained by using a linear phase filter design. In all cases, the insertion loss method allows filter performance to be improved in a straightforward manner, at high order filter. In the insertion loss method a filter response is defined by its insertion loss or power loss ratio P_{LR},

$$P_{LR} = \frac{P_S}{P_l} = \frac{1}{1-|\Gamma(\omega)|^2}$$

(2.5)

where; P_l = Power delivered to the load, P_s = Power available from the source and Γ = the reflection coefficient at the input port. The insertion loss (IL) in dB is

$$IL = 10 \log P_{LR}$$

(2.6)

We know that $|\Gamma(\omega)|^2$ is an even function of ω, therefore it can be expressed as a polynomial in ω^2 .We can write

$$|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)}$$

(2.7)

Where M and N are real polynomials in ω^2.If we substitute this form in power loss ratio gives following form.

$$P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

(2.8)

For a filter to be physically realizable its power loss ratio must be given in this form.

- **Maximally Flat:**

This characteristic is also called the binomial or Butterworth response. It provides the flattest possible pass band response for a given filter complexity or order. For a low-pass filter, it is specified by

$$P_{LR} = 1 + k^2 (\omega / \omega_c)^{2n}$$

(2.9)

where n is the order of filter and ω_c is the cutoff frequency. The passband extends from ω=0 to $\omega = \omega_c$, at the band edge the power loss ratio is $1 + k^2$.If we choose this as -3 dB point, we have k=1. For $\omega \rangle \omega_c$, the attenuation increase monotonically with frequency. For $\omega \rangle\rangle \omega_c$ $P_{LR} \cong k^2 (\omega / \omega_c)^{2n}$ which shows that the insertion loss increases at the rate of 20 dB /decade.

- **Equal Ripple:**

This characteristic is also called the Chebyshev response. If Chebyshev polynomial is used to specify the insertion loss of an n order low-pass filter as

$$P_{LR} = 1 + k^2 T_n (\omega / \omega_c)$$

(2.10)

The pass band response will have ripples of amplitude $1 + k^2$, since $T_n(x)$ oscillates between ± 1 for $|x| \leq 1$.Thus k^2 determines the pass band ripple level. For large x, $T_n(x) \cong 1/2(2x)^n$ so for $\omega \rangle\rangle \omega_c$, the insertion loss becomes

$$P_{LR} \cong \frac{k^2}{4} \left(\frac{2\omega}{\omega_c} \right)^{2n}$$

(2.11)

The insertion loss for the Chebyshev case is $(2^{2n})/4$ greater than Butterworth response, at any given frequency where $\omega \rangle\rangle \omega_c$.

2.3 Low-Pass Lumped Prototype Filter Design

Insertion loss based design method uses network synthesis techniques to design a filter with a completely specified frequency response. The design is simplified by beginning with a low-pass filter prototype that is normalized with respect to impedance and frequency. Once the low-pass prototype is obtained, transformations are then applied to convert the prototype design to the desired frequency range and impedance level. Two fundamental low-pass characteristics are commonly preferred for the prototype filter design. These are Butterworth and Chebyshev type filters.

2.3.1 Butterworth Filter

The Butterworth, or "maximally flat" response provides the flattest possible pass band response for a given filter complexity. A filter with many reactive elements would be expected to more closely approximate an ideal filter with rectangular shape than one with few reactive elements. For a filter with n poles (n reactive elements), the low pass Butterworth approximation provides the maximum flatness in its passband near $\omega = 0$.

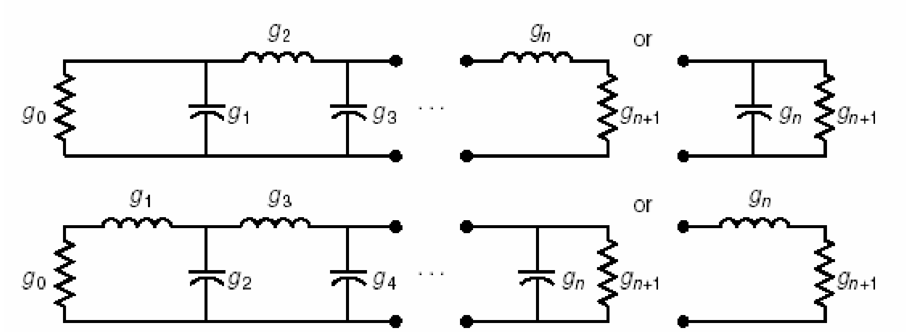


Figure 2.2 Low-pass filter network topology

The gain function for this type of filter is given by

$$\left|H(j\omega)\right|^2 = G_T = \frac{H_0}{1+(\omega/\omega_c)^{2n}} \tag{2.12}$$

where $H_0 \le 1$. The first 2n-1 derivatives of the denominator of this function are all zero at ω=0, implying that it is maximally flat. The poles of this function all have a magnitude of 1 and are separated from one another on the unit circle by π/n radians. Furthermore there are no poles on the jω axis.

Often minimum requirements are placed on the shape of the passband. In this instance the minimum number of poles needed to produce a desired specification is

$$n = \frac{\log[(10^{\alpha_{\min}/10} - 1)(10^{\alpha_{\max}/10} - 1)]}{2\log(\omega_s / \omega_p)}$$

(2.13)

In this expression the maximum attenuation in the passband $0 \leq \omega \leq \omega_p$ is $\alpha_{\max}$. The minimum attenuation in the stop band, $\omega_s \leq \omega < \infty$, is $\alpha_{\min}$.

At the edge of the passband, the filter attenuates the power by ½ or -3 dB. A recursion formula for the filter elements g_k as indicated in Figure 2.2 that would produce this response can be found in a variety of references [2].

$$g_0 = g_{n+1} = 1, \quad g_k = 2\sin\left[\frac{(2k-1)\pi}{2n}\right] \qquad k=1,2,3,\ldots,n$$

(2.14)

For a normalized low-pass design where the source and load terminations are 1 Ω, if the cutoff frequency is set as ω_c=1 rad/s and passband ripple is assumed 3 dB, the element values are calculated as in Table 2.1 [2].

It is possible to scale the response to have other attenuation levels at ω=1 rad/s. For an attenuation of K_p in dB:

$$\omega_{K_p} = (10^{0.1K_p} - 1)^{1/(2n)}$$

(2.15)

In order for the filter to have K_p attenuation at ω=1 rad/s , the 3 dB case pole positions or component values must be scaled ω_{K_p} value.

Table 2.1 Normalized low pass Butterworth filter element values (ω_c =1 rad/s ; 3 dB passband ripple)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}
1	2.000									
2	1.414	1.414								
3	1.000	2.000	1.000							
4	0.765	1.847	1.847	0.765						
5	0.618	1.618	2.000	1.618	0.618					
6	0.517	1.414	1.931	1.931	1.414	0.517				
7	0.445	1.246	1.801	2.000	1.801	1.246	0.445			
8	0.390	1.111	1.662	1.961	1.961	1.662	1.111	0.390		
9	0.347	1.000	1.532	1.879	2.000	1.879	1.532	1.000	0.347	
10	0.312	0.907	1.414	1.782	1.975	1.975	1.782	1.414	0.907	0.312
	L_1	C_2	L_3	C_4	L_5	C_6	L_7	C_8	L_9	C_{10}

Referring to Figure 2.2 , for a normalized low-pass design where the source and load terminations are 1 Ω , the cutoff frequency is ω_c=1 rad/s and passband ripple 1 dB , the element values are calculated as in Table 2.2 [2].

Table 2.2 Normalized low pass Butterworth filter element values (ω_c =1 rad/s; 1 dB passband ripple)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}
1	1.017									
2	1.008	1.008								
3	0.798	1.596	0.798							
4	0.646	1.560	1.560	0.646						
5	0.539	1.413	1.747	1.413	0.539					
6	0.562	1.263	1.726	1.726	1.263	0.562				
7	0.404	1.132	1.636	1.815	1.636	1.132	0.404			
8	0.358	1.021	1.528	1.802	1.802	1.528	1.021	0.358		
9	0.322	0.927	1.421	1.743	1.855	1.743	1.421	0.927	0.322	
10	0.292	0.848	1.321	1.665	1.846	1.846	1.665	1.321	0.848	0.292
	L_1	C_2	L_3	C_4	L_5	C_6	L_7	C_8	L_9	C_{10}

Example 2.1

According to Table 2.1 normalized Butterworth filter structure and element values for order 5 are given in Figure 2.3 and the transfer function plot is as shown in Figure 2.4.

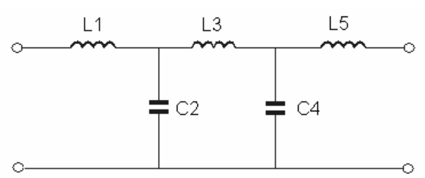


Figure 2.3 Low-pass Butterworth filter network topology for order 5 with normalized element values: L1= 0.61803 H, C2=1.61803 F, L3= 2.0 H, C4=1.61803 F, L5= 0.61803 H

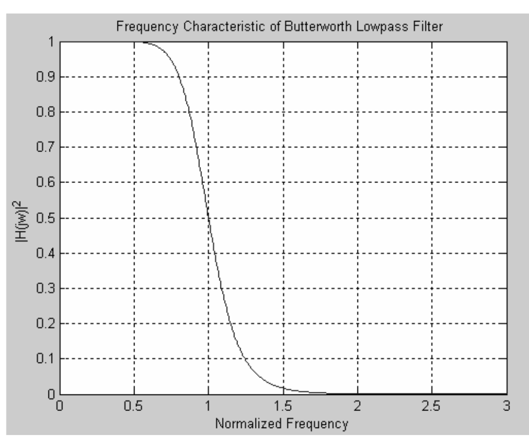


Figure 2.4: Frequency response of low-pass Butterworth filter for order 5

2.3.2 The Chebyshev Filter

In filter design the Chebyshev function provides the maximum possible bandwidth for a given passband ripple or the minimum possible passband ripple for a given bandwidth. The Chebyshev (equal ripple) low-pass filter transducer gain function is

$$\left|H(j\omega)\right|^2 = G_T = \frac{H_0}{1+\varepsilon^2 T^2{}_n(\omega/\omega_c)}$$

(2.16)

where ω_c is the low-pass cutoff frequency. The value ε is a number < 1 and is a measure of the passband ripple. The Chebyshev function, $T_n(x)$, oscillates between +1 and -1 when its argument is less than 1. The poles of this transfer function lie on an ellipse with no jω axis poles. For x > 1, $T_n(x)$ rapidly becomes large. The Chebyshev function can be written in a form that clearly shows this characteristic:

$$T_n(x) = \cos[n \times \arccos(x)] \qquad 0 \le x \le 1$$
$$T_n(x) = \cosh[n \times \arccos h(x)] \qquad x>1$$

(2.17)

Since $T_n(x) < 1$ in the passband, the passband transfer function is

$$\frac{1}{1+\varepsilon^2} \le \left|H(j\omega)\right|^2 \le 1$$

(2.18)

For an attenuation of K_p in dB, ε can be calculated as

$$\varepsilon = \sqrt{10^{0.1Kp} - 1}$$

(2.19)

Outside the passband, $T_n(x)$ increases approximately exponentially. The Chebyshev functions can be found in terms of a polynomial of its argument from a recursion formula:

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$$

(2.20)

The formula begins by setting $T_0(x) = 1$ and $T_1(x) = x$. Furthermore for n odd $T_n(0) = 0$ and $T_n(\pm 1) = \pm 1$, while for n even $T_n(0) = (-1)^{n/2}$ and $T_n(\pm 1) = 1$.

The next few Chebyshev functions are shown below:

$$T_2(x) = 2x^2 - 1$$
$$T_3(x) = 4x^3 - 3x$$
$$T_4(x) = 8x^4 - 8x^2 + 1$$
$$T_5(x) = 16x^5 - 20x^3 + 5x$$
$$T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1$$
$$T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x$$
$$T_8(x) = 128x^8 - 256x^6 + 160x^4 - 32x^2 + 1$$
$$T_9(x) = 256x^9 - 576x^7 + 432x^5 - 120x^3 + 9x$$

(2.21)

Just as in the Butterworth approximation, there is a set of recursion formulas for the Chebyshev filter. Finding expressions for the g values for the filter requires first expanding the Chebyshev functions by its own set of recursion formulas. The low-pass prototype filter structure (for a given number n of reactive elements) is then equated to the nth order filter function so that a correlation is made between the circuit and the function.

One important difference between the Butterworth and Chebyshev approximations is the value for g_{n+1}. The unequal impedance levels for the even-order Chebyshev termination impedances is often avoided by simply restricting the choices of n for the Chebyshev function to odd values. The circuit element values for these two filter functions were found by using network synthesis techniques after determining the poles of the transfer function.

The recursive element value expression for Chebyshev filters with prescribed ripple and order are calculated as given below[2]

$$g_1 = \frac{2a_1}{\sinh \beta l 2N},$$

$$\beta = \ln \frac{\sqrt{1+k^2}+1}{\sqrt{1+k^2}-1},$$
$$b_k = \sinh^2 \frac{\beta}{2N} + \sin^2 \frac{k\pi}{N},$$
$$a_k = \sin \frac{2k-1}{2N} \pi,$$
$$g_k = \frac{4a_{k-1}ak}{b_{k-1} g_{k-1}} \qquad \text{k=1,2,3,…,n}$$

(2.22)

For different passband ripple levels (which is a function of ε) there exist tabularized filter element values for a Chebyshev response [1]. A typical ripple case with 1 dB passband ripple is given in Table 2.3 [1].

Table 2.3 Normalized low pass Chebyshev filter element values(ω_c=1 rad/s; 1 dB pass band ripple)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}
1	1.017									
2	1.821	0.685								
3	2.023	0.994	2.023							
4	2.099	1.064	2.831	0.789						
5	2.134	1.091	3.000	1.091	2.134					
6	2.154	1.104	3.063	1.157	2.936	0.810				
7	2.166	1.111	3.093	1.173	3.093	1.111	2.166			
8	2.174	1.116	3.110	1.183	3.148	1.169	2.968	0.817		
9	2.179	1.119	3.121	1.189	3.174	1.189	3.121	1.119	2.179	
10	3.538	0.777	4.676	0.813	4.742	0.816	4.726	0.805	4.514	0.609
	L_1	C_2	L_3	C_4	L_5	C_6	L_7	C_8	L_9	C_{10}

If the maximum passband frequency is ω_c and the minimum stopband frequency beyond which the attenuation is always greater than $\alpha_{\min}$, is ω_s, then the number of poles required in the function is n :

$$n = \frac{\operatorname{arccos}h\left[\frac{1}{\varepsilon}(10^{\alpha \max/10} - 1)^{-1/2}\right]}{\operatorname{arccos}h(\omega_s / \omega_c)}$$

(2.23)

Example 2.2

According to Table 2.3 normalized Chebyshev element values for order 5 are given in Figure 2.5 and the transfer function plot is as shown in Figure 2.6.

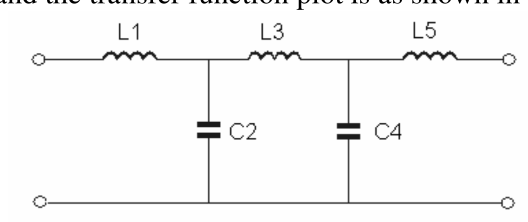


Figure 2.5 Low pass Chebyshev network topology for order 5 with normalized element values:L1= 1.58846 H, C2=1.78565 F, L3= 1.83856 H, C4=1.48558 F, L5= 0.76996 H

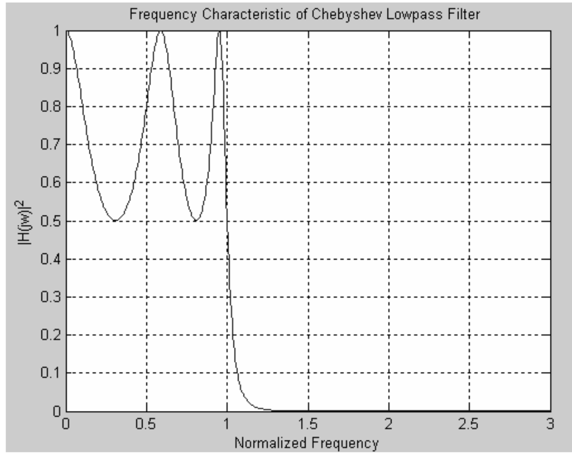


Figure 2.6 Frequency response of low-pass Chebyshev filter for order 5 with 3 dB ripple

2.4 Distributed Element Filter Design

The lumped filter design generally works well at low frequencies, but two problem arise at microwave frequencies. First, lumped elements such as inductor and capacitors are available only for a limited range of values and are difficult to implement at microwave frequencies and they must be approximated with distributed components. In addition, at microwave frequencies the distances between components is not negligible. Richard's transformation is used to convert lumped elements to transmission line sections, while Kuroda's identities can be used to separate filter elements by using transmission line sections. Such additional transmission line sections do not affect the filter response.

- **Richard's Transformation**

The transformation

$$\Omega = \tan \beta l \tag{2.24}$$

This transformation introduced by Richard to synthesize an LC network using open and short circuited transmission lines. If we replace the frequency variable ω with Ω, the reactance of an inductor can be written as

$$jX_L = j\Omega L = jL \tan \beta l \tag{2.25}$$

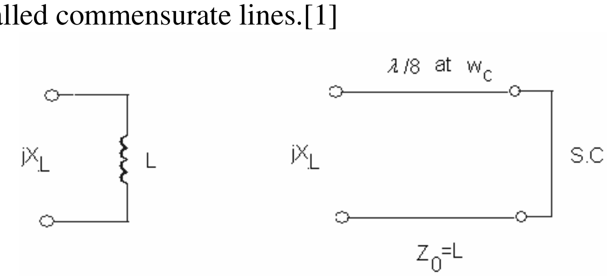
and the susceptance of a capacitor can be written as

$$jB_C = j\Omega C = jC \tan \beta l \tag{2.26}$$

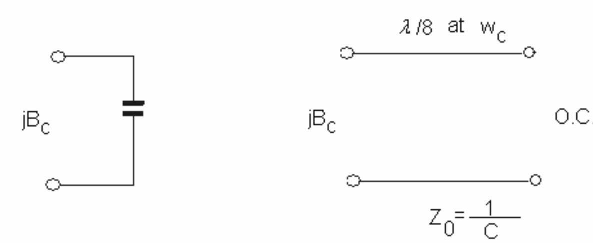
These results indicate that an inductor can be replaced with a short circuited stub of length βl and characteristic impedance L, while capacitor can be replaced with an open circuited stub of length βl and characteristic impedance 1/C. Cutoff occur at unity frequency for low-pass filter prototype. To obtain the same cutoff frequency for Richard's transformed filter, we used this equation.

$$\Omega = 1 = \tan \beta l \tag{2.27}$$

which gives a stub length of $l = \lambda / 8$ where λ is the wavelength of the line at cutoff frequency ω_c. At the frequency $\omega_0 = 2\omega_c$, the lines will be $\lambda/4$ long, and attenuation pole will occur. At frequencies away from ω_c, the impedances and the filter response will differ from desired prototype response. Also, the response will be periodic in frequency repeating every ω_c .The inductors and capacitors of lumped element filter design can be replaced with short circuited and open circuited stubs as illustrated in Figure 2.7.Since the lengths of all the stubs are the same $\lambda/8$ at ω_c, these lines are called commensurate lines.[1]



(a)



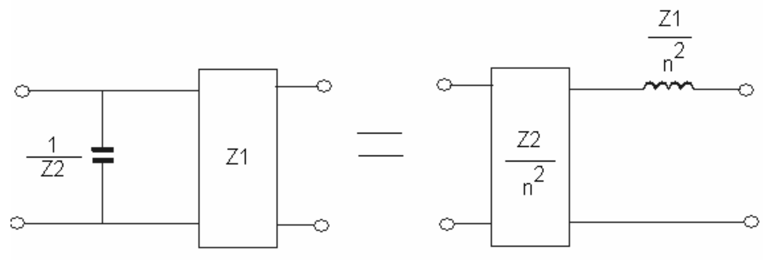
(b)

Figure 2.7 Richard's transformation (a) For inductor to short circuited stub (b)For a capacitor to an open circuited stub

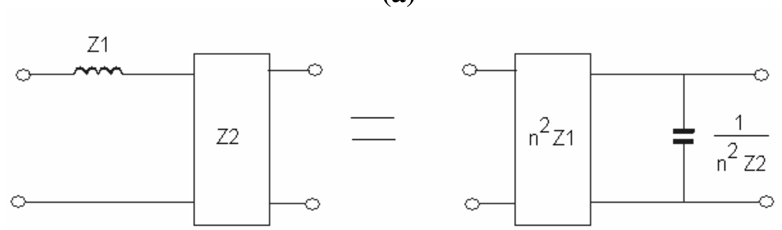
- **Kuroda's Identities**

The four Kuroda identities utilize redundant transmission line sections to achieve a more practical microwave filter implementation. Those additional transmission line sections are called unit elements and are $\lambda/8$ long at ω_c, the unit elements are commensurate with the stubs used to implement the inductors and capacitors of the prototype design.

