A COOPERATIVE COMMUNICATION APPROACH TO COGNITIVE RADIO

MURAT İŞLEYEN

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APPROVED BY:

Assist. Prof. Onur Kaya (Thesis Supervisor)	Işık University	____________________
Assoc. Prof. Mengüç Öner	Işık University	____________________
Assist. Prof. Ali İnan	Işık University	____________________

APPROVAL DATE: ..../..../....

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Abstract

Cognitive radio, whose main idea is to have intelligent devices utilize the limited resources, such as frequency spectrum in wireless channels, is becoming a groundbreaking technology. The aim of this thesis is to apply the cooperative communication techniques to the systems which consist of cognitive radio terminals and to develop new cooperative encoding and decoding protocols to reach higher data rates. Accordingly, the performance of a cognitive system where there is one primary (licensed) user and one secondary (unlicensed) user in a network is analyzed considering two approaches: maximization of the sum rate, and maximization of the rates achievable of the secondary user. Optimal power allocation policies for the cooperative cognitive multiple access channel (MAC) are obtained, and the resulting power distributions and achievable rate regions are computed via simulations.

İŞBİRLİKLİ HABERLEŞME YAKLAŞIMI İLE BİLİŞSEL TELSİZ

Özet

Temel fikri kablosuz kanallardaki frekans tayfı gibi sınırlı kaynakları verimli kullanmak olan bilişsel telsizler yeni bir çığır açan teknoloji olma yolunda ilerlemektedir. Bu tez çalışmasında işbirlikli haberleşme modellerinde kullanılan tekniklerin bilişsel telsiz içeren ağlarda uygulanması ele alınarak daha yüksek veri hızlarına erişim için yeni bir kodlama ve kodçözme protokolü geliştirilmesi hedeflenmektedir. Bu amaçla bir birincil (lisanslı) ve bir ikincil (bilişsel) kullanıcıdan oluşan bir bilişsel ağın başarımı iki farklı açıdan ele alınmıştır: toplam veri hızının ve ikincil kullanıcının veri hızının en iyilemesi. İşbirlikli bilişsel çoklu erişim kanalları için özkaynakların en iyi dağılım politikaları incelenmiş, bunlara karşı gelen güç dağılımları ve erişilebilir veri hızı bölgeleri bilgisayar benzetimleri aracılığıyla hesaplanmıştır.

Acknowledgements

There are many people who helped to make my years at the graduate school most valuable. First, I would like to express my deepest acknowledgements to my supervisor Assist. Prof. Onur Kaya for his support and guidance throughout my thesis work. I am also thankful to Işık University for providing me all the facilities and for being a part of such a wonderful environment. I would like to thank my friends and colleagues, Res. Assist. Mehmet Güneş, Burak Çizmeçi, Engin Tamer, Res. Assist. Teoman Mert, and Res. Assist. Doğan Kırcalı and my professors at Işık University for their lifelong support. In addition, I am grateful of TÜBİTAK for their financial support through this long journey. Finally, I will always be indebted to my parents Hamide and Cengiz for their patience and encouragement.

To my family…

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List of Symbols

C	Capacity
$E[\bullet]$	Expected Value
I	Mutual Information
$\max(\bullet)$	Maximum Value
$\min(\bullet)$	Minimum Value

List of Abbreviations

AF	Amplify-and-Forward
AWGN	Additive White Gaussian Noise
CF	Compress-and-Forward
CR	Cognitive Radio
CCR	Causal Cognitive Radio
DF	Decode-and-Forward
DPC	Dirty Paper Coding
DSA	Dynamic Spectrum Access
FCC	Federal Communications Commission
GP	Gel'fand-Pinsker
KKT	Karush-Kuhn-Tucker
MAC	Multiple Access Channel
MAC-GF	Multiple Access Channel with Generalized Feedback
PT	Primary Transmitter
PR	Primary Receiver
RF	Radio Frequency
SDR	Software Defined Radio
SINR	Signal-to-Interference Noise Ratio
SNR	Signal-to-Noise Ratio
SR	Secondary Receiver
ST	Secondary Transmitter
UWB	Ultra-Wide-Band

Chapter 1

Introduction

1.1 Motivation

Wireless networks have been expanding very rapidly, and technology holders and researchers are challenged to develop new equipments and standardizations in wireless services, to meet this demand. The most crucial bottlenecks for the growth in wireless networks is the interference, transmitting power and available spectrum limitations.

Several users in a network may be willing to coexist in the same frequency band and at the same time, thereby causing interfering to each other. Some frequency bands are also reserved to some services or service providers in order to compensate this issue and to better manage the licensed wireless spectrum. Accordingly, several coding strategies and diversity techniques are developed in the field of communications to achieve users' demands.