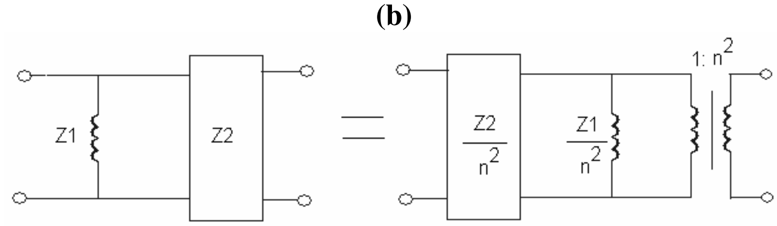
The four identities are illustrated in Figure 2.8 where each box represents a unit element or transmission line of the indicated characteristic impedance and length ($\lambda/8$ at ω_c).The inductors and capacitors represents short circuit and open circuit stubs respectively.[1]



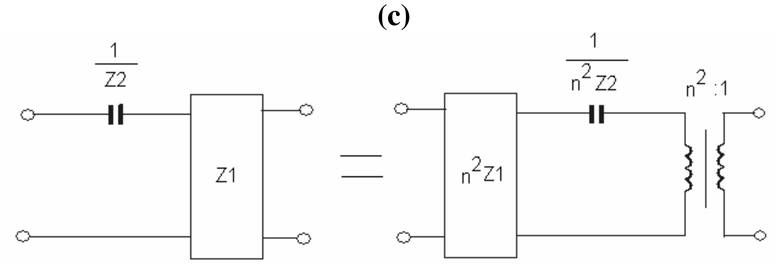
(a)



(b)



(c)



(d)

Figure 2.8 The Four Kuroda identities

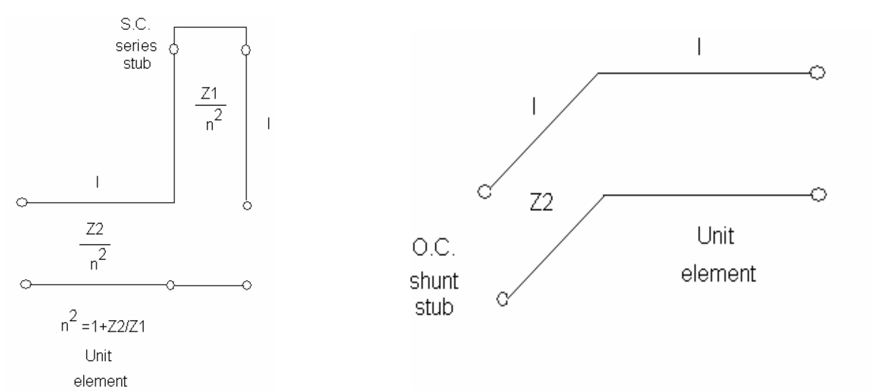


Figure 2.9 Equivalent circuit Kuroda identity of Figure 2.8(a)

2.4.1 Filter Design with Cascaded Transmission Lines

The most important application of commensurate line or unit element synthesis methods, particularly in the microwave region, has been to problems of insertion loss design. The scattering formalism is most useful in considering the insertion loss problem. If we consider a general lossless reciprocal two-port in the λ domain The scattering matrix of the two-port is

$$S(\lambda)=\begin{bmatrix} s_{11}(\lambda) & s_{12}(\lambda) \\ s_{21}(\lambda) & s_{22}(\lambda) \end{bmatrix} \tag{2.28}$$

Except for a possible branch point pair at $\lambda = \pm 1$, the $s_{ij}(\lambda)$ are analytic in Re $\lambda \geq 0$. As we see for a cascade of lines forming a two-port, under $\lambda = \tanh\Gamma$, $s_{12}(\lambda)$ may be irrational but $s_{11}(\lambda)$, $s_{22}(\lambda)$ are rational. However, all products $s_{ij}(\lambda)\, s_{ij}(-\lambda)$ are even and rational. The unitary requirement demands

$$\left|s_{11}(j\Omega)\right|^2 = \left|s_{22}(j\Omega)\right|^2 = 1 - \left|s_{12}(j\Omega)\right|^2$$

or

$$s_{11}(\lambda)\, s_{11}(-\lambda) = s_{22}(\lambda)\, s_{22}(-\lambda) = 1 - s_{12}(\lambda)\, s_{12}(-\lambda) \tag{2.29}$$

If the scattering matrix normalization numbers at the two-port r_1 and r_2 are real and positive then if r_1, r_2 terminate the two port, we have

$$\left|s_{12}(j\Omega)\right|^2 = \frac{P_2(\Omega)}{P_{A1}(\Omega)} = \text{Available gain} \tag{2.30}$$

where P_{A1} is the available r_1 generator power at port 1 and P_2 is the power delivered to r_2 .Since 10 log $\left|1/s_{12}\right|^2$ is generally defined as the insertion loss. It is clear that we can examine the properties of $s_{12}(\lambda)$ for a cascade of unit elements.

If the unit element (UE) is lossless then generally in a passband region $\lambda = \tanh\Gamma(p)$ takes the real frequency axis $j\omega$ into $j\Omega$ in the λ domain. Specifications in true frequency ω are merely transformed into a similar characteristic but plotted against a distorted frequency scale in $j\Omega$**.** The change in frequency scaling is generally no impediment to rational design. On the other hand, if the UE is dissipative the imaginary axes in Richard's transformation do not map into each other. Even in the lossless case we have the problem that the reference UE characteristic impedance r(p) may be frequency dependent even though real.

The synthesis process has been extended by Kinariwala to a cascade of unequal length lines. The method uses an exponential frequency transformation [3].

For commensurate line structure ,we consider available gain function $|s_{12}(j\Omega)|^2$

$$\lambda = \tanh\Gamma = \tanh\gamma L \tag{2.31}$$

and at real radian frequencies $\omega = 2\pi f$

$$\lambda = j\Omega = j\tanh\beta L = j\tan\omega\tau \tag{2.32}$$

where the fixed delay length of the UE is $\tau = \frac{L}{v}$ with v the propagation velocity on the line.At real frequencies we can write the polynomial $P_n(\Omega^2)$ as

$$P_n(\Omega^2) = C_0 + C_2 + ...+ C_{2n}\Omega^{2n} \tag{2.33}$$

Hence by,

$$P_n(\Omega^2) = C_0 + C_2\tan^2\omega\tau + ...+ C_{2n}\tan^{2n}\omega\tau \tag{2.34}$$

Furthermore, the numerator of $|s_{12}(j\Omega)|^2$ is $(1+\Omega^2)^n = \sec^{2n}\omega\tau$ (2.35)

Dividing numerator and denominator of $|s_{12}(j\Omega)|^2 = \frac{(1+\Omega^2)^n}{P_n(\Omega^2)}$ by $(1+\Omega^2)^n = \sec^{2n}\omega\tau$ yields (2.36)

$$|s_{12}(j\Omega)|^2 = \frac{1}{C_0\cos^{2n}\omega\tau + C_2\sin^2\omega\tau\cos^{2n-1}\omega\tau + ...+ C_{2n}\sin^{2n}\omega\tau} \tag{2.37}$$

Since each denominator term is even in both sin($\omega\tau$) and cos($\omega\tau$), one can make this denominator an even polynomial in either of these quantities by using

$$\sin^2\omega\tau + \cos^2\omega\tau = 1 \tag{2.38}$$

Thus the expression is completely equivalent to

$$|s_{12}|^2 = \frac{1}{A_0 + A_2x^2 + ... + A_{2n}x^{2n}} \tag{2.39}$$

where the new variable x can have either form

$$x = \alpha \cos \omega\tau \qquad \text{or} \qquad x = \alpha \sin \omega\tau \tag{2.40}$$

with α any real positive constant. The advantage of this form in x is that the approximation problem is reduced to determining a polynomial, the denominator.[3]

- **Design of an Equal Ripple Low Pass Filter with UEs**

Low-pass filter can be constructed with cascaded lines. The filter is to have Chebyshev performance in the pass band. A typical insertion gain shape $|s_{12}|^2$ is drawn in Figure 2.11.

The response characteristic must be periodic in the variable ω . This means that the point $\omega\tau = \frac{\pi}{2} \equiv \omega_e\tau$ must be chosen at a high enough frequency to provide a satisfactory upper limit for the end of the stop band. In other words, this point sets the entire useful operating frequency range of the filter.

To construct an appropriate insertion gain function which will have the desired filtering shape and be physically realizable, the polynomial representation of $|s_{12}|^2$ given in

$$|s_{12}|^2 = \frac{1}{A_0 + A_2x^2 + ... + A_{2n}x^{2n}} \tag{2.41}$$

is used with the variable $x = \alpha \sin \omega\tau$

This frequency variable is chosen so that the middle of the passband for $\omega\tau$ (dc) also goes into the origin in the transformed frequency scale Ω, simplifying the determination of an analytic insertion gain characteristic. The parameter α is used to adjust the normalized cutoff frequency to occur at x= ± 1. At cutoff ($\omega = \pm\ \omega_c$) then

(2.42)

$$x = \alpha \sin(\pm\ \omega_c) = \pm 1$$

Thus

$$\alpha = \frac{1}{\sin \omega_c \tau} \geq 1 \tag{2.43}$$

The length of each line section is chosen to be 1/4 wavelength at the prescribed frequency ω_e, which occurs at the high end of the stopband.

The value of L hence is τ determined. The parameters associated with x=$\alpha\sin(\omega\tau)$ are completely defined. Lumped parameter insertion gain functions suggest that

$$|s_{12}(x)|^2 = \frac{1}{1+\varepsilon^2 T^2{}_n(x)} \tag{2.44}$$

where T(x) is the Chebyshev polynomial of order n, with $\varepsilon < 1$.

This has equal ripple response in the normalized passband $-1 \le x \le 1$ and the gain falls off monotonically outside this region. The values of ε and n are to be adjusted to control the ripple in the passband as well as the insertion loss at x=α, the end of the stop band which the function repeats in the frequency variable $\omega\tau$.

Example 2.3

Consider the design of an equal ripple filter with the following specifications:

1. Ripple factor to give 0.4 dB maximum insertion loss in passband.
2. End of useful frequency range to be 3000 MHz, corresponding to $\omega_e\tau = \pi/2$, x=α.
3. Cutoff frequency = l000 MHz, corresponding to x = 1.

Minimum gain in the passband occurs when T(x)= 1. At the maximum pass band loss point, insertion loss = 10 log$|1/s_{12}|^2$ = 10 log (1+ε^2)=0,4 ε^2=0,1

Since L=1/4 wavelength at 3000 MHz, L=2.5 cm, $\omega\tau = \pi/6$, and $\alpha = \frac{1}{\sin \pi/6}$=2

When x>>1 $T_n(x) = 2^{n-1} x^n$ and x=α requirement leads to

10 log (1+$\varepsilon^2 (2^{n-1}\ \alpha^n)^2$)> 40 dB

N=5 is smallest odd integer which satisfies this requirement. In this case

$T_5(x) = 16x^5 - 20x^3 + 5x$

10 log [(1+ε^2 ($T^2{}_5$(x)]= 41.2 dB

The insertion gain function is

$$|s_{12}(x)|^2 = \frac{1}{1+0.1[16x^5 - 20x^3 + 5x]^2}$$

where

$$x^2 = \alpha^2 \sin^2 \omega\tau = \frac{4\tan^2 \omega\tau}{1+\tan^2 \omega\tau} = \frac{4\lambda^2}{\lambda^2 - 1} \text{ and } \lambda^2 = -\Omega^2 = -\tan^2 \omega\tau$$

If this is substituted in the equation we obtain $s_{12}(x)^2$ and in turn we obtain the equation for $s_{11}(\lambda)\, s_{11}(-\lambda) = 1 - s_{12}(\lambda)\, s_{12}(-\lambda)$ which is then factored and the left half plane denominator roots are used for $s_{11}(\lambda)$. The reflection factor obtained is

$$s_{11}(\lambda) = \frac{114.5\lambda^5 + 44.27\lambda^3 + 3.16\lambda}{114.5\lambda^5 + 83.21\lambda^4 + 74.48\lambda^3 + 28.89\lambda^2 + 8.53\lambda + 1}$$

The extraction of lines from the obtained reflection function can be carried out using the Richards extraction technique until the final 1 **Ω** termination is reached.. The filters insertion loss characteristic is shown in Figure 2.11 [3].

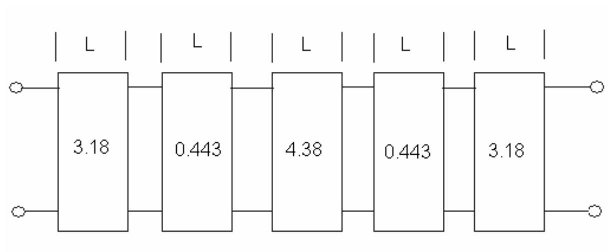


Figure 2.10 5 section Chebyshev filter circuit with transmission lines (L=2.5cm)

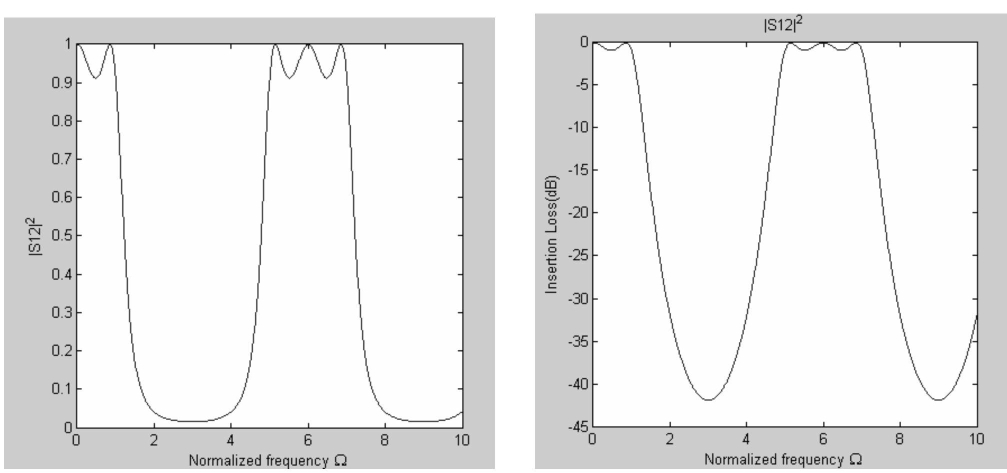


Figure 2.11 Low-pass cascaded line 5 section Chebyshev filter frequency response and insertion loss

2.4.2 Filter Design with Commensurate Lines and Stubs

An optimum multi-pole is defined as a two-port network, which is constructed with minimal number of elements (L's, C's and Unit Elements) with the given specifications and whose element values are chosen such that the transfer response most resembles to rectangle in a Butterworth or Chebyshev sense. An optimum multi-pole filter is obtained by combination of non-redundant number of quarter-wave stubs (LC elements) and unit elements. All filters employing only quarter-wave lines can be reduced to non-redundant form by suitable application of Kuroda's identities and/or series parallel reduction. It should be noted that introduction of redundant elements does not improve the response of the filter but in practical cases just enough redundancy is introduced to be able to construct the filter.

The design of optimum multi-pole filters involves three distinct steps.

1. Determination of the polynomial form of the ratio of reflection to transmission coefficients for a composite two-port filter containing both short or open circuited quarter-wave stubs and unit elements.
2. Development of the approximation function, usually chosen as maximally flat (Butterworth) or equal ripple (Chebyshev), used to approximate a rectangular low-pass or high-pass prototype power transmission characteristic.
3. Synthesis and physical realization of practical network in the form of distributed quarter wave lines.

Step 1 Polynomial Ratio of Reflected to Transmitted Power

Referring to the two-port network representation shown in Figure 2.12, the wave cascading matrix R is defined as

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \qquad (2.45)$$

where a_1, a_2, and b_1, b_2 are the incident and reflected waves as shown in Figure 2.12.

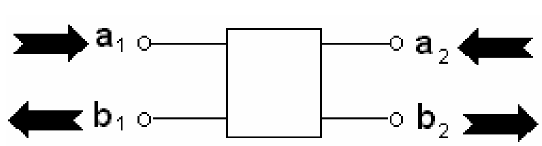


Figure 2.12 Two-port network

Scattering matrix can also be written for the same network as

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

(2.46)

where

$$b_1 = s_{11}a_1 + s_{12}a_2 \ , \ b_2 = s_{21}a_1 + s_{22}a_2$$

(2.47)

if equation (2.47) is rearranged to look like equation (2.45), that is,

$$a_1 = \frac{1}{s_{21}} b_2 - \frac{s_{22}}{s_{21}} a_2$$

$$b_1 = \left(s_{12} - \frac{s_{11}s_{22}}{s_{21}} \right) a_2 - \frac{s_{11}}{s_{21}} b_2$$

(2.48)

thus, the wave cascading matrix R is obtained as follows

$$R = \frac{1}{S_{21}} \begin{bmatrix} -\Delta_s & s_{11} \\ -s_{22} & 1 \end{bmatrix}$$

(2.49)

where $\Delta_s = s_{11}s_{22} - s_{12}s_{21}$ is the scattering matrix determinant. The individual R matrices of cascaded two-ports can be multiplied to give the overall R matrix of the cascade.

As an example, wave cascading matrix R can be easily derived for distributed LC ladder as

$$R = \begin{bmatrix} 1 - \frac{sL}{2} & \frac{sL}{2} \\ -\frac{sL}{2} & 1 + \frac{sL}{2} \end{bmatrix}$$

(2.50)

which can be written as in the form

$$R = sL(\frac{1}{sL} I + A^T)$$

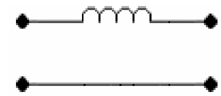
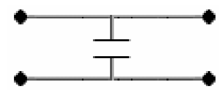
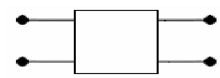
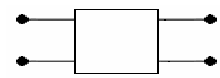
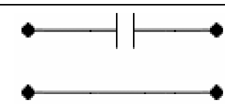
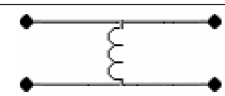
(2.51)

where $A = \frac{1}{2}\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$; and ; $A^T = \frac{1}{2}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$

(2.52)

The constant matrix A with transpose A^T appears in the R matrix for each of the distributed L's, C's and U.E.'s as shown Table 2.4.

Table 2.4 Wave cascade matrix R for distributed LC ladder and unit elements

Filter Elements		SCHEMATIC	R-MATRIX
LOW PASS	L		$LS\left(\frac{1}{Ls}I + A^T\right)$
	C		$CS\left(\frac{1}{Cs}I + A^T\right)$
	U.E.		$\frac{s}{\sqrt{1-s^2}}\left[\frac{1}{s}I + (ZA^T + Z^{-1}A)\right]$
HIGH PASS	U.E.		$\frac{1}{\sqrt{1-s^2}}\left[s(ZA^T + Z^{-1}A) + I\right]$
	C		$\frac{1}{Cs}\left(CsI + A^T\right)$
	L		$\frac{1}{Ls}\left(LsI + A^T\right)$

- **High Pass Prototype**

High pass filters are comprised of distributed series C's, shunt L's, U.E.'s and a unit terminating load. The C's, L's, and U.E.'s may occur in random sequence. However, in order to be non-redundant filter, no two C's nor L's may occur adjacent to each other even separated by one or more U.E.'s. Otherwise the elements may be combined, reducing the total number, by simple parallel or series combinations possibly in conjunction with use of one of

Kuroda's identities. An optimum highpass filter, having a mixed cascade of m high-pass ladder elements and n unit elements terminated in a unit load, will have an overall R-matrix, by taking into account the form of the individual R-matrix of the lossless high pass elements in Table 2.4, of the form,

$$R = \left(\frac{1}{s}\right)^m \left(\frac{1}{\sqrt{1-s^2}}\right)^n B_{m+n}(s) \tag{2.53}$$

where $B_{m+n}(s)$ is an $(m+n)^{th}$ degree 2x2 matrix polynomial in s. The R-matrix element of interest in equation (2.50) is $r_{12} = s_{11}/s_{21}$ representing the ratio of input reflected wave to that transmitted in to load.

$$r_{12} = \frac{s_{11}}{s_{21}} = \left(\frac{1}{s}\right)^m \left(\frac{1}{\sqrt{1-s^2}}\right)^n b_{12_{m+n}}(s)$$

(2.54)

For simplicity s_{11} and s_{21} will be renamed as ρ and *t* respectively. Then total power into the filter is conseved, thus

$$|\rho|^2 + |t|^2 = 1$$

(2.55)

rearranging to show the dependence of the power transmission response on $r_{21} = \rho/t$

$$|t|^2 = \frac{1}{1+|\rho|^2/|t|^2} = \frac{1}{1+|r_{12}|^2}$$

(2.56)

using equation (2.54), we obtain

$$|r_{12}|^2 = r_{12}(s)r_{12}(-s) = (\frac{1}{-s^2})^m \left(\frac{1}{1-s^2}\right)^n b_{12_{m+n}}(s) b_{12_{m+n}}(-s)$$

(2.57)

The general form of the resultant numerator polynomial, which has real coefficients, will not change if each term is multiplied by a real constant involving $s_c^2 = (j\tan\theta_c)^2$ where $\theta_c = \frac{\pi\omega_c}{2\omega_0}$ and ω_c is designated to be the filter cutoff frequency. Then,

$$\frac{|\rho|^2}{|t|^2} = \left(\frac{-s_c^2}{-s^2}\right)^m \left(\frac{1-s_c^2}{1-s^2}\right)^n P_{m+n}\left(\frac{-s^2}{-s_c^2}\right)$$

(2.58)

where P_{m+n} is a $(m+n)^{th}$ degree polynomial in $-s^2/-s_c^2$

- **Low-Pass Prototype**

A low-pass optimum filter can be comprised of series L's, shunt C's, and U.E 's in random sequence. However, to be non redundant L's must be adjacent to C's if not separated by a U.E., or L's must be adjacent to C's if not separated by a U.E. or L's must be adjacent to each other (and likewise C's) if separated by a U.E. By applying a procedure similar to that used above for the high-pass filter, the low-pass prototype response ratio of reflected to transmitted power is given by:

$$\frac{|\rho|^2}{|t|^2} = \left(\frac{-s^2}{-s_c^2}\right)^m \left(\frac{-s^2(1-s_c^2)}{-s_c^2(1-s^2)}\right)^n Q_{m+n}\left(\frac{-s_c^2}{-s^2}\right)$$

(2.59)

where Q_{m+n} is a $(m+n)^{th}$ degree polynomial in $-s_c^2/-s^2$

Step 2 Approximation Functions

There are two common approximations, which are the maximally flat (Butterworth) and the equal ripple (Chebyshev). They are given as follows [4];

Butterworth:

High-pass: $$\frac{|\rho|^2}{|t|^2} = \left(\frac{s_c}{s}\right)^{2m}\left(\frac{\sqrt{1-s_c^2}}{\sqrt{1-s^2}}\right)^{2n}$$

(2.60)

Low-pass: $$\frac{|\rho|^2}{|t|^2} = \left(\frac{s}{s_c}\right)^{2m}\left(\frac{s\sqrt{1-s_c^2}}{s_c\sqrt{1-s^2}}\right)^{2n}$$

(2.61)

Chebyshev:

High-pass: $$\frac{|\rho|^2}{|t|^2} = \varepsilon^2\left[T_m\left(\frac{s_c}{s}\right)T_n\left(\frac{\sqrt{1-s_c^2}}{\sqrt{1-s^2}}\right) - u_m\left(\frac{s_c}{s}\right)u_n\left(\frac{\sqrt{1-s_c^2}}{\sqrt{1-s^2}}\right)\right]^2$$

(2.62)

Low-pass: $$\frac{|\rho|^2}{|t|^2} = \varepsilon^2\left[T_m\left(\frac{s}{s_c}\right)T_n\left(\frac{s\sqrt{1-s_c^2}}{s_c\sqrt{1-s^2}}\right) - u_m\left(\frac{s}{s_c}\right)u_n\left(\frac{s\sqrt{1-s_c^2}}{s_c\sqrt{1-s^2}}\right)\right]^2$$

(2.63)

where $T_m(x) = \cos(m \arccos x)$ and $U_m(x) = \sin(m \arccos x)$ are unnormalized mth degree Chebyshev polynomials of the first and second kinds, respectively.

Step 3 Network Synthesis

Solving the realization problem is the final step for obtaining optimum filter. As stated earlier the optimum filter is consist of cascaded unit elements and distributed L's and C's which obey the form of approximation functions given in equations 15, 16, 17 and 18. If the input impedance of this cascade is determined from the specified power function, Richard's theorem can be applied to determine unit element values and pole-removing techniques can be used to determine the LC values. The power reflection coefficient $|\rho|^2$ can be written as

$$|\rho|^2 = \frac{|\rho|^2/|t|^2}{1+|\rho|^2/|t|^2}$$

(2.64)

Then the desired reflection coefficients are determined from the squared approximating function $|\rho|^2$ by finding roots of the numerator and denominator polynomials and associating thee left half-plane poles with ρ. Then applying the transformation

$$Z_{in} = \frac{1+\rho}{1-\rho} \tag{2.65}$$

the input impedance is obtained. Once input impedance is obtained the element values can be determined by using Richard theorem and pole zero remove techniques [4].

Example 2.4

As an example we consider the design of a three-section maximally flat low pass filter of 70 percent bandwidth. Using the reflected to transmitted power ratio approximation functions given by equation (16), for n=3, first all stub structure is obtained from the obtained impedance function.

$$Z_{in}(\lambda) = \frac{2.74\lambda^2 + 3.05\lambda + 1.19}{2\lambda^3 + 2.74\lambda^2 + 3.05\lambda + 1.19}$$

Then Kuroda identities are applied to obtain the final circuit. The output screen of the developed toolbox yields the filter structure as shown in Figure 2.13 and the transfer function characteristic as shown in Figure 2.14.

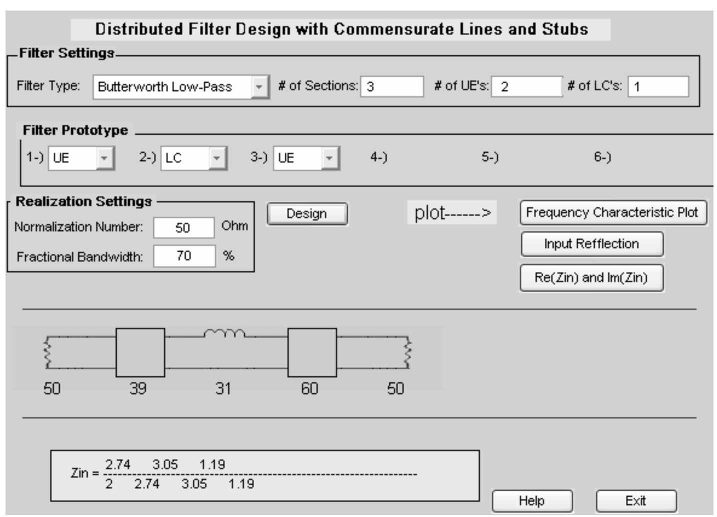


Figure 2.13 Low-pass Butterworth distributed filter with commensurate lines and stubs for order 3

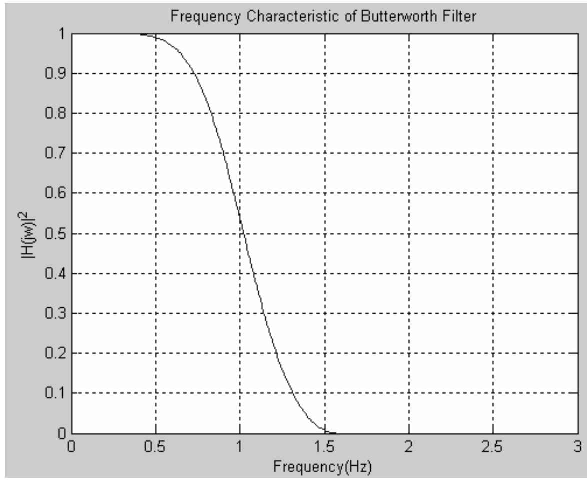


Figure 2.14 Frequency response of low-pass Butterworth filter for order 3

CHAPTER 3

Construction of Mixed Lumped-Distributed Filters

In the literature, solution to filter problems with lumped elements alone are well established. Especially at microwave and millimeter-wave frequencies, use of lumped elements alone in the circuit realization presents serious implementation problems, because of the difficulties regarding the physical interconnection of components and the associated parasitic effects. Therefore, it is inevitable to use distributed structures composed of transmission lines, although they penalize the performance of the system due to the resulting undesired higher harmonics. On the other hand, from the physical realization point of view, there exist neither ideal lumped capacitors and inductors nor ideal transmission lines in the actual world. Hence, the physical realization of the lumped and distributed network elements during the implementation of microwave discrete, hybrid or monolithic integrated circuits (MIC) is associated with several problem. In general, lumped elements can be represented with distributed ones due to their physical sizes and the real world transmission lines can be modeled with ideal lines and fringing lumped elements. Therefore, in the microwave circuit designs, such as filters, an important problem is to construct lossless two-ports with ideal lumped and distributed elements, so that the physical parameters which arise during the implementation process can easily be absorbed in the resulting circuit structure and the actual connections are thereby made possible. In this regard, the utilization of mixed lumped and distributed circuits would appear to offer several advantages and flexibilities in MIC layouts.

It has long been appreciated that the cascade of reciprocal two-port networks connected by means of equi-delay ideal transmission lines constitutes a reasonable model for real distributed structures. The sub networks which also compensate junction and discontinuity effects may consist of lumped elements or may be mixed lumped and distributed in nature. Microwave filters incorporating such cascaded structures combine obviously the properties of both lumped and distributed networks and offer advantages over those designed with lines or lumped elements alone. One of the most important advantages is the harmonic filtering property of the mixed structure. Additionally, the required physical circuit interconnections in MIC layouts is provided by no redundant transmission line elements which also contribute to the filtering performance of the structure. In the literature, there exist no general procedure to design filters with mixed lumped and distributed elements. Exact formulation of the synthesis problem of mixed-element structures require solutions to transcendental or multivariable approximation problems. Over several decades, the approximation problem of multivariable transfer functions which results in the lossless two-ports consisting of mixed, lumped and distributed elements was of serious concern in the literature. This problem has not yet been solved analytically. Rather, research efforts have been concentrated on the realizability conditions of the restricted class of two variable functions and some synthesis procedures had been devised by [12], [13], [14].

The basic idea in obtaining a filter network which incorporates both lumped and distributed elements is to construct realizable two-variable network functions from those of the single variable ones. For filter problems, there are a number of efforts in the literature on the one to two-variable reactance function transformations. Especially in conjunction with the design of reference circuits for the multidimensional wave digital filters, some synthesis procedures have been proposed by [15] and [16]. Because of the problems associated with the factorization of multivariable polynomials and the complex realizability conditions in the multivariable synthesis procedures, it is difficult to use these approaches in filter design. Regarding the practical design of microwave filters, [3] and [11] and [17] have proposed different procedures in which a cascaded commensurate transmission line prototype filter is employed. In both of these approaches, it was aimed to obtain an approximate equivalent of the transmission line cascade with parasitic junction capacitors which can be considered as a quite special class of mixed networks.

In practice, starting from a chosen topology with unknown mixed elements, an attempt is made to optimize the transducer gain of the system under consideration which in turn yields the element values. As a matter of fact, an optimization technique specifically devised for the design of lumped-distributed two-ports are suggested by [18]. In a similar manner, widely known and commercially available computer packages can be used as well. In this case, the problem is highly nonlinear and one needs to initiate the optimization with very good guesses on the element values, no matter how excellent the optimization algorithm is.

As a consequence of the above considerations, for the design of filters with mixed lumped-distributed elements, an organized combination of the approximate methods with CAD techniques possibly provides the best approach utilizing the presently available mathematical techniques.

In this chapter, an integrated design tool to construct filters with mixed lumped and distributed elements is presented. The design tool gathers the insertion loss design technique with an appropriate application of two-variable transformation and network replacement techniques.

3.1 Proposed Design Method for Mixed Lumped Distributed Filters

In the first step of the design method, the prototype filter transfer function and the network (lumped or distributed) is generated on an insertion loss basis. In this study, the prototype is assumed to be lumped for the sake of simplicity. But it should be obvious that the same procedure can be extended to the case where the prototype is chosen to be distributed.
In the second step, applying a two-variable reactance transformation two-variable transfer function preserving the given pass band specifications is generated.
In the third step, the realization of two-variable transfer function in two-variable ladder forms is obtained.

In the forth step, the two-variable ladder prototype networks are transformed into mixed lumped and distributed element filters. At this step the possible mixed element implementation forms based on the applied transformations are studied.

Here, the two-variable prototype ladder is decomposed into cascaded sub-sections of T-type. The two-variable T-type sections are exchanged by their *almost equivalent* mixed element networks using the replacement techniques. Here, the term *almost equivalent* is used since the swapped distributed networks do not possess exactly the same electrical description of their lumped counterparts; Rather an approximation is made over the prescribed frequencies. Hence, an initial design is generated at the end of this step with mixed lumped and distributed two-ports in tandem connections. Finally, the initial design may be optimized to yield the desired gain performance. In the following sections the steps of the proposed approach is described using illustrative examples.

3.2 Generation of Prototype Transfer Function on an Insertion Loss Basis

In the first step of the design method, the low-pass prototype filter transfer function. For this purpose, the single-variable insertion loss design approaches discussed in chapter 2 can directly be utilized. Based on the design specifications, once the single-variable transfer function is obtained (in the complex frequency variable p), then this transfer function characteristic will be preserved for the remaining steps.

3.3 Generation of Two-Variable Transfer Function using Reactance Transformation

Two-variable reactance functions have been used in the literature to synthesize lumped networks with time-varying elements. These functions have found extensive applications in the realization of lumped-distributed networks and also have been widely employed for studying the properties of 2-dimensional digital filters. However there is approximation problem for 2-dimensional digital filters. The types of approximations have been mainly restricted to FIR low-pass digital filters with circular symmetry. In the case of the IIR filters, complicated computational procedures have to be followed. The basis of this method is a concept called g-correspondence between a single-variable and a two-variable function.

This concept was used earlier for the design of certain very restricted classes of digital filters. For a single-variable function H(p) and two-variable function $g(p_1,p_2)$, the composite function $H(g(p_1,p_2))$ will be said to be g-correspondent to H(p) and vice versa. It is shown that if H(p) is a stable transfer function and $g(p_1,p_2)$, is a two-variable positive real function (2-PRF), then $H(g(p_1,p_2))$ is always stable. Further, if $g(p_1,p_2)$ is chosen to be a two-variable reactance function (2-RF), a point (an ordered pair of real's), (ω_1,ω_2), in the 2-dimensional frequency region can be uniquely related to a point, ω , on the l-dimensional frequency axis. In addition, for any physically realizable transfer function H(p), if is a $g(p_1,p_2)$ realizable driving-point function with at least one known realization, the method of approximation given here automatically guarantees at least one realization for $H(g(p_1,p_2))$ [5].

l-dimensional slices of the magnitude characteristic of a two-variable filter of a given type, such as low-pass, high-pass, etc., can be considered to be the corresponding characteristic of a single-variable filter of the same type. For a suitable 1-dimensional slice of the two-variable characteristic, an optimal single-variable transfer function H(p) can be obtained using known techniques. A two-variable transfer function, $H(g(p_1,p_2))$, can now be generated from H(p) via g-correspondence where is $g(p_1,p_2)$ a two-variable reactance function which is the solution of a contour approximation problem. Given a realization of H(p), if a realization $g(p_1,p_2)$ is known, a realization of $H(g(p_1,p_2))$ can be obtained by simply replacing the impedance blocks of the form ks (inductors) and k/p (capacitors) with two-variable reactance blocks of the form $kg(p_1,p_2)$ and k/g (p_1,p_2) [5].

H(p) of a certain type, low pass $H(g(p_1,p_2))$, will not necessarily be of the same type for any arbitrary reactance function $g(p_1,p_2)$. The necessary and sufficient conditions for $g(p_1,p_2)$ have been obtained such that H(p) and $H(g(p_1,p_2))$ are of the same type, or of opposite type, in a local region of the frequency domain, or even in the entire frequency domain.
Let H(p) be the transfer function of a stable, linear lumped finite time-invariant network, where *p* is the complex frequency variable. Then the map H: $C \longrightarrow C$ can be represented as a ratio of two rational polynomials in p where the denominator polynomial must be strictly Hurwitz. Here C denotes the the field of complex numbers. Let g: $C \times C \longrightarrow C$ be a map where g is defined for all $p_1, p_2 \in C$. In that case, the composition map $C \times C \xrightarrow{g} C \xrightarrow{H} C = C \times C \xrightarrow{H_g} C$ is obtained by replacing *p* everywhere in H(p) with $g(p_1,p_2)$. This resultant two-variable function $H_g(p_1,p_2) = H(g(p_1,p_2))$ will be said to be generated by the one-variable transfer function H(p) under the map g. It is obvious that if $g(p_1,p_2)$ is a driving-point function, $H_g(p_1,p_2)$ is a transfer function whenever H(p) and $g(p_1,p_2)$ are realizable, $H_g(p_1,p_2)$ is also realizable and referred to as the g-correspondent of H(p) [5].

In the single-variable case, the behavior of any network function is usually studied under sinusoidal excitations. In that case, the values of p on the $j\omega$ -axis are of interest. For p=$j\omega$ the magnitude and the phase functions are the quantities involved in the steady-state response to the sinusoidal excitations. Correspondingly, in the two-variable cases, for the complex frequencies p_1 and p_2, values on the $j\omega_1$ axis and the $j\omega_2$ axis are the important ones. Therefore, in order to relate a point on the (ω_1, ω_2)-plane uniquely to a point on the w axis, it is convenient to select only those maps g for which $g(j\omega_1, j\omega_2) = j\omega$ Then the imaginary axes of the (p_1,p_2) plane will be mapped into the imaginary axis of the *p*-plane.

In generating the two-variable transfer function $H_g(p_1,p_2)$ from H(p) it is essential to consider the stability properties of $H_g(p_1,p_2)$. I have been proved in [5] that if H(p) is stable and $g(p_1,p_2)$ is a two-variable positive real function, then $H_g(p_1,p_2)$ is also stable.

In the case of l-dimensional filters ,the passband of a low-pass filter is the closed interval $[0,w_c]$; the passband of a bandpass (l-BP) filter is the closed interval $[\omega_{c1},\omega_{c2}]$; the passband of a high-pass filter is the closed-open interval $[\omega_c,\infty]$. A parallel set of definitions can be stated such that it will be possible to establish a direct correspondence between the response characteristics of l-dimensional and 2-dimensional filters. For example, a two-variable transfer function will be called an "ideal" 2-LP, if the corresponding magnitude function, $M(\omega_1, \omega_2)$, as a real-valued nonnegative function of two real variables, satisfies the condition

$$M(\omega_{1,}, \omega_2) = \begin{cases} H, \forall < \omega_1, \omega_2 > \in \Omega^0 \\ \\ 0, otherwise \end{cases}$$

where Ω^0, is a nonempty compact subset of Ω, containing <0,0>. Figure 3.1 illustrates a typical ideal two-variable LP characteristic [5].

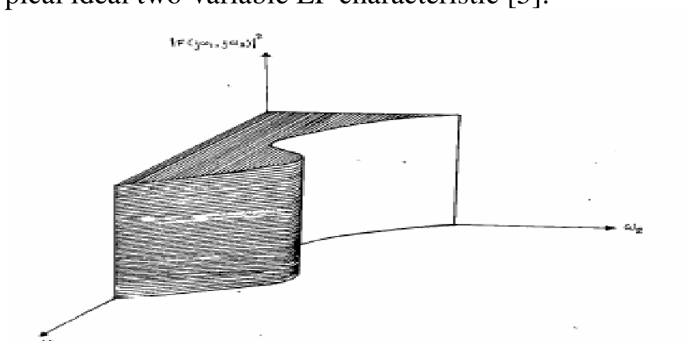


Figure 3.1 Typical ideal two-variable low-pass response characteristic

It is obvious that, the ideal filter characteristic can not be exactly realized in practice and hence, have to be approximated with certain tolerances. At this point, proper selection of the two-variable positive real transformation reactance function $g(p_1,p_2)$, controls the tolerances of the approximation while preserving the LP characteristic. The properties and possible forms of the reactance function $g(p_1,p_2)$ leading to different filter characteristics have been discussed in [5]. A simple but effective two-variable reactance map for LP characteristic control in two-variable domain is defined as

$$g(p_1,p_2) = \alpha\, p_1 + \beta\, p_2 \tag{3.1}$$

where α and β are real positive constants. By proper control of the constants α and β, the response characteristic in the one-variable transfer function can be preserved within prescribed tolerances in the two-variable domain.

The generation two-variable filter transfer function under g transformation is shown in the illustrative example given below.

Example 3.1

Obtain a two-variable maximally flat low pass filter function which is flat to 0.1 dB at least in the region [0,1 kHz]×[0,2 kHz] and is more than 60 dB down in the region [f_1>13 kHz, f_2>15 kHz].From the specifications, we can choose the ap_1+bp_2 as two-variable reactance map g. We can choose a=1 b=1 for simplicity.

Then it is sufficient to design a two-variable low-pass filter which is flat to 0.1 dB in the region below line $f_1+f_2=3$ kHz including the line itself and is more than 60 dB down in the region above the line $f_1+f_2=12$ kHz including the line. The f_1 intercepts of these lines are at <3 kHz, 0> and <12 kHz, 0>.We need only design one variable low-pass maximally flat filter which is flat to 0.1 dB up to 3 kHz and is down to 60 dB in the frequency range f≥12 kHz. A 7th order Butterworth filter with a unit radian frequency corresponding to 25 rad/s satisfies the specification. Let the corresponding angular frequency be denoted by ω_0.The one-variable low-pass transfer function is given by

$$H(p)=\frac{1}{1+4.494p+10.0978p^2+14.592p^3+14.592p^4+10.0978p^5+4.494p^6+p^7}$$

where the normalization frequency is ω_0. The corresponding normalized expression for the desired two-variable low pass function is then obtained as

$$H_g(p_1,p_2)=[1+4.494(p_1+p_2)+10.0978(p_1+p_2)^2+14.592(p_1+p_2)^3$$
$$+14.592(p_1+p_2)^4+10.0978(p_1+p_2)^5+4.494(p_1+p_2)^6+(p_1+p_2)^7]^{-1}$$

The denormalized expression for $H_g(p_1,p_2)$ may obtained by replacing (p_1+p_2) with $(p_1+p_2)/\omega_0$ in the above expression. 2-dimensional normalized frequency region shown in Figure 3.2.

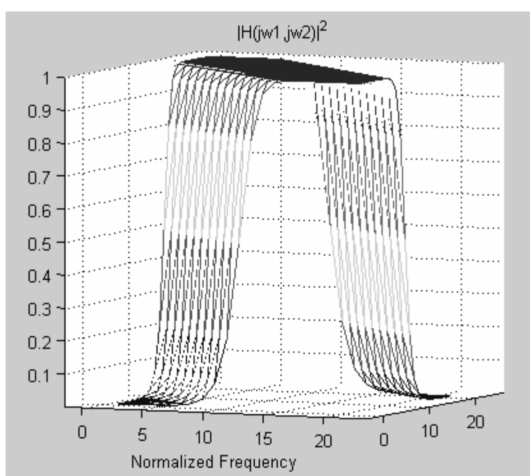


Figure 3.2 3-dimensional transfer function frequency characteristic plot of example 3.1

3.4 Two-Variable Ladder Prototype Realization

The realization of the transfer function $H_g(p_1, p_2)$ in ladder form can directly be obtained from the single variable realization of H(p). Once the realization of H(p) is obtained in ladder form, the reactance transformation $p = p_1 + p_2$ implies the replacement of each p blocks by $(p_1 + p_2)$ blocks of the same type [5]. That is to say, a (pL) reactance is replaced by the series connection of a $(p_1 L)$ and $(p_2 L)$ reactance's; a (pC) susceptance is replaced by the parallel connection of $(p_1 C)$ and $(p_2 C)$ susceptance's. The resultant two-variable ladder realization is shown in Figure 3.3.