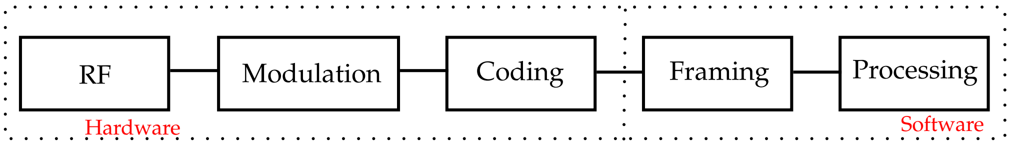
Traditionally, the most popular diversity techniques (time, frequency, space) have been tried to overcome the impediments in a wireless channel, but spatial diversity, specifically dealt with in this thesis, generally requires the multiple antennas. Unfortunately, implementation of multiple antenna technique is hard for small-sized devices. Yet, through cooperative communication strategies, each user in the same network may act as an antenna, so that they generate a virtual multiple antenna array.

However, better quality of communication is not the only problem for telecommunications systems also need to operate with high efficiency, Federal Communications Commission (FCC), which is charged with regulating all of the radio spectrum and all international communications that originate or terminate in the United States published a snapshot of the spectrum usage [1], showing that there is a severe under-utilization of the licensed spectrum by primary users. In order to achieve higher efficiency, new spectrum management approaches are needed. These new approaches are based on the opportunistic access to the spectrum, and are sometimes called dynamic spectrum access (DSA) or cognitive radio (CR). In this thesis, the aim is to propose cooperative communication strategies in networks involving cognitive radios.

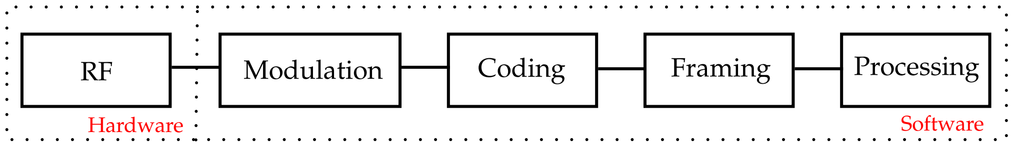
1.2 Software Defined Radio and Cognitive Radio

Traditional radios have been transforming into more intelligent devices over the past two decades. The distinct difference is that radios are now a combination of hardware and software instead of the fact that they are pure hardware-based radios. This devices are referred to as software defined radio (SDR). SDRs are radios which have a configurable behavior, i.e., radio parameters can be adapted to suit the changes in the surrounding radio environment, such as modulation, coding scheme, and transmitting power [2].

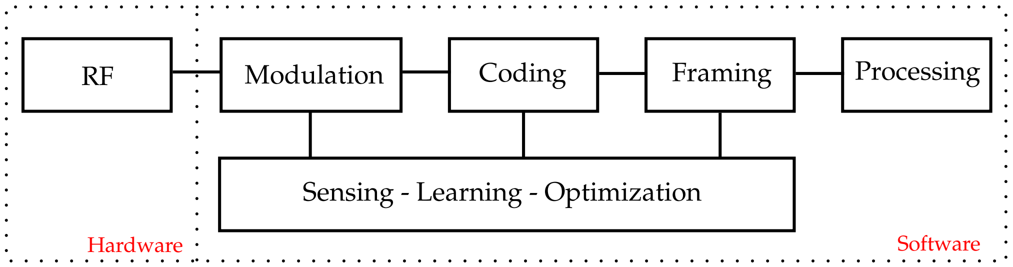
Cognitive radios (CRs) are a type of SDRs and they are capable of sensing and adapting the parameters to environmental changes like in SDR technology: power control, modulation and coding adaptation [2]. However the difference occurs in the intelligence on sensing. A block diagram contrasting traditional radio, SDR, and CR is given in Figure 1.1 [3].



Traditional Radio



Software Radio



Cognitive Radio

Figure 1.1: Comparison of Types of Radios

1.3 Related Works

The term of cognitive radio was firstly introduced by Joseph Mitola [4] in his PhD thesis. CR is defined as an intelligent radio that is capable of analyzing the environment, learning, predicting the most suitable way of the available spectrum and adapting all of its operational parameters [4]. In a cognitive radio network there are three main types of communication techniques: underlaying the signal, overlaying the signal, and interweaving the signal [5]. The main focus on cognitive radios is detecting spectrum holes and communicating at those frequency bands. However, this technique brings with complexity of the devices [6].

The work [5] shows that the communication between cognitive radio's transmitter and receiver does not impact the primary user's communication, significantly. Considering these techniques, we can summarize that an underlay system requires the side information about the interference caused by the cognitive transmitter to the primary receiver. Overlay systems require the side information about the codebook and the possible message of the noncognitive user. In interweave technique, the side information required is the user activity (primary user sensing). On the other hand, while cognitive user's transmit power is limited by the interference constraint in underlay technique, there is no such a constraint in overlay technique. Furthermore, the overlay technique can increase the throughput of secondary communications significantly over the interweave technique [7].