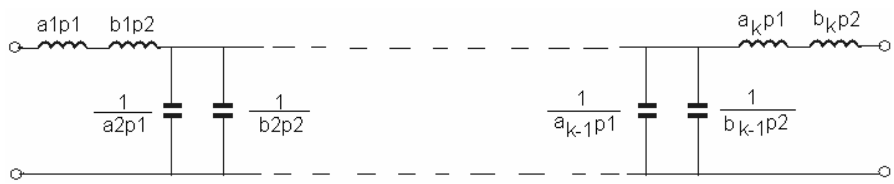


Figure 3.3 Two-variable ladder prototype

Generation of two-variable ladders is illustrated in the following example.

Example 3.2

Let H(p) be the transfer function of the network shown in Figure 3.4 which corresponds to a 7th-order Butterworth filter. The normalized transfer function of this filter (with respect to the cutoff frequency) is given by

$$H(p) = \frac{1}{1 + 4.494p + 10.0978p^2 + 14.592p^3 + 14.592p^4 + 10.0978p^5 + 4.494p^6 + p^7}$$

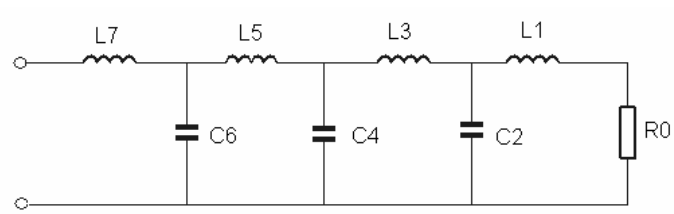


Figure 3.4 7th order normalized Butterworth filter network topology (L1 = 0.22254 H, C2= 0.6560F, L3 = 1.05504 H, C4= 1.3972 F, L5 = 1.65884H, C6= 1.7988 F, L7 = 1.5576 R_o =1 Ω)

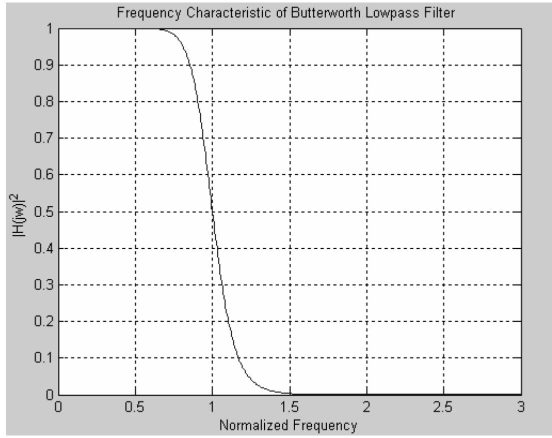


Figure 3.5 Frequency response of example 3.2

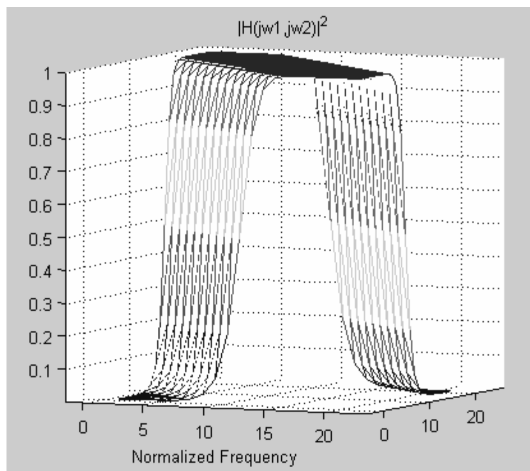


Figure 3.6 3-dimensional transfer function characteristic of example 3.2

Straight line contours which are conspicuous in regions where almost vertical part of response surface begins to flatten out.2-dimensional normalized frequency region shown in Figure 3.6.

Let $g(p_1,p_2) = p_1+p_2$. Then $H_g(p_1,p_2)$ is obtained as

$$H(p) = \frac{1}{1+4.494p+10.0978p^2+14.592p^3+14.592p^4+10.0978p^5+4.494p^6+p^7}$$

$$H_g(p_1,p_2) = [1+4.494(p_1+p_2)+10.0978(p_1+p_2)^2+14.592(p_1+p_2)^3$$
$$+14.592(p_1+p_2)^4+10.0978(p_1+p_2)^5+4.494(p_1+p_2)^6+(p_1+p_2)^7]^{-1}$$

Figure 3.6 shows the plot $\left|H_g(j\omega_1, j\omega_2)\right|^2$ versus $<\omega_1,\omega_2>$ which can be identified as a two-variable low-pass characteristic.

The realization of the transfer function $H_g(p_1,p_2)$ is shown in Figure 3.7 which is obtained directly from Figure 3.4 by replacing the inductors k's with $k(p_1+p_2)$ and the capacitors k/p with $k/(p_1+p_2)$ for suitable real positive constant k's [5].

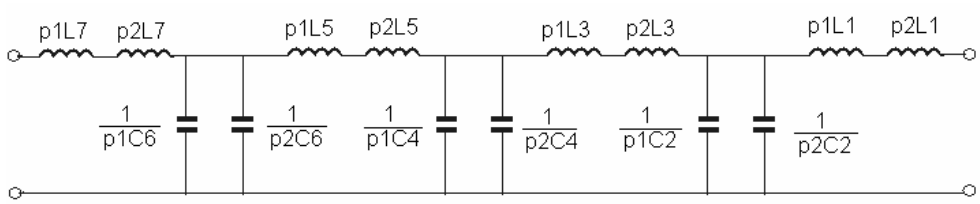


Figure 3.7 Realization of transfer function $H_g(p_1,p_2)$ in Example 3.2

3.5 Transformation of Two-Variable Ladders into Lumped Distributed Filters

Lumped-distributed networks are extensively used in microwave applications. While multivariable techniques have been used for the synthesis of such networks from a given transfer function, one of the main unsolved problems in this area about which very little work has been done is the approximation of a prescribed frequency characteristic by a suitable transfer function. The objective of this section is to present a simple solution to the above problem by utilizing the ideas developed in the previous sections.

The single-variable reactance function can always be synthesized as a low-pass ladder network by a continued fraction expansion. The connection between the ladder network elements and the Routh-Hurwitz array is well established. It is also known that not every single-variable positive real function can be realized by a continued-fraction expansion.

Hence, it is natural to predict that not all two variable reactance functions being generalizations of single-variable positive-real functions and hence, not all the multivariable reactance functions, are realizable by continued fraction expansion as ladder networks, which are corroborated by different synthesis procedures . Furthermore, for multivariable network functions, the conditions for the continued fraction expansion and those for the realizability of ladder networks are unavailable in the literature .In this thesis, we propose a multivariable array from which the realizability conditions for the multivariable low-pass ladder networks (MLPL's) consisting of series inductors and shunt capacitors are obtained. This array ,for a single variable, reduces to the Routh-Hurwitz array. By suitable transformations, several other types of multivariable ladder networks and their realizability conditions are derived starting from the MLPL. We consider two applications of the ladder networks.

- The realization of two-variable ladder networks using lumped lossless elements and commensurate short circuited and open circuited stubs.

- The ladder realization with cascade of UE s, where the equivalence relation between the cascade of unit elements (UE's) separated by series lumped inductors on one side and shunt lumped capacitors on the other side is utilized..

3.5.1 Lumped Element and Stub Realization of Two-Variable Ladders

For p_1 = s and p_i = tanh sξ_i ($2 \le i \le n$), where s is the complex frequency variable and ξ_i >0 are the time delays of the transmission lines, the p_i ($2 \le i \le n$)-type inductors and capacitors can, be replaced by noncommensurate short circuited and open circuited stubs. Thus various types of filters can be developed with mixed lumped-distributed elements from the above derived ladder structures.

Suppose the driving-point impedance Z(p), not identically 0 or ∞, of a finite network of lossless transmission lines of commensurate delays, ideal transformers, and lumped inductances and capacitances, is expressed as a real rational expression in s and e^{bp}, where b is some positive constant. Then the formal replacement of every e^{bp} portion with the expression (1 + p')/(1- p'), leaving the powers of p intact, defines a function of two independent complex variables Z (p,p') with two-variable reactance property.

The transformation $e^{bp_1} \longrightarrow (1 + p_2)/(1 - p_2)$ is nothing but the well-known Richard's transformation. defined by p_2 = tanh (τ p_1/2) where $\tau = 2l\sqrt{LC}$ is the round-trip delay time for the shortest commensurate length line, l is the length of the line, and L, C are the inductance and the capacitance per-unit-length of the transmission line conductors.
In that case, for a two-variable realizable network, whenever one of the complex variables, say p_2, is related to the variable p_1, through Richards transformation, we can conclude that a lumped-distributed realization for the network exists, where each p_2 inductance (p_2 capacitance) is replaced by a short-circuited (open-circuited) transmission line.

At this stage it may be pointed out that a lumped-distributed filter network containing commensurate transmission lines has a periodic response with a normalized period π. Hence, any such filter may be considered as either bandpass or band elimination. However, usually such filters are classified according to their low-frequency behavior. In that case, by convention, the filter response is classified by the response type in the transformed frequency interval [0, π/2], and depending on the applications, a band-elimination filter may be considered low pass and a band pass filter may be considered high pass.

If we consider the GTP function $g(p_1,p_2) = p_1+p_2$. Then the correspondence of p_1, and p_2 through Richards transformation $p_2=\tanh \tau p_1/2$ yields the relationship

$$p_1+\tanh(\tau p_1/2)= p \tag{3.2}$$

Replacing p= $j\omega_1$ by the symbol p =jq we have g(p)=p+tanh (rp/2)=p and g(q)=q+tan(τ q/2) $=\omega$. Since the tangent function is a periodic function, g(q) has a repetitive character with respect to q. Let q_{10}, q_{20} . . . denote the lst, 2nd... zeroes of g(q) in $R^+{}_0$ .Then q_{10}=0 Similarly, let $q_{1\infty}, q_{2\infty} \ldots$ denote the lst, 2nd,. . . values of $q\in R^+{}_0$ for which g(q) is infinity.

Since g(q) is repetitive, as q ranges over $0\leq q<\infty$, $\left|H_g(jq)\right|^2 = \left|H(j\omega)\right|^2$ obtained from a given H(p) will also be repetitive. In the first interval, $[q_{10}, q_{1\infty})$, for every q, there exists a unique w, given by q+ tan (τ q/2)=ω , such that $\left|H_g(jq)\right|^2 = \left|H(j\omega)\right|^2$.

Thus over any such interval

$$\left|H_g(jq)\right|^2 = \left|H(j\omega)\right|^2 \quad w\in(-\infty,\infty)$$

If we call the interval $[q_{10},\ q_{1\infty})=[0,\pi/\tau)$ as the primary band. All other intervals $(q_{k\infty}, q_{k+1,\infty})$ will be called the secondary bands

For a lumped-distributed filter to be useful, the input signal has to be band limited so as to mainly lie within the primary band. For a given input spectrum, the g-correspondent filter function can be designed to achieve this end by controlling the parameter τ so that the first of the secondary bands is sufficiently away from the primary band. The parameter τ can be controlled by varying either L or LC product or both of the lossless lines.For the exact design of a lumped-distributed filter function in the primary band, the familiar pre-warping techniques can be used. Through the substitution q+ tan(τ q/2) $=\omega$, the design of a g-correspondent lumped-distributed filter function reduces to the problem of the design of a lumped filter function. The design procedure may be listed as follows.

1) First we must ensure that the primary band is wide enough.

$$H(p) = \frac{1}{1+4.494p+10.0978p^2+14.592p^3+14.592p^4+10.0978p^5+4.494p^6+p^7}$$

2) Then we denote the normalized critical frequencies by $q_{ci} = q_i/q_0$, where q_0 is suitably chosen. We compute a new set of transformed critical frequencies for the analog filter, ω_{ci}, by $\omega_{ci} = q_{ci} + \tan(\tau q_{ci}/2)$

3) We obtain a lumped transfer function and its realization with the properties of the lumped-distributed filter at the new frequencies ω_{ci}, and the transformed ranges.

4) Then the desired lumped-distributed transfer function is $H_g(p)$ which is obtained by replacing s with p + tanh(τ q/2) in H(p). A realization of $H_g(p)$ may now be obtained by replacing the different inductors and capacitors of the form kp and (k/p), respectively, by blocks with driving-point impedances of the form k(p + tanh (pτ/2)) and k/(p + tanh (pτ/2)), respectively.

$$H_g(p_1, p_2) = [1 + 4.494(p_1 + p_2) + 10.0978(p_1 + p_2)^2 + 14.592(p_1 + p_2)^3$$
$$+14.592(p_1 + p_2)^4 + 10.0978(p_1 + p_2)^5 + 4.494(p_1 + p_2)^6 + (p_1 + p_2)^7]^{-1}$$

Example 3.3

Obtain the lumped-distributed realization of a maximally flat low-pass filter with monotone characteristics which is flat to 0.1 dB in the pass band 0 to 500 MHz and is more than 60 dB down at frequencies beyond 1.7 GHz. This specification can be met by a suitable Butterworth function. Choosing $q_0/2\pi$=1 GHz, the normalized critical frequencies are 0.5 and 1.7. Hence, the transformed frequencies, ω_{c1} and ω_{c2}, are obtained as

ω_{c1} = 0.5 + tan (0.25) = 0.76
ω_{c2}= 1.7 + tan (0.85) = 2.84

where τ is assumed to be unity.

The lumped filter must be maximally flat low-pass with monotone characteristic which is flat to 0.1 dB in the interval [0, 0.76] and is more than 60 dB down at frequencies beyond 2.84.

If we apply following transformation

$$p = p_1 + p_2 \quad \text{where} \quad p_1 = j\omega, \quad p_2 = j\tan(\omega\tau)$$

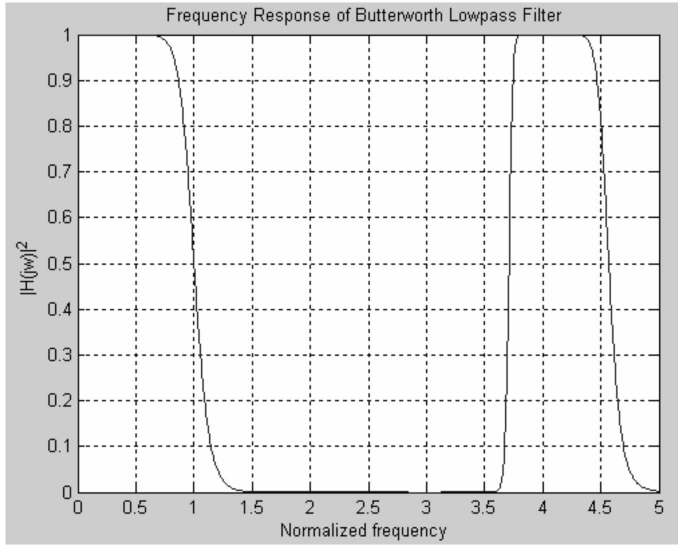


Figure 3.8 Transfer function frequency characteristic of example 3.3 for τ =3

The above requirements may be met by a 7th-order lumped Butterworth filter. The desired transfer function, H(p),

$$H(p) = \frac{1}{1 + 4.494p + 10.0978p^2 + 14.592p^3 + 14.592p^4 + 10.0978p^5 + 4.494p^6 + p^7}$$

The typical realization in the form of a singly terminated ladder network may be obtained as shown in Figure 3.9 where L_1, = 0.2225R_0, C_2,= 0.6560/ R_0, L_3 = 1.0550 R_0, C_4 = 1.3972/ R_0, L_5 =1.6588/ R_0, C_6 = 1.7988/ R_0, and L_7 = 1 .5576 R_0, R_0 being the magnitude of the load impedance (resistance) expressed in ohms and all L's (C's) are in henrys (farads).
In that case, the lumped-distributed realization (for the normalized filter) is immediately obtained as in Figure 3.9 which has the transfer function [5]

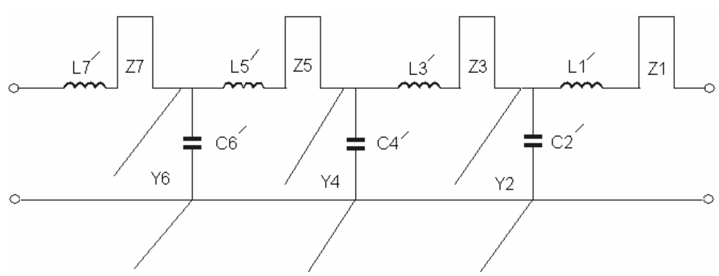


Figure 3.9 Lumped-distributed realization of transfer function H_g (p) of example 3.3

In Figure 3.9 L_i'= L_i , Z_i= L_i i=1,3,5,7 and C_i'= α_i C_i , Y_i=(1- α_i) C_i i=2, 4 and 6 , Z_i(Y_i) are the appropriate characteristic impedance (admittance) of the respective lines, where each line has τ = 1. For any ith line, 1_i and the per-unit-length inductance (capacitance) L_i(C_i). where τ_i = 1=2$1_i$ $\sqrt{L_i C_i}$ and Z_i $\sqrt{L_i / C_i}$, can be chosen depending on other design requirements. Finally, the denormalized expression for the lumped- distributed transfer function may be obtained by replacing p in H_g(p) with (p /q_0).

3.5.2 Low-Pass Ladder Realization with UE Separations (LPLU)

It has been shown that the synthesis of cascaded UE's can be performed by means of two-element kind ladder networks. By defining p_1= tanh pξ and p_2=p cosh pξ, the equivalent relation between the cascade of UE's separated by series lumped inductors on one side and shunt lumped capacitors on the other side and the TLPL is shown to exist [6]. Let

$$\bar{a}_1(p_1) = \begin{bmatrix} A_1(p_1) & \dfrac{B_1(p_1)}{F(s)} \\ C_1(p_1)F(s) & D_1(s\ ) \end{bmatrix} \tag{3.3}$$

be the transmission matrix of UE, where p_1= sinh pτ and F(p) =cosh pτ, and that of the LC network be

$$a(p_1) = \begin{bmatrix} A(p_1) & B(p_1) \\ C(p_1) & D(p_1) \end{bmatrix} \tag{3.4}$$

Then the LC network is said to be equivalent to the UE. Figure 3.10 shows the UE and its equivalent LC network.

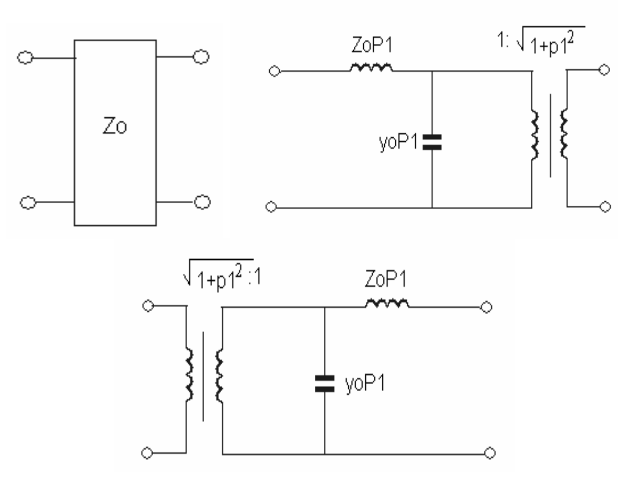


Figure 3.10 UE and its equivalents ($Z_0 = 1/Y_0$ **)**

Thus by means of the transformation $p_1 = \sinh p\tau$, the uniform lossless transmission line is transformed into an equivalent LC network in the p_1 plane, keeping the lossless nature in both the planes. The T-equivalent relation remains invariant when the UE's are connected in cascade. Thus the synthesis can be carried out by T-equivalent LC ladder network. The input admittances of these networks are given below.

$$y_{11}(p_1) = \frac{D(p_1)}{B(p_1)} = \bar{y}_{11}(p_1) / F(s) \tag{3.5}$$

$$z_{11}(p_1) = \frac{D(p_1)}{C(p_1)} = \bar{z}_{11}(p_1) F(s) \tag{3.6}$$

where $y_{11}(p_1)$ and $z_{11}(p_1)$ are the short-circuited and open circuited driving point functions of the LC equivalent networks and $\bar{y}_{11}(p_1)$ and $\bar{z}_{11}(p_1)$ are the corresponding admittances of the cascaded UE's.

Using the same idea, let

$$\bar{a}(p_1, p_2) = \begin{bmatrix} A(p_1, p_2) & \dfrac{B(p_1, p_2)}{F(s)} \\ C(p_1, p_2)F(s) & D(p_1, p_2) \end{bmatrix} \tag{3.7}$$

be the matrix of a UE with the shunt lumped capacitor termination at the second port as shown in Figure 3.11 (a) and let

$$a(p_1, p_2) = \begin{bmatrix} A(p_1, p_2) & B(p_1, p_2) \\ C(p_1, p_2) & D(p_1, p_2) \end{bmatrix} \tag{3.8}$$

be the matrix of the two-variable ladder of Figure 3.11(b). In such a case, the two networks are considered to be equivalent.

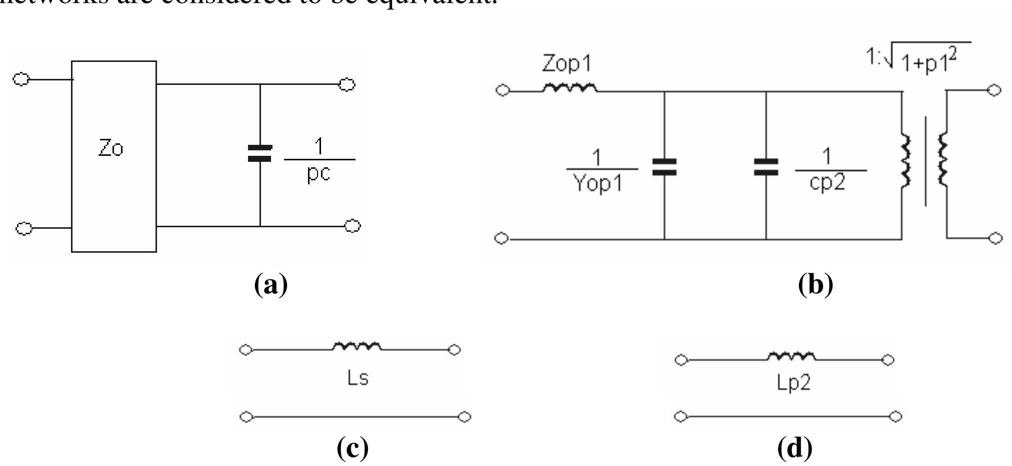


Figure 3.11 (a) UE terminated in capacitance. (b)Equivalent network of (a) (c) and (d) series inductor equivalents.

Similarly, we can see that the series inductors of Figure 3.11 (c) and (d) are equivalent, their respective matrices being

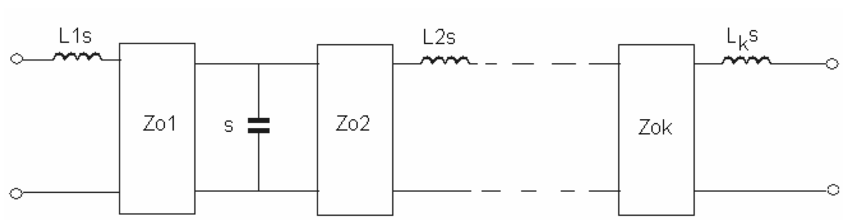
$$\overline{a}_1(p_2)=\begin{bmatrix}1 & \dfrac{p_2L}{F(p)}\\ 0 & 1\end{bmatrix}=\begin{bmatrix}A(p_2) & \dfrac{B(p_2)}{F(p)}\\ C(p_2) & D(p_2)\end{bmatrix}$$

(3.9)

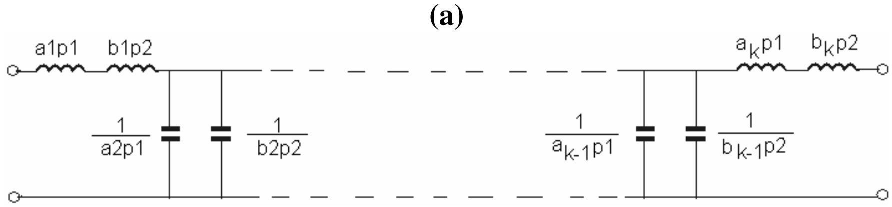
$$a_1(p_2)=\begin{bmatrix}1 & p_2L\\ 0 & 1\end{bmatrix}=\begin{bmatrix}A(p_2) & B(p_2)\\ C(p_2) & D(p_2)\end{bmatrix}$$

(3.10)

Using these equivalences it is possible to show that the equivalence relations remain invariant when the corresponding elements are connected in cascade. Utilizing the above derived equivalent relations, the realizability conditions for the cascade of UE's separated by series lumped inductors on one side and shunt lumped capacitors on the other side, and with a resistive termination as shown in Figure 3.12(a), are derived in terms of the two-variable ladder with resistive termination. In the cascaded structure of Figure 3.12(a), if the UE and capacitor are replaced by Figure 3.11(b), the series inductors by Figure 3.11(d), and the UE by Figure 3.10(c), the two-variable ladder of Figure 3.12(a) is obtained. In a similar manner, a two-variable lumped ladder in p_1, p_2 type elements can be transformed into a lumped ladder structure with UE separations as shown in Figure 3.12 [6].



(a)



(b)

Figure 3.12 (a) Cascade of UE's separated by series lumped inductors on one side and shunt lumped capacitors on the other side (LPLU)
(b) Equivalent two-variable ladder

3.5.3 Design algorithm for Low Pass Ladders with UE

Step 1 p-domain design:

From insertion loss specification of transfer function for an nth-order filter (Butterworth or Chebyshev) obtain the transfer function in single-variable and the corresponding prototype (Figure 3.13).

$$H(p)= \frac{1}{1+A_0 p + A_1 p^2 ....+A_n p^n} \tag{3.11}$$

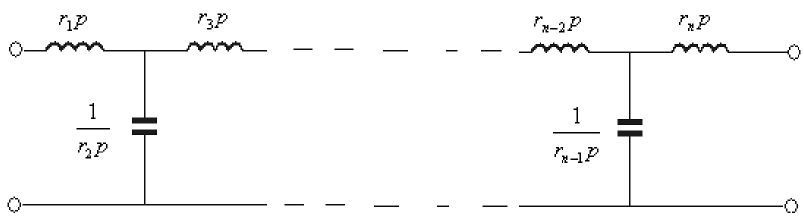


Figure 3.13 Low-pass filter topology at p domain

Step 2 Single-Variable to Two-Variable Transformation

By applying the reactance transformation p= α p_1+ β p_2 , transfer function of the two-variable low-pass characteristic is obtained as

$$H(p_1, p_2)= \frac{1}{1+A_0(p_1+p_2)+A_1(p_1+p_2)^2 ....+A_n(p_1+p_2)^n} \tag{3.12}$$

By proper control of the positive real constants α and β, the response characteristic in the one-variable transfer function can be preserved within prescribed tolerances in the two-variable domain. Here, choose p_1=sinh(pτ) , p_2=p. cosh(pτ), where τ is the delay length which is a function of the cutoff frequency.

For real frequencies the transformation function f(ω) can be written as

$$x = f(\omega) = \alpha \sin(\omega\tau) + \beta\omega\cos(\omega\tau) \tag{3.13}$$

To adjust normalized cut off of the two-variable transfer function at x=+1, corresponding to ω_c=1 Hz in the single variable prototype, we can calculate the weighting constants α and β. In particular for a preselected fixed value of α, β parameter can be used to control the passband edge and can be computed as

$$1= x=\alpha \sin(\omega_c\tau) + \beta\omega_c \cos(\omega_c\tau)$$

$$\text{for } \omega_c = 1,\ \beta = \frac{1}{\cos(\tau)} - \alpha \tan(\tau)$$

where $\tau = \dfrac{\pi}{2\omega_e}$ and ω_e represents the attenuation pole point or the repetition frequency for the resulting periodic transfer function. The resulting two-variable ladder is as shown in Figure 3.14.

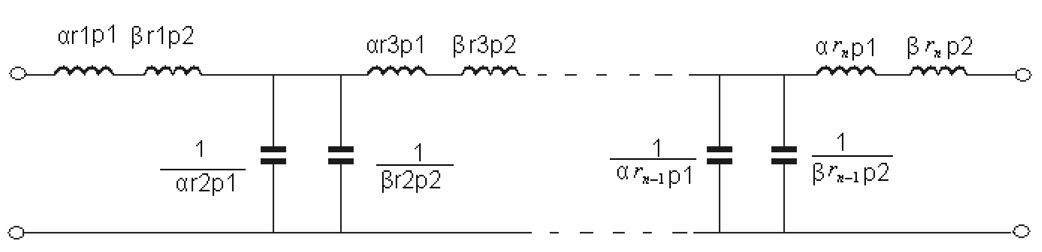


Figure 3.14 Two-variable ladder prototype

Step 3 Decomposition of Two-Variable Ladder into Unit -T Sections

Using a partial decomposition scheme, two-variable ladder can be put into periodic connection of T-type subsections, which we call Unit-T sections, as shown in Figure 3.15.

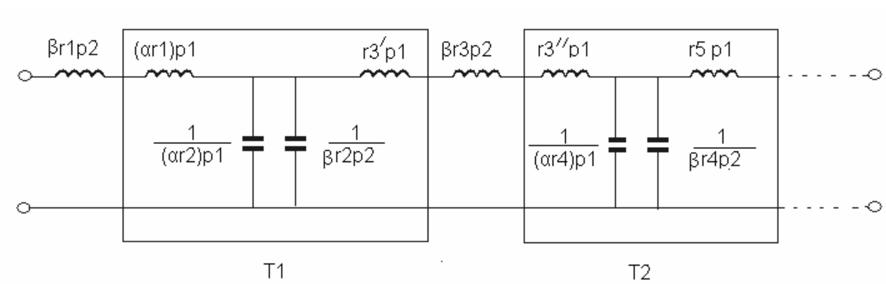


Figure 3.15 Decomposition of two-variable ladder network into unit-T sections

At this point, in order to ensure positive element values, the partial decomposition should satisfy the constraints

$$r_3{'} + r_3{''} = \alpha\, r_3$$
$$r_5{'} + r_5{''} = \alpha\, r_5$$
$$r_7{'} + r_7{''} = \alpha\, r_7$$
$$r_9{'} + r_9{''} = \alpha\, r_9$$
$$\ldots$$
$$r_n{'} + r_n{''} = \alpha\, r_n \qquad \text{for } n = 3,5,7,9\ldots\ldots \text{ (odd numbers)} \tag{3.14}$$

- **Unit-T and UCU Equivalence**

Using the two-variable ladder equivalences defined in section 3.5.2 it can easily be shown that a Unit-T section p_1 and p_2 elements type is equivalent to a UE-C-UE section (UCU) in the real frequency domain p, which is represented in Figure 3.16(b).

In the UCU equivalence the element value correspondence is as described below:

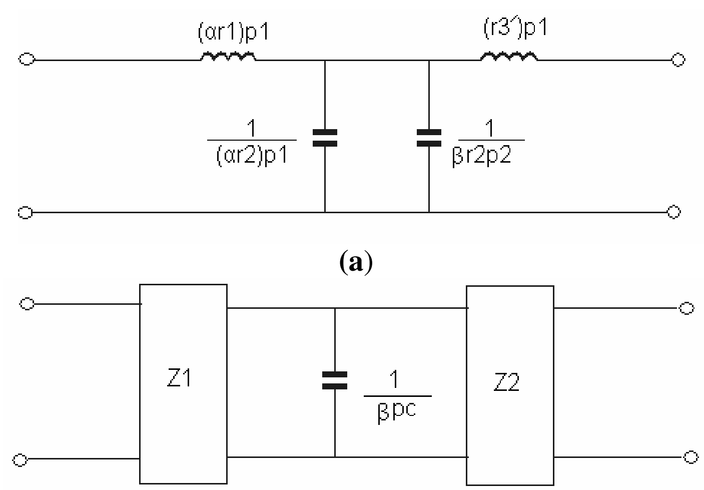
1. $\alpha r_1 = Z_1 > 0$ (3.15)

2. $\alpha\ r_2 = (Y_1 + Y_2) = \dfrac{1}{\alpha\,\mathrm{r}_1} + Y_2$ (3.16)

3. $Y_2 = \alpha\ r_2 - \dfrac{1}{\alpha\,\mathrm{r}_1} > 0$ which implies $\alpha > \dfrac{1}{\sqrt{r_1 r_2}}$ (3.17)

4. $r_3' = \dfrac{1}{Y_2} > 0$ (3.18)

5. $r_3'' = \alpha\ r_3 - r_3' > 0$ (3.19)



(b)

Figure 3.16 Unit-T section(a) and its UCU Equivalent(b)

Utilizing the Unit-T and UCU equivalences for each T-sections generated in the decomposition step in Figure 3.16, the two-variable ladder is transformed sequentially to a low-pass ladder with Unit Element separations resulting in the desired LPLU structure of Figure 3.12(a).

In the sequential application of Unit-T and UCU replacements for n-number of Unit-T sections, one should compute α parameter value for each T-subsection separately using

$$\alpha_i > \frac{1}{\sqrt{r_{n-2} r_{n-1}}} \tag{3.20}$$

and select the maximum α value to ensure positive element values in decomposition step.

As a result of sequential process, there will be a remainder inductance in p_1 variable at the final step as shown in Figure 3.17.

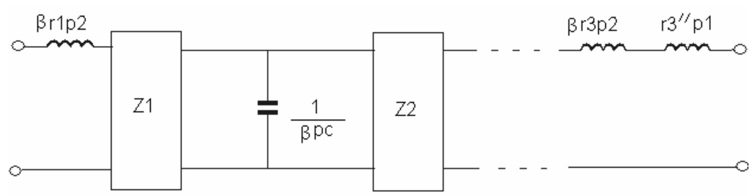


Figure 3.17 UCU equivalent of T network for nth-order

We use final element transformation to combine p_1 and p_2 type inductors to obtain an approximate equivalence by adding these inductors as shown in Figure 3.18. The approximate equivalence at this point requires :

$$p_1 \overset{\Delta}{=} p_2, \ \text{Sinw}\,\tau = \omega \cos \omega\tau \ , \ \ \omega_c \overset{\Delta}{=} \tan \omega_c \tag{3.21}$$

This approximation will result in small perturbation for small values of delay length or $\omega_c \tau = \omega_c \dfrac{\pi}{2\omega_e} << \dfrac{\pi}{8}$

(3.22)

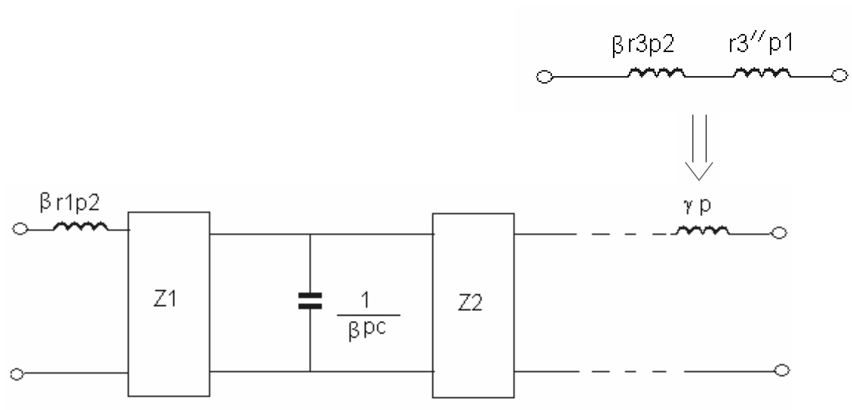


Figure 3.18 LPLU equivalent for order 3

Example 3.4

According to our algorithm, the first step is selecting the appropriate circuit topology for Butterworth or Chebyshev filter .3rd order Butterworth filter topology shown in Figure 3.19.Transfer function of this network

$$H(p)=\frac{1}{P^3+2p^2+2p+1}$$

3rd order Butterworth filter topology in p domain is shown in Figure 3.19

r_1=1 H, r_2=2 F, r_3=1 H

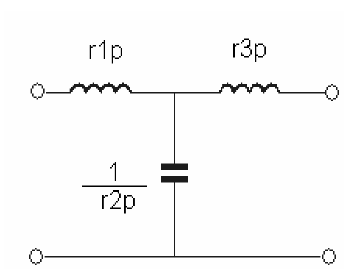


Figure 3.19 Butterworth filter topology for order 3

Using the transformation $p=\alpha p_1+\beta p_2$ when $p_1=\sinh(p\tau)$, $p_2=p.\cosh(p\tau)$

$$H(p_1,p_2)=\frac{1}{(\alpha p_1+\beta p_2)^3+2(\alpha p_1+\beta p_2)^2+2(\alpha p_1+\beta p_2)+1}$$

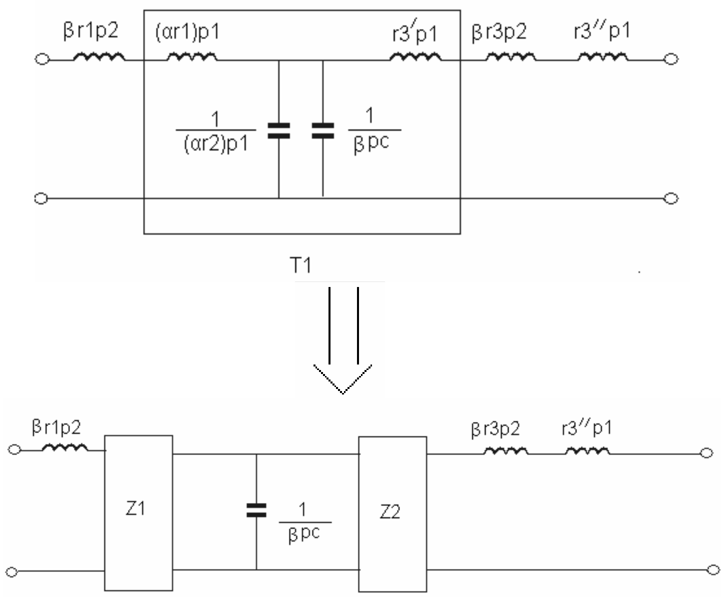
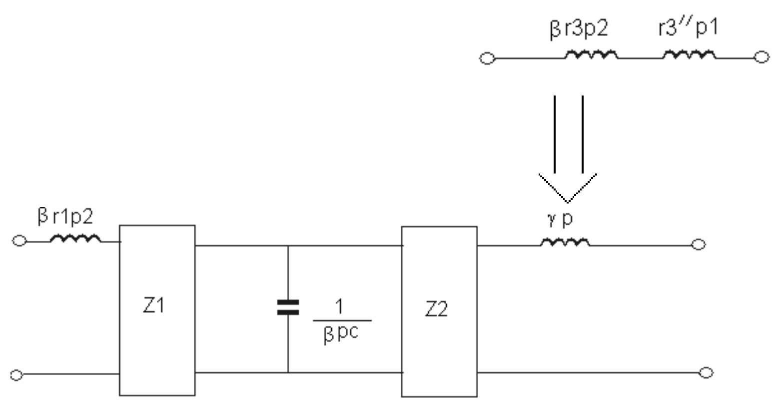


Figure 3.20 (a) Unit-T, UCU transformation steps for order 3



(b)

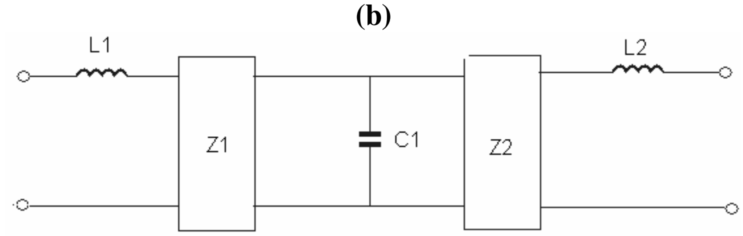


Figure 3.20 (b) LPLU equivalent for order 3 filter

For $\alpha=2, \beta=0.6957$

L1=0.6951 H, Z1=1.990 Ω, C1=1.39 F, Z2=0.2860 Ω ,L2=0.9923 H

Minimum value for α is computed as 1.5 . Selecting $\alpha=2$, β is computed as $\beta=0.6957$ .For repetition frequency=10 Hz , n=3 lumped Butterworth low-pass transfer function characteristic is shown in Figure 3.21, two-variable transformed Butterworth filter frequency characteristic is shown in Figure 3.22,Butterworth filter frequency characteristic of LPLU transformation shown in Figure 3.23.

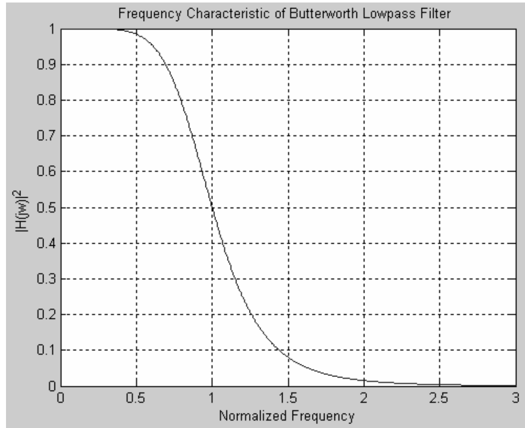


Figure 3.21 Lumped Butterworth low-pass filter frequency characteristic for order 3

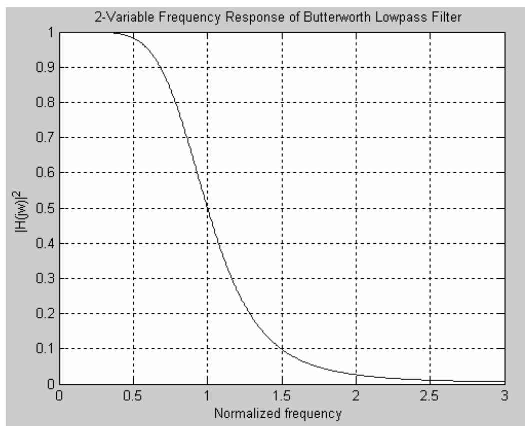


Figure 3.22 Two-variable transformed filter frequency characteristic for order 3

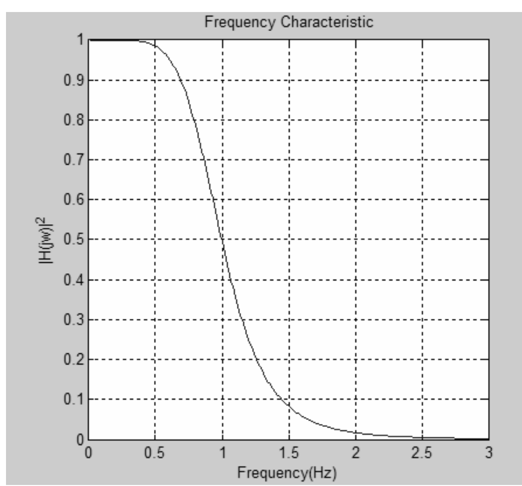


Figure 3.23 Filter frequency characteristic of LPLU transformation for order 3

Example 3.5

Obtain the lumped-distributed realization of a maximally flat low-pass filter with monotone characteristics for order 7 and cutoff frequency 1 Hz. Transfer function of Butterworth low-pass filter for order 7 is

$$H(p) = \frac{1}{1 + 4.494p + 10.0978p^2 + 14.592p^3 + 14.592p^4 + 10.0978p^5 + 4.494p^6 + p^7}$$

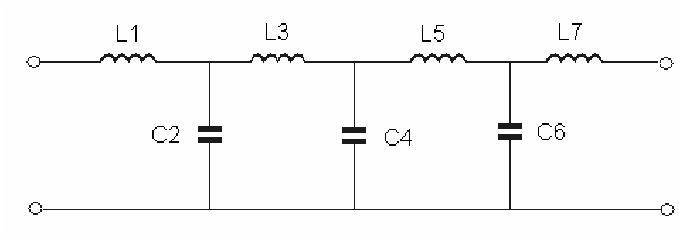


Figure 3.24 (a) Lumped Butterworth low-pass filter topology

L1=0.44504 H ,C1=1.24698 F, L3=1.80154 H, C4=2 F, L5=1.80154 H,
C6=1.24698 F, L7=0.44504 H

By applying the reactance transformation p= α p_1+ βp_2 when p_1=sinh(pτ), p_2=p.cosh(pτ), transfer function of the two-variable low-pass characteristic is obtained as

$$H(p_1, p_2) = [1 + 4.494(\alpha p_1 + \beta p_2) + 10.0978(\alpha p_1 + \beta p_2)^2 + 14.592(\alpha p_1 + \beta p_2)^3$$
$$+ 14.592(\alpha p_1 + \beta p_2)^4 + 10.0978(\alpha p_1 + \beta p_2)^5 + 4.494(\alpha p_1 + \beta p_2)^6 + (\alpha p_1 + \beta p_2)^7]^{-1}$$

Selecting α =2 , β is computed as β=0.6957 repetition frequency=10 Hz, n=7, lumped Butterworth low-pass transfer function characteristic is shown in Figure 3.24(b), two-variable transformed Butterworth filter frequency characteristic is shown in Figure 3.25, Butterworth filter unit-T section LPLU transformation topology shown in Figure 3.26(a), Butterworth filter frequency characteristic of LPLU transformation shown in Figure 3.26(b)

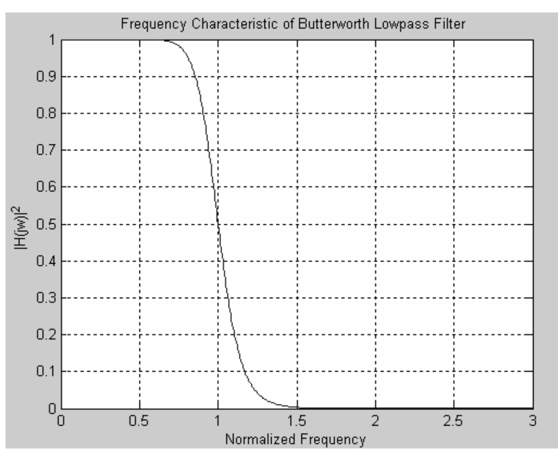


Figure 3.24 (b) Lumped Butterworth low-pass filter frequency characteristic for order 7

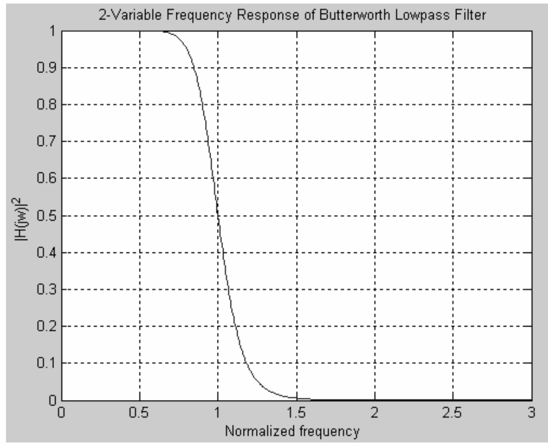


Figure 3.25 Two-variable transformed Butterworth filter frequency characteristic for order 7

Selecting $\alpha = 2$, β is computed as $\beta = 0.6957$

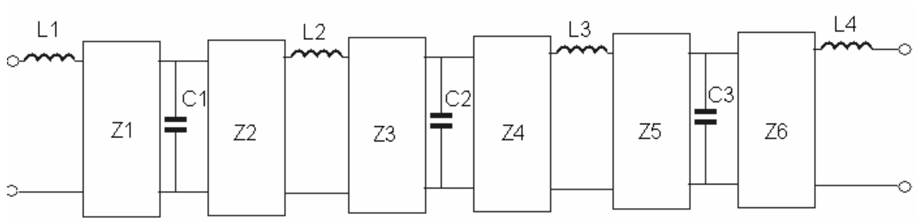


Figure 3.26 (a) Butterworth filter unit-T section LPLU transformation topology for order 7

L1=0.3095 H L2=1.2531 H L3=0.1994 H L4=0.0219 H
Z1=0.8897 Ω Z3=2.1410 Ω Z5=3.0710 Ω
C1=0.8672 F C2=1.3903 F C3=0.8672 F
Z2=0.7033 Ω Z4=0.2655 Ω Z6=0.4291 Ω

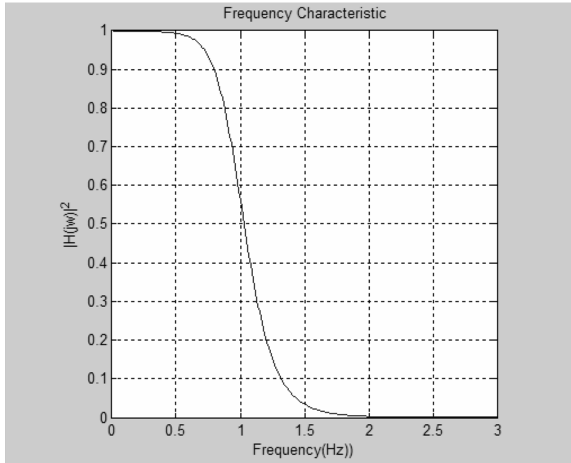


Figure 3.26 (b) Butterworth filter frequency characteristic of LPLU transformation for order 7

3.5.4 LPLU Element Value Tables for Butterworth and Chebyshev Filters

Using the proposed approach, we can generate the LPLU structure element values from prototype lumped filters. Thus for different ripple specifications, LPLU filter element values can be tabularized.

In the following, the LPLU equivalent circuit element values are derived from the corresponding lumped filter for a 1 dB passband ripple specification and given in table forms

Table 3.1 Normalized Butterworth Element Value(1dB pass band ripple)

N	g1	g2	g3	g4	g5	g6	g7	g8	g9	g10
1	1.017									
2	1.008	1.008								
3	0.798	1.596	0.798							
4	0.646	1.560	1.560	0.646						
5	0.539	1.413	1.747	1.413	0.539					
6	0.562	1.263	1.726	1.726	1.263	0.562				
7	0.404	1.132	1.636	1.815	1.636	1.132	0.404			
8	0.358	1.021	1.528	1.802	1.802	1.528	1.021	0.358		
9	0.322	0.927	1.421	1.743	1.855	1.743	1.421	0.927	0.322	
10	0.292	0.848	1.321	1.665	1.846	1.846	1.665	1.321	0.848	0.292
	L1	C2	L3	C4	L5	C6	L7	C8	L9	C10

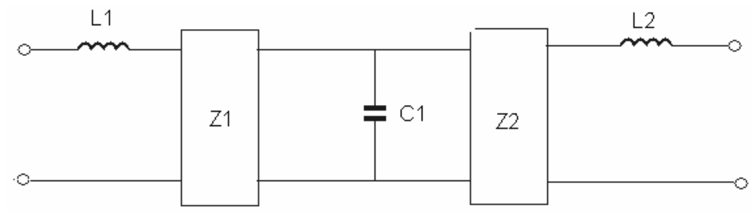


Figure 3.27 Transformed Butterworth filter network topology For Odd Degrees

Table 3.2 Transformed Butterworth Element Values For Odd Degrees (alfa=2; β=0.6957; 1 dB pass band ripple; repetition frequency=10 Hz)

Order	3	5	7	9
L1	0.5554	0.3756	0.2811	0.2241
Z1	1.5967	1.0790	0.8081	0.6765
C1	1.1108	0.9833	0.7877	0.6453
Z2	0.3895	0.5260	0.9735	2.1271
L2	0.5688	1.2155	1.1382	0.9887
Z3		2.4423	2.2987	0.8575
C2		0.9833	1.2633	1.2129
Z4		0.3813	0.2928	0.3219
L3		0.2206	0.1811	1.2907
Z5			2.9794	3.5700
C3			0.7877	1.2129
Z6			0.4769	0.2834
L4			0.2304	0.9887
Z7				2.7012
C4				0.6453
Z8				0.5643
L9				0.0780

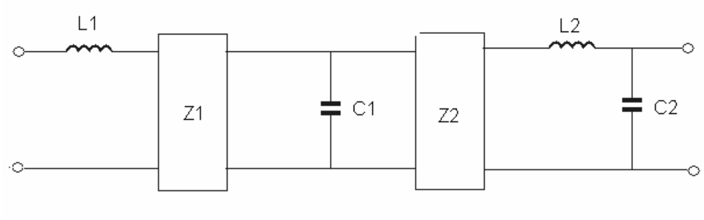


Figure 3.28 Transformed Butterworth Filter network topology For Even Degrees

Table 3.3 Transformed Butterworth Element Values For Even Degrees (alfa=2; β=0.6957; 1 dB pass band ripple; repetition frequency=10 Hz)

Order	4	6	8	10
L1	0.4497	0.3217	0.2494	0.2034
Z1	1.2928	0.9250	0.7171	0.6725
C1	1.0857	0.8790	0.7104	0.5904
Z2	0.4259	0.6914	1.5423	2.1459
L2	1.5787	1.2008	1.0632	0.9195
C2	0.6464			
Z3		2.0692	1.5131	0.8902
C3		1.2008	1.2541	1.1587
Z4		0.3114	0.3053	0.2991
L3		1.3248	1.2541	1.2844
C4		0.4623		
Z5			3.3000	3.9473
C5			1.0632	1.2844
Z6			0.3442	0.2417
L4			1.1813	0.5793
C6			0.3585	
Z7				3.5891
C7				0.9195
Z8				0.3425
L5				1.1196
C8				0.2924

Table 3.4:Normalized Chebyshev Filter Element Values(1dB pass band ripple)

N	g1	g2	g3	g4	g5	g6	g7	g8	g9	g10
1	1.017									
2	1.821	0.685								
3	2.023	0.994	2.023							
4	2.099	1.064	2.831	0.789						
5	2.134	1.091	3.000	1.091	2.134					
6	2.154	1.104	3.063	1.157	2.936	0.810				
7	2.166	1.111	3.093	1.173	3.093	1.111	2.166			
8	2.174	1.116	3.110	1.183	3.148	1.169	2.968	0.817		
9	2.179	1.119	3.121	1.189	3.174	1.189	3.121	1.119	2.179	
10	3.538	0.777	4.676	0.813	4.742	0.816	4.726	0.805	4.514	0.609
	L1	C2	L3	C4	L5	C6	L7	C8	L9	C10

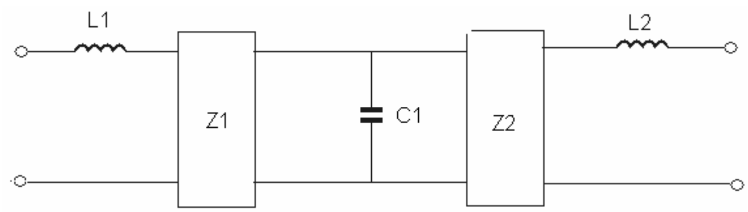


Figure 3.29 Transformed Chebyshev Filter network topology For Odd Degrees

Table 3.5 Transformed Chebyshev Filter Element Values For Odd Degrees (alfa=2; β=0.6957; 1 dB pass band ripple; repetition frequency=10 Hz)

Order	3	5	7	9
L1	1.4078	1.4852	1.5072	1.5164
Z1	4.0471	4.2697	4.3331	4.3594
C1	0.6915	0.7590	0.7732	0.7786
Z2	0.5743	0.5133	0.5019	0.4977
L2	2.0164	2.0877	2.1522	2.1715
Z3		4.9751	5.1833	5.2473
C2		0.7590	0.8164	0.8276
Z4		0.4803	0.4433	0.4378
L3		2.3020	2.1522	2.2085
Z5			5.2986	5.4736
C3			0.7732	0.8276
Z6			0.4697	0.4370
L4			2.3608	2.1715
Z7				5.3687
C4				0.7786
Z8				0.4661
L9				2.3842

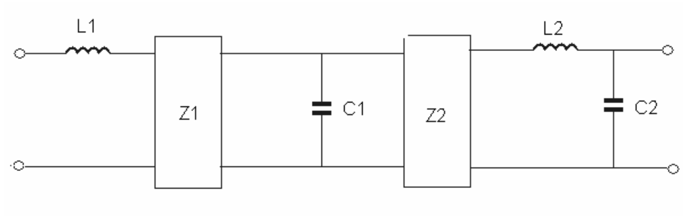


Figure 3.30 Transformed Chebyshev Filter Network Topology For Even Degrees

Table 3.6 Transformed Chebyshev Filter Element Values For Even Degrees (alfa=2; β=0.6957; 1 dB pass band ripple; repetition frequency=10 Hz)

Order	4	6	8	10
L1	1.4603	1.4989	1.5126	1.5151
Z1	4.1981	4.3091	4.3487	4.3671
C1	0.7405	0.7681	0.7764	0.7801
Z2	0.5289	0.5060	0.4994	0.4966
L2	1.9696	2.1312	2.1641	2.1765
C2	0.7891			
Z3		5.1148	5.2226	5.2640
C3		0.8012	0.8236	0.8301
Z4		0.4533	0.4401	0.4363
L3		2.0430	2.1905	2.2185
C4		0.8100		
Z5			5.4172	5.5052
C5			0.8136	0.8341
Z6			0.4450	0.4334
L4			2.0652	2.2080
C6			0.8175	
Z7				5.4807
C7				0.8183
Z8				0.4422
L5				2.0748
C8				0.8209

CHAPTER 4

MICROWAVE FILTER DESIGNER TOOLBOX

4.1 Microwave Filter Designer Toolbox Program Description

Microwave filter design toolbox is a tool for filter design (lumped, distributed, mixed)and analysis under a single GUI window. It has 4 design options. These are lumped filter design, distributed filter design, mixed filter design, analysis module.

Given a type of filter (Butterworth or Chebyshev low-pass), filter order, pass band, stopband and repetition frequency, passband ripple, termination impedance it can list transfer function, plot frequency characteristic of filter and calculate needed inductor and capacitor values for circuit implementations at every design parts. The screenshot of main program module is as shown in Figure 4.1

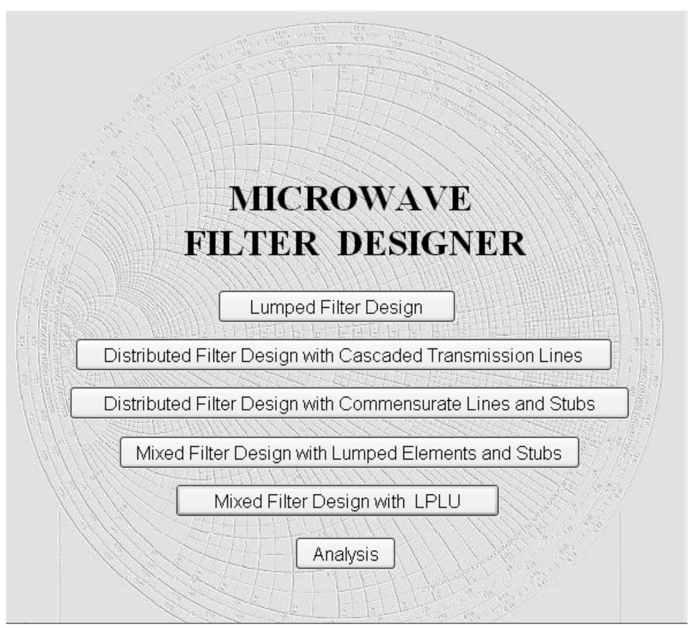
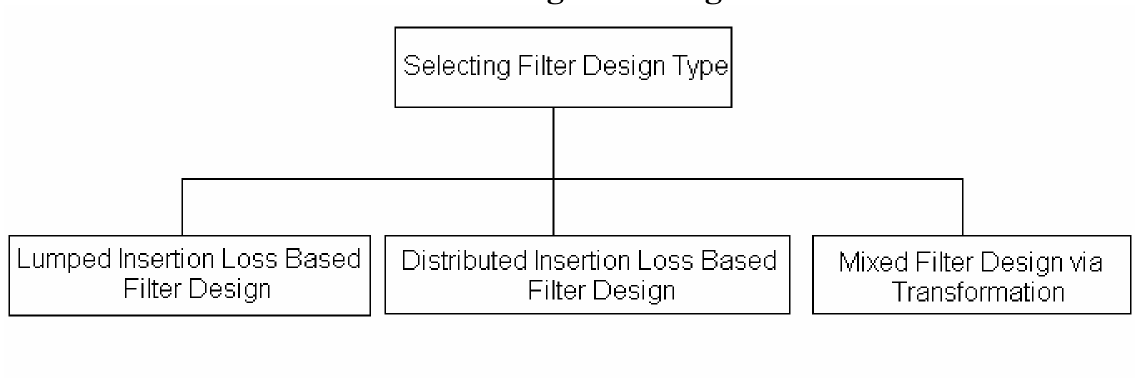
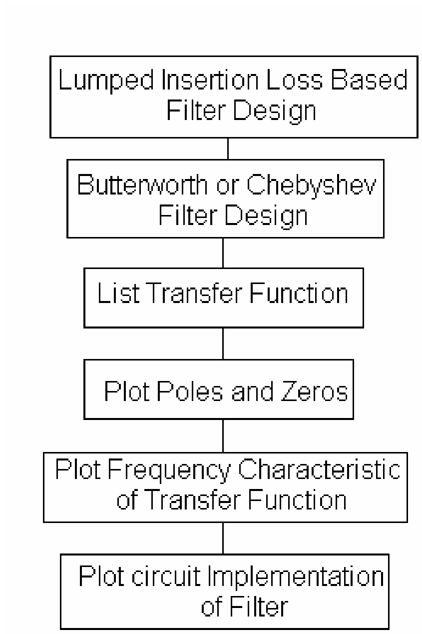


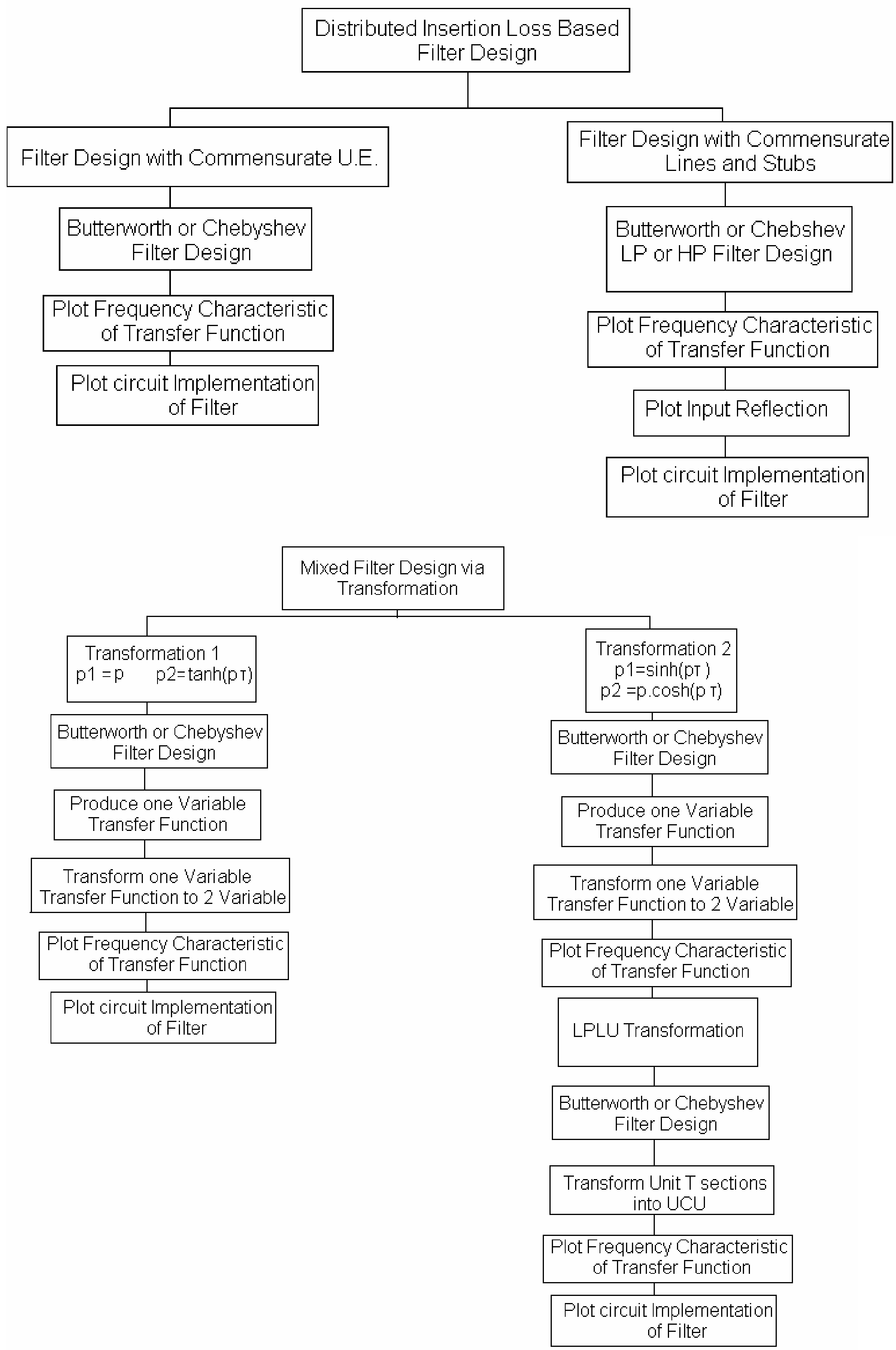
Figure 4.1 Microwave filter designer main module

Microwave filter design program algorithm can be shown as

Microwave Filter Design Tool Algorithm






Distributed Insertion Loss Based Filter Design
Filter Design with Commensurate U.E.
Butterworth or Chebyshev Filter Design
Plot Frequency Characteristic of Transfer Function
Plot circuit Implementation of Filter
Filter Design with Commensurate Lines and Stubs
Butterworth or Chebshev LP or HP Filter Design
Plot Frequency Characteristic of Transfer Function
Plot Input Reflection
Plot circuit Implementation of Filter
Mixed Filter Design via Transformation
Transformation 1
p1 =p p2=tanh(pτ)
Butterworth or Chebyshev Filter Design
Produce one Variable Transfer Function
Transform one Variable Transfer Function to 2 Variable
Plot Frequency Characteristic of Transfer Function
Plot circuit Implementation of Filter
Transformation 2
p1=sinh(pτ)
p2 =p.cosh(p τ)
Butterworth or Chebyshev Filter Design
Produce one Variable Transfer Function
Transform one Variable Transfer Function to 2 Variable
Plot Frequency Characteristic of Transfer Function
LPLU Transformation
Butterworth or Chebyshev Filter Design
Transform Unit T sections into UCU
Plot Frequency Characteristic of Transfer Function
Plot circuit Implementation of Filter

4.1.1 Lumped Filter Design Module:

The inputs of of this module are filter order, passband, stopband and repetition frequency, passband ripple, termination impedance. To obtain normalized frequency characteristic of filter, user must be choose passband frequency and termination impedance 1 Hz and 1 ohm. Transfer function displays the transfer function H(p) required to implement lumped filter .It display transfer function in polynomial form. User can compare filter performance by plotting frequency characteristic of filter. The frequency response window allows the user to compare filter performance. The screenshot of lumped filter design module examples is shown in Figure 4.2 and 4.3 for Butterworth and Chebyshev filter characteristic.

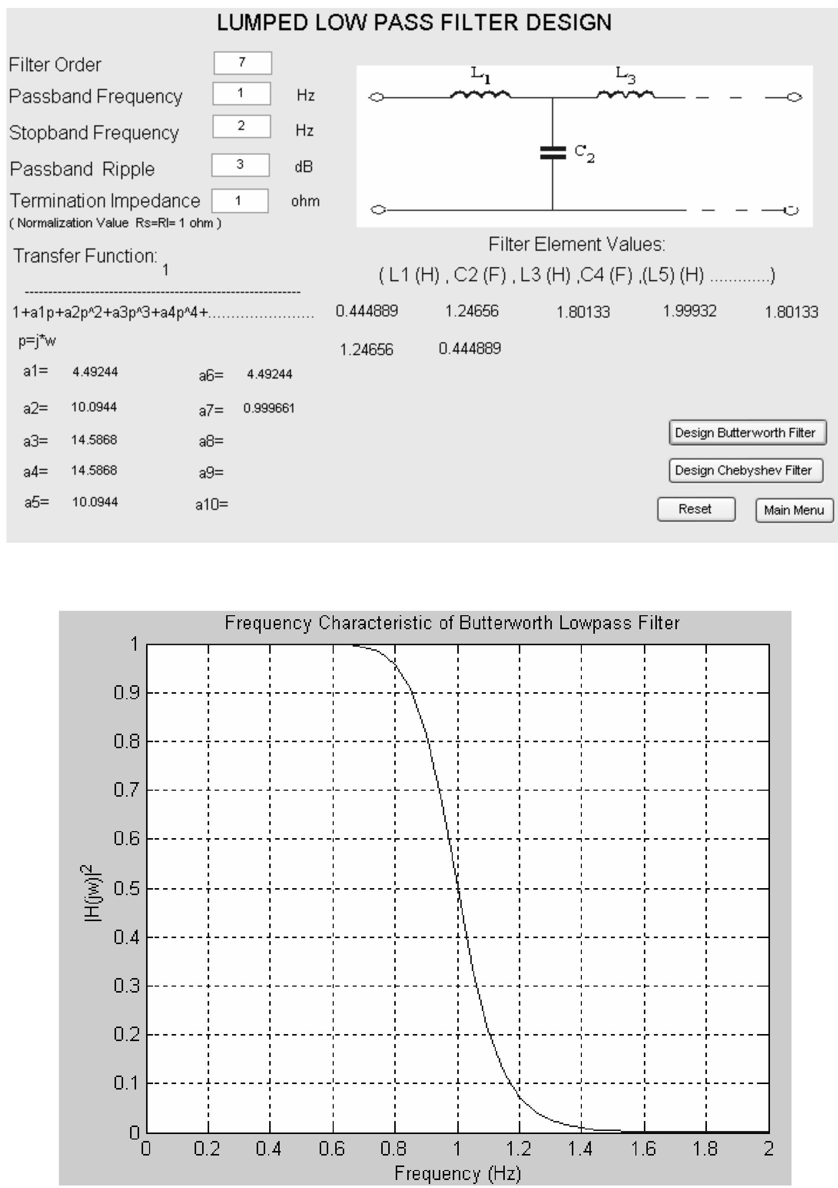
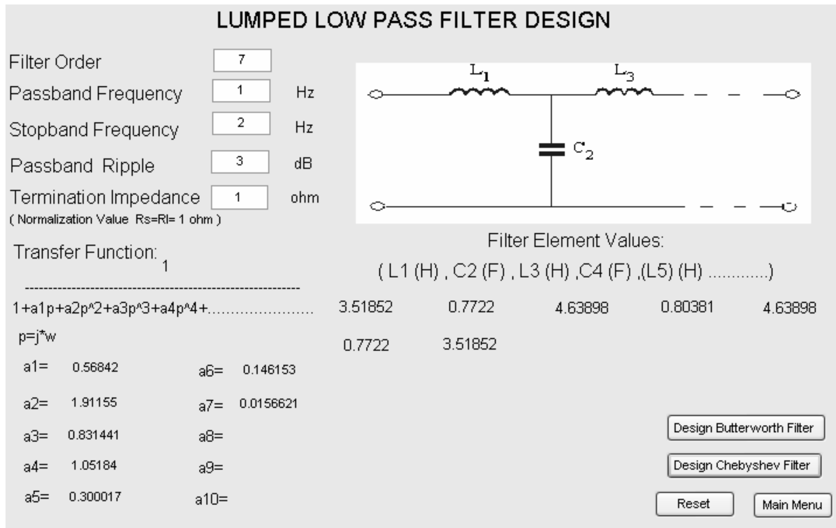


Figure 4.2 Lumped filter design module (Butterworth filter example)



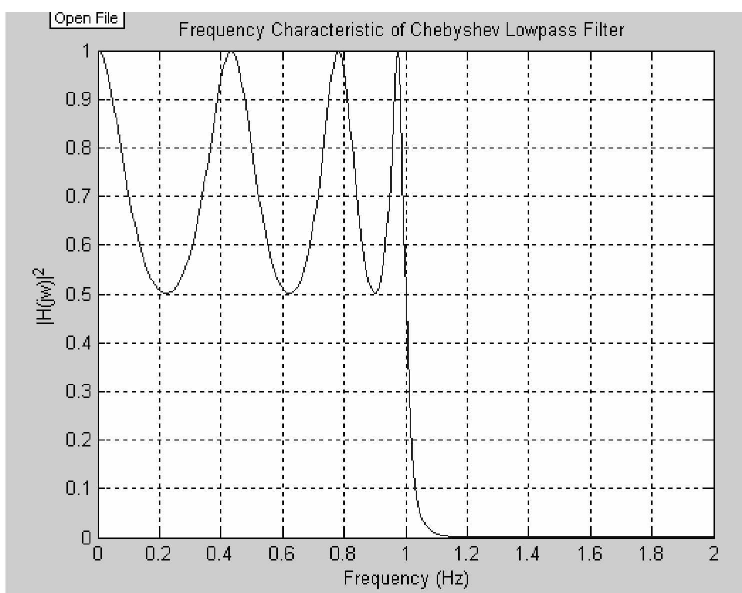


Figure 4.3 Lumped filter design module (Chebyshev filter example)

4.1.2 Distributed Filter Design with Cascaded Transmission Lines Module

At distributed filter design with transmission lines module, given filter order, passband, stopband and repetition frequency, passband ripple, termination impedance , it can synthesis realization of practical network in the form of transmission lines, plot frequency characteristic of transfer function .In this module, $p = \sinh(p\tau)$ transformation is used to obtain frequency characteristic. The screenshot of lumped filter design module examples is shown in Figure 4.4 and 4.5 for Butterworth and Chebyshev filter characteristic.

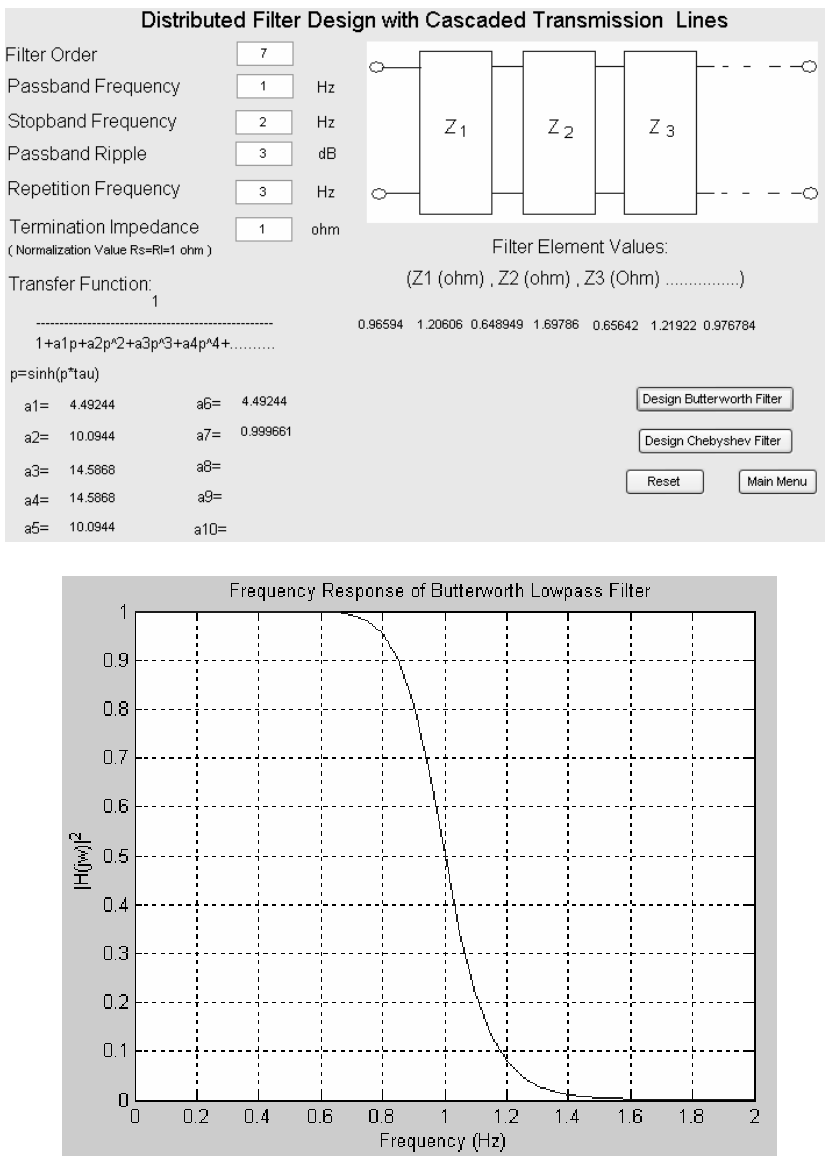
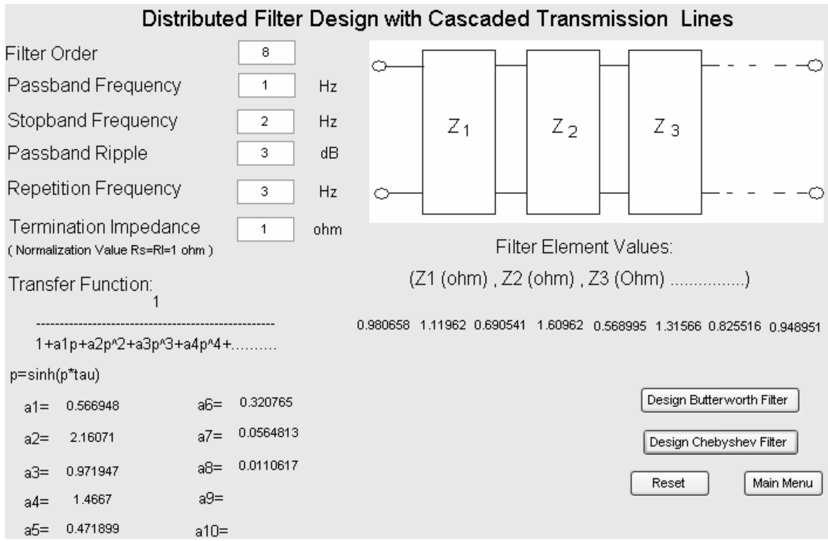


Figure 4.4 Distributed filter design with transmission lines module (Butterworth filter example)



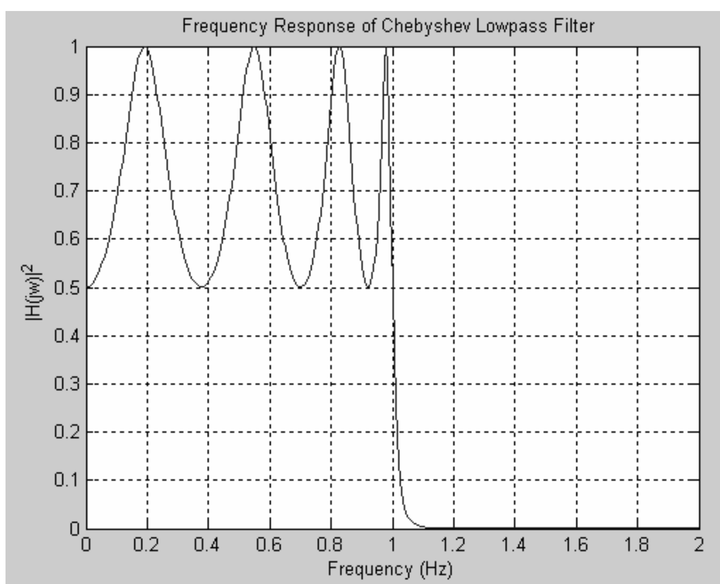


Figure 4.5 Distributed filter design with transmission lines module (Chebyshev filter example)

4.1.3 Distributed Filter Design with Commensurate Lines and Stubs Module

The wizard is developed in Matlab with graphical user interface. The inputs of the wizard are filter type, number of unit elements, number of stubs (LC) elements, filter prototype, normalization number and fractional bandwidth. The filter type is Butterworth or Chebyshev and low-pass. Number of unit elements and LC's determines the number of corresponding elements, the filter contains. Filter prototype enables to arrange filter elements in the desired order. Normalization number is necessary for denormalization at the final step and fractional bandwidth determines the bandwidth of the filter.

When we click on the design button the output of the program is shown in Figure 4.6.Where corresponding circuit is drawn for visualizations. It should be noted that calculated elements values may not physically realizable. As stated earlier Kuroda's identities are used to obtained realizable element values. The screenshot of distributed filter with commensurate lines and stubs design module examples is shown in Figure 4.6.

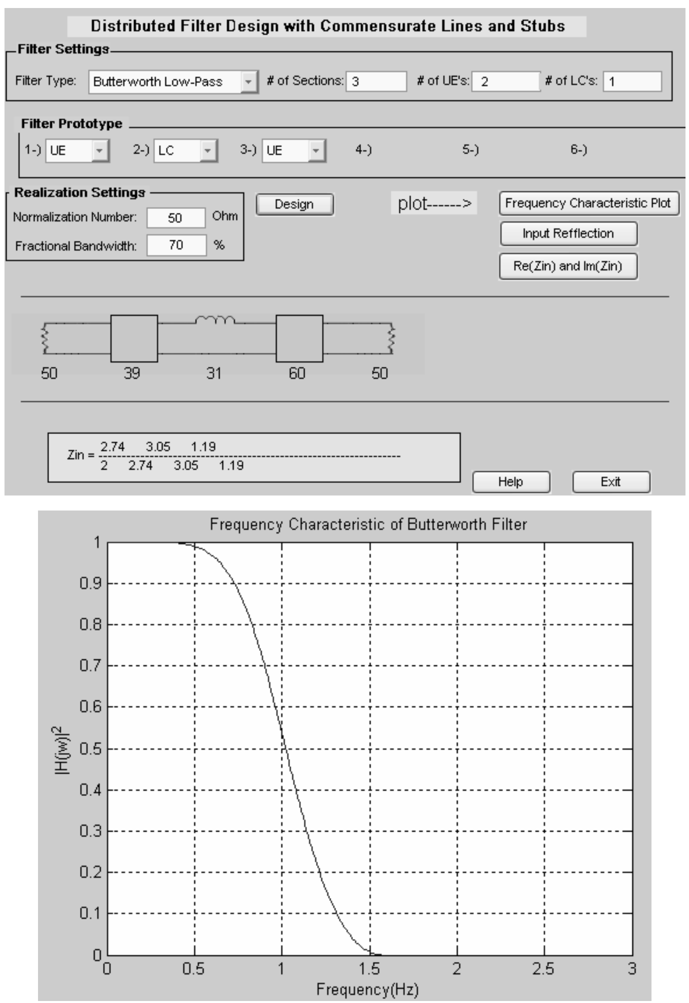


Figure 4.6 Distributed filter design with commensurate lines and stubs module (Butterworth filter example)

4.1.4 Mixed Filter Design with Lumped Elements and Stubs Module

At mixed filter design with stubs module, given filter order, passband, stop band and repetition frequency, passband ripple, termination impedance, it can synthesis realization of practical network and plot frequency characteristic of filter. In this module, transformation 1 (p_1 =p and $p_2 = \tanh(p\tau)$) is used to obtain frequency characteristic. The screenshot of mixed filter design with stubs module examples is shown in Figure 4.7 and 4.8 for Butterworth and Chebyshev filter characteristic.

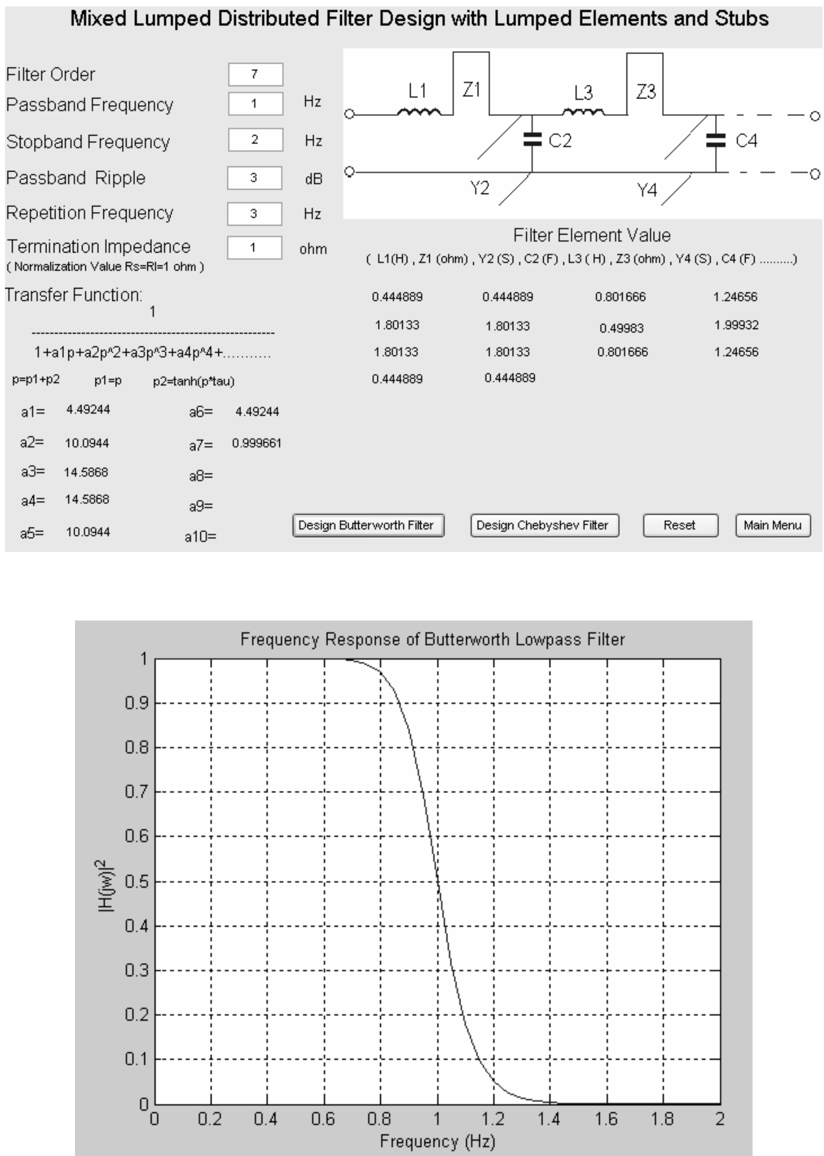


Figure 4.7 Mixed filter design with stubs module (Butterworth filter example)

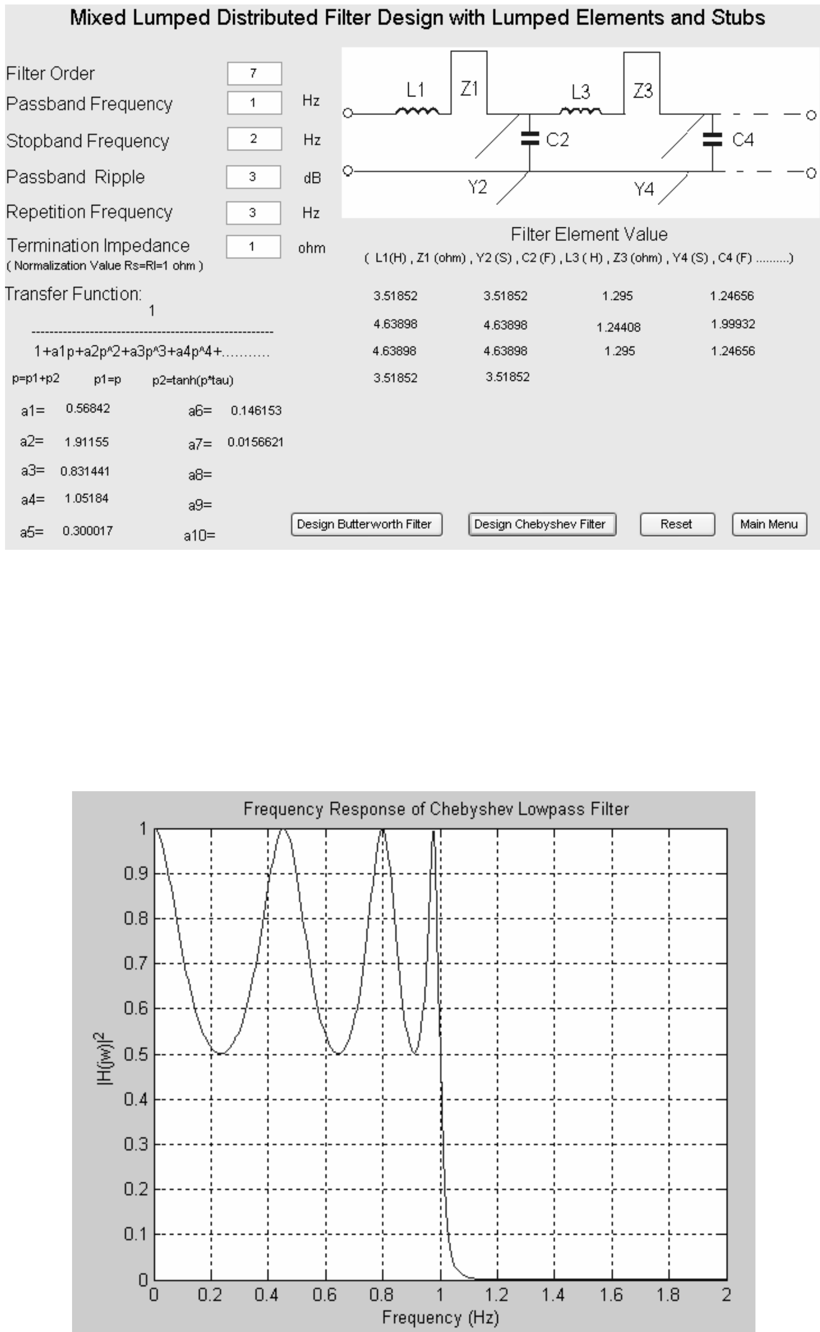


Figure 4.8 Mixed filter design with lumped elements and stubs module (Chebyshev filter example)

4.1.5 Mixed Filter Design with LPLU Module

At mixed filter design with stubs module, given filter order, passband, stop band and repetition frequency, passband ripple, termination impedance and bandwidth control parameter, user can synthesis realization of practical network and plot frequency characteristic of filter. In this module, transformation 2 ($p_1 = \sinh(p\tau)$ and $p_2 = p.\cosh(p\tau)$) and LPLU transformations is used to obtain frequency characteristic. The screenshot of mixed filter design with LPLU module examples is shown in Figure 4.9 and 4.10 for Butterworth and Chebyshev filter characteristic.

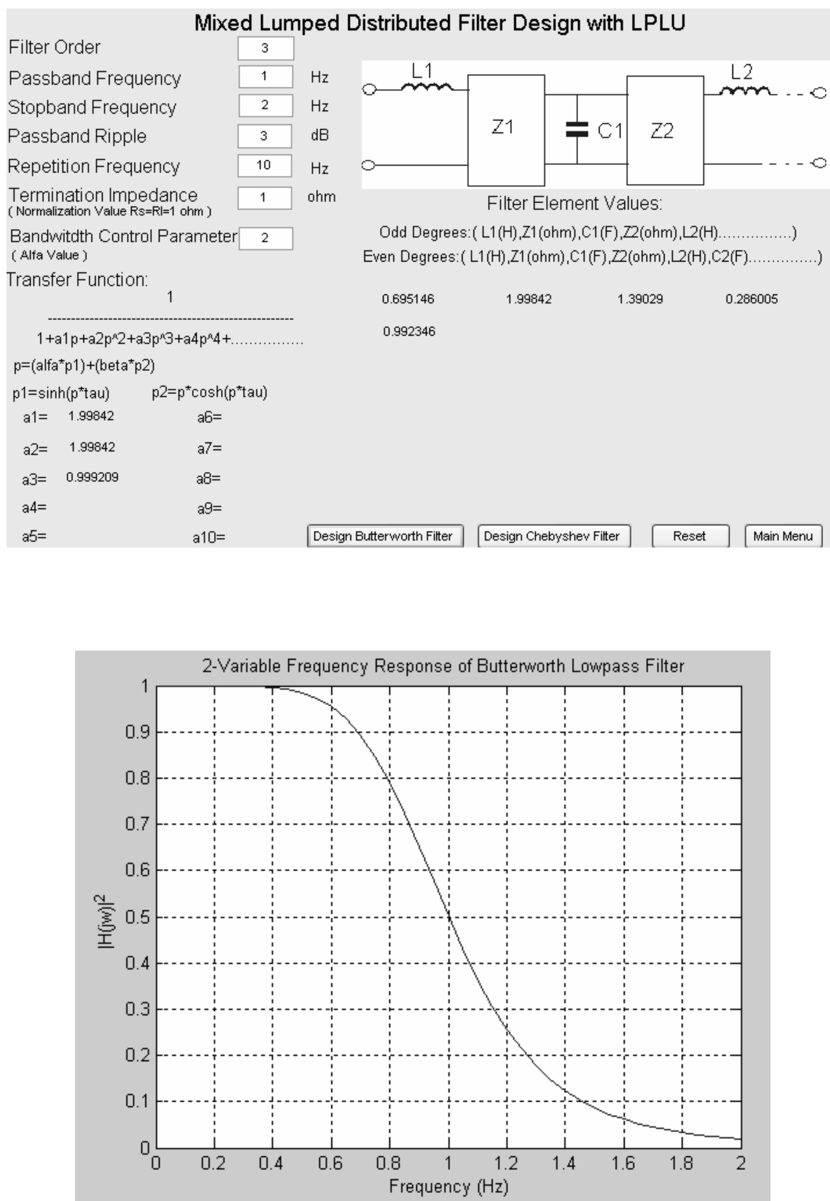
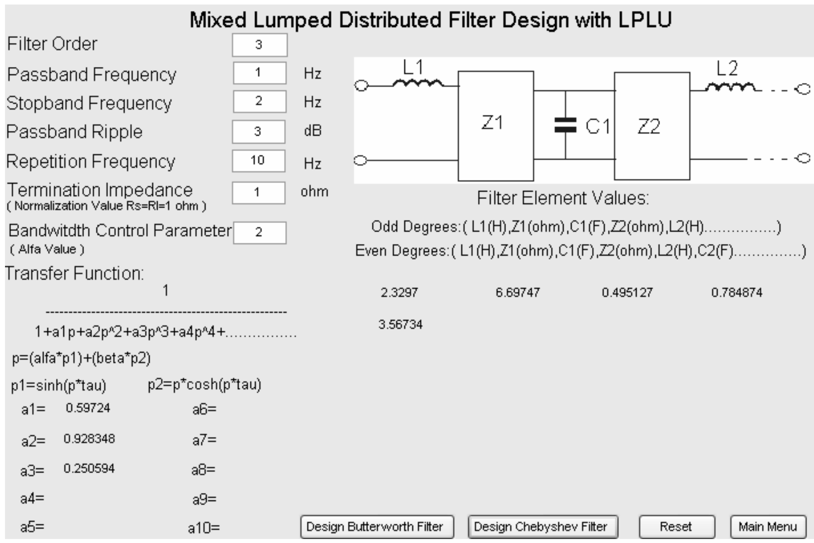


Figure 4.9 Mixed filter design with LPLU module (Butterworth filter example)



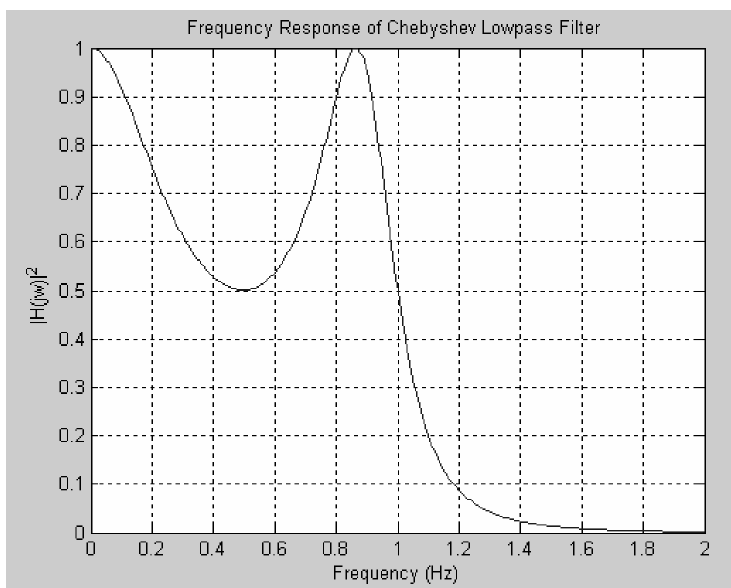
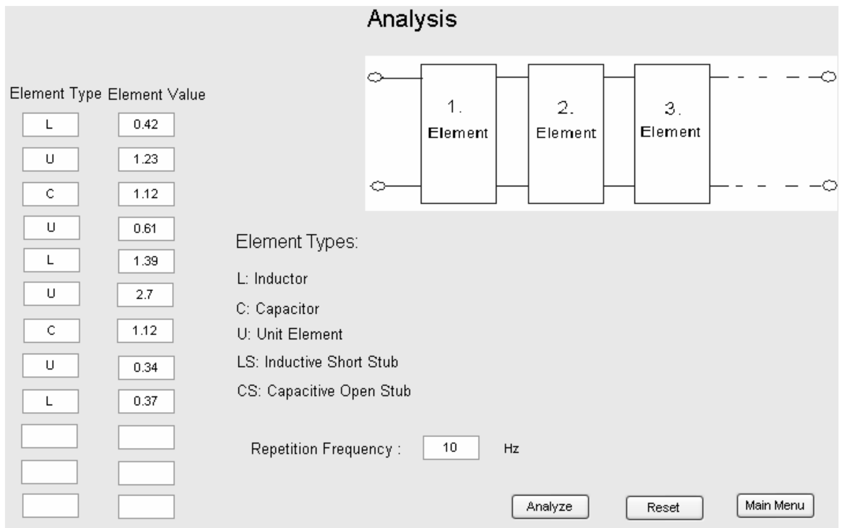


Figure 4.10 Mixed filter design with LPLU module (Chebyshev filter example)

4.1.6-Analysis Module

At analysis module, given filter circuit element type, element value and repetition frequency, user can obtain frequency characteristic of filter. User can enter inductor, capacitor, unit element, inductive short stub and capacitive open stub in element type blocks to compare performance of filter characteristic. The screenshot of analysis module examples is shown in Figure 4.11 and 4.12 for Butterworth and Chebyshev filter.



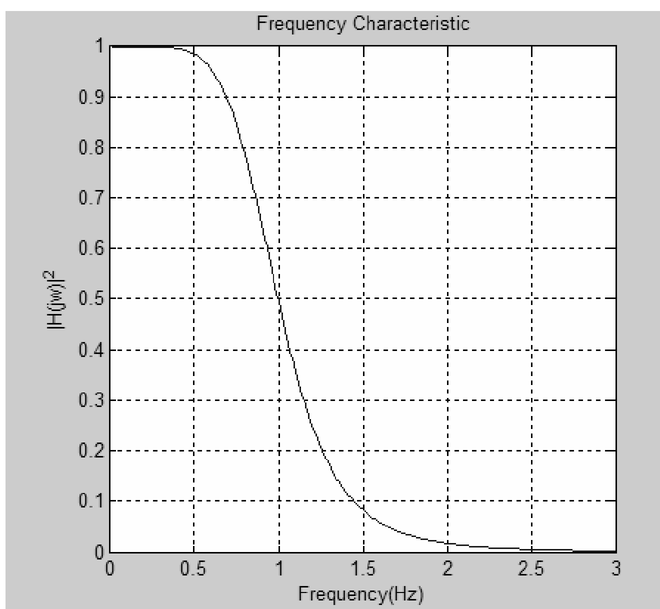
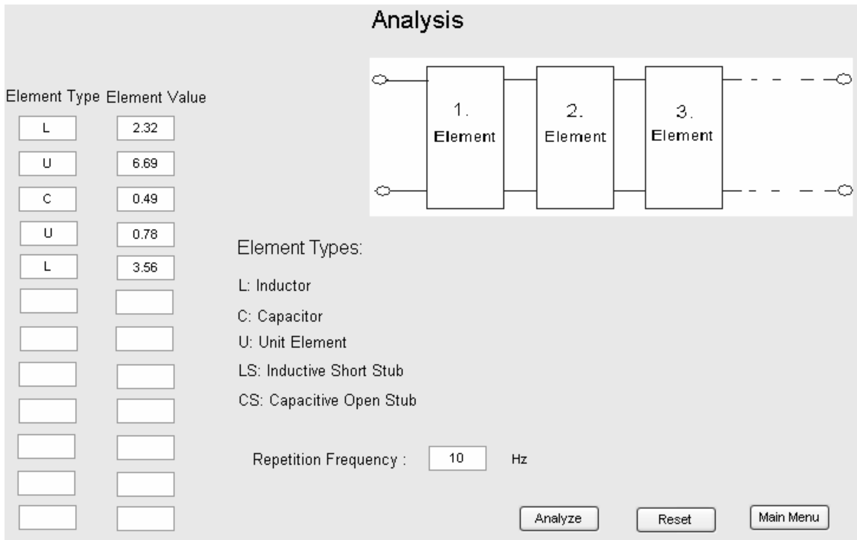


Figure 4.11 Analysis module
(Butterworth filter example)



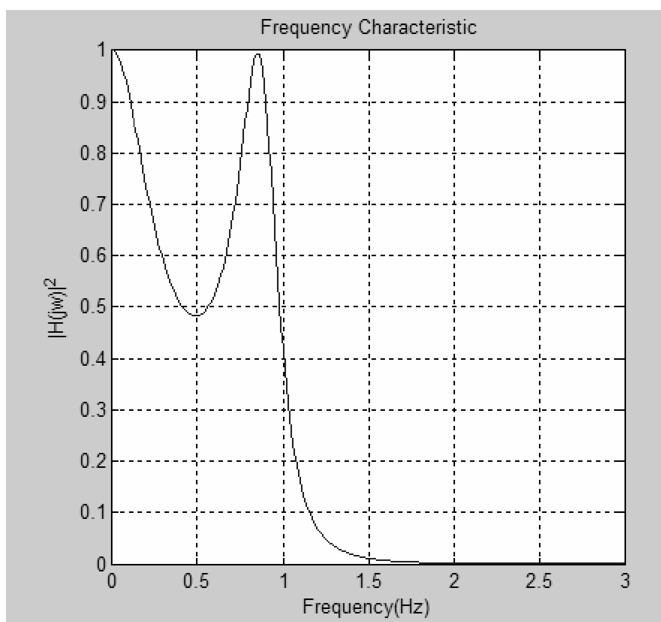


Figure 4.12 Analysis module (Chebyshev filter example)

CHAPTER 5

CONCLUSION

In this thesis, insertion loss based design of microwave filters with lumped and distributed elements is studied. The purpose was to develop a user friendly software tool which enables one to design microwave filters with lumped circuit elements, distributed transmission lines and mixed lumped-distributed elements. For the design tool, modern insertion loss approach is preferred because of several advantages such as flexible filter specification and easy control of filter characteristic. The work is mainly concentrated on Butterworth and Chebyshev type filter designs. First of all low-pass filter prototypes with maximally flat or equal ripple characteristics are designed. Transformation can be then applied to convert the other type of filters such as band-pass, band-stop or stop-band. As far as the realization of filters are concerned, the objective was to come up with designs with lumped elements, distributed elements and mixed lumped-distributed elements.

Design of filters incorporating solely lumped reactive elements is well elaborated in the literature and explicit design equations are already available. Similarly, for the design of transmission line filters, there exist well formulated insertion loss design techniques.

It is well known that lumped element design has serious implementation problems at microwave frequencies, such as difficulties of interconnection of components and parasitic effects. Because of these disadvantages, we studied distributed filter design technique at second part of the thesis. In that part, two different design method is investigated. These are distributed filter design with transmission lines and distributed filter design with commensurate lines and stubs. In the work, first these techniques are studied to form the basis of the filter design theory. Then the filter design techniques with only lumped and only distributed elements are developed and integrated in the design tool developed.

Because of the periodic nature of the transfer function of distributed structures, distributed element filters result in undesired harmonics in the frequency response of the filter. Because of this reason, to obtain better performance at microwave frequencies we examined lumped and distributed (mixed) filter network structures. Both lumped and distributed (mixed) filter design problem has not been solved analytically yet in literature. We examined two different design method of mixed filter design. These are mixed filter design with stubs and mixed filter design with low pass unit element (LPLU). For this purpose, a novel design method for mixed lumped distributed filters is proposed. Our proposed design method for mixed filter with LPLU has four steps. In the first step of the design method, the prototype filter transfer function and the network (lumped or distributed) is generated on an insertion loss basis. Then applying a two-variable reactance transformation two-variable transfer function preserving the given pass band specifications is generated.

The realization of two-variable transfer function in mixed lumped distributed ladder forms is obtained. Here, the two-variable prototype ladder is decomposed into cascaded sub-sections of T-type. The two-variable T-type sections are exchanged by their *almost equivalent* mixed element networks using the replacement techniques. Thus, using our new approximation, a new approach for the transformed mixed element Butterworth and Chebyshev filter circuit elements is deviced. It is believed that the new mixed element ladder structures and the design tables will provide new possibilities and flexibilities in designing microwave filters.

The developed mixed element design approach for Butterworth and Chebyshev type filters is integrated to the developed design tool. The design tool is developed in GUI supported MATLAB environment. The developed software tool, 'Microwave Filter Designer' consist of five major user friendly interactive design modules, which enables one to obtain complete design results for different practical implementation choices of Butterworth and Chebyshev filters. The design options offered by the design tool are *Lumped Filter Design, Distributed Filter Design with Transmission Lines, Distributed Filter Design with Commensurate Lines and Stubs, Mixed Filter Design with Stubs, Mixed Filter Design with LPLU* and *the Analysis Module.*

In the work different design examples are studied for each design module and the resultant performance characteristics are compared with those of the ideal prototype designs. The tool presents an integrated usage of different filter design algorithms and provides complete solutions. Hence it is believed that it will be very useful for the successive works concentrated in filter design area. Also the user friendly nature of the tool is thought to be useful for generating different design topologies leading to the easy comparison of different filter implementations. Easy usage features can also be utilized for instructive purposes. A possible future extension of the tool would be the addition of new modules for pass-band or stop-band type transformations. Also, for practical design problems the tool can be complemented with the inclusion of physical implementation parameters for a typical planar realization technique such as microstrip realization.

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