In many works, the channel model is accepted as an interference channel which consists of two transmitters (cognitive and noncognitive) and two intended receivers. Its characteristics can be changed according to the cognitive communication channel techniques mentioned above. Most of the works in literature are based on interweave techniques, but overlay techniques nowadays become more popular yielding cooperative communication advantages [8].

Interference channel where the cognitive transmitter has noncausal knowledge of the primary user's message is the most encountered scenario in the literature. This type of systems are referred to as an interference channel with genie-aided message knowledge [9], degraded message sets [10], [11], and one cooperating encoder [12]. All these works are based on an unrealistic assumption that cognitive user has a priori (non-causal) knowledge of the message of the primary user.

Causal Cognitive Radio (CCR) was first investigated in [9]. Later, an improved rate region was introduced in [13] by introducing the use of delays. [9] proposes a two-phase protocol and the first phase is the transmission of the message from the primary user to the cognitive encoder while [13] employs rate-splitting even with delay. Moreover the impact of feedback and the common information for the channel model in [9] has also been investigated in [14] and [15]. One step further is the bidirectional cooperation between transmitters [16], so both transmitters are cognitive, and coding strategy has changed to the combination of decode-and-forward (DF) and compress-and-forward (CF). In [17], the authors investigated the causal cognitive radio channel where the cognitive source is subjected to a half-duplex constraint in contrast to the full-duplex manner in [13] and four two-phase protocols in [9]. Besides the achievable rate regions for different scenarios, the other important constraint is the limited transmitting power. The work [18] considered the long-term or the short-term transmit-power constraint over the fading states at each non-cognitive transmitter, combined with the interference power constraint at each primary receiver, and for each case, the optimal power allocation scheme is derived. For multi-transmitter multi-receiver channel, [19] has suggested a power allocation algorithm to increase the channel capacity by 50 %. Recent works have been also investigated for the perfect message knowledge at the cognitive encoder but lack of partial channel knowledge.

The situation of the knowledge of the primary user's message enables the cognitive transmitter to apply several encoding schemes that will improve the rates of both cognitive and noncognitive users [5], [9], [10] and [13]. Each receiver observes a noisy copy of the transmitted signal in an interference channel, so the energy of the received signal depends on the path gain from the transmitter to the intended receiver. Using superposition coding [20], information for a worse receiver is first encoded, and a signal for a better user is then superimposed on it. Better receivers can eliminate interference by decoding first and by subtracting their effect from the received signal. However the users cannot be ordered by the quality of their received signals, the interference can still be partially eliminated. The transmitter can apply the precoding technique referred to as Gel'fand-Pinsker (GP) binning [21] and Dirty Paper Coding (DPC) [22]. Moreover, rate-splitting is the best known encoding technique for interference channels. This technique was applied to overlay networks in [9], [11] and [12]. However additional techniques beyond rate-splitting can be used to best exploit the cognitive transmitter's message knowledge.

1.4 Outline of Thesis

Overlay technique provides simultaneous communication in a network and requires considering application of user cooperation and spatial diversity. Chapter 2 aims to give some basic knowledge on cooperative communications and fundamental concepts of information theory, based on different channel models. Particularly, channel models with additive white Gaussian noise (AWGN), are used to introduce these topics in Chapter 2.

Chapter 3 describes several techniques to communicate in a cognitive radio network. The interference channel, where there is one primary and one cognitive transmitter with the irrespective intended receiver pairs, takes large place in the literature. The channel capacity of the interference channel can be found under some special conditions, this chapter tries to take a picture of the works in the literature and their relationships, detailing the different techniques of the communication for the cognitive channel model. Moreover the encoding and decoding strategies and the methodologies used to improve the achievable rates are given.

In Chapter 4, the problem of our thesis is stated. The proposed model of multiple access channel with two transmitters (one is cognitive and the other one is primary) is investigated. Accordingly, the problem is formulated by taking into account of two different approaches: optimization of the sum rate of the system and maximization of the rate achievable by the cognitive user. This optimization problems are solved by using Lagrangian methods, and KKT conditions. For both optimization problems, iterative algorithms are proposed, based on the structure of the optimal power allocation policies. Resultant achievable rate regions are introduced and corresponding power distributions of the primary user and the secondary user are given with respect to the different channel states.

Chapter 2

Background Theory

This chapter of the thesis includes the fundamental theory. It is given what information capacity means and several channel models of interest, such as parallel Gaussian channel, multiple user channels. Karush-Kuhn-Tucker (KKT) conditions should be also considered to solve the optimization problem in this chapter and the next ones of the thesis. Last part of this chapter mentions diversity techniques and user cooperation. For this purpose, relay channels and main strategies are described. Notations used in this thesis follow those used in [23].

2.1 Capacity and Gaussian Channel

Information theory is developed by Claude E. Shannon in his seminal work, "A Mathematical Theory of Communication" to find fundamental limits on signal processing operations such as compressing data, storing and communicating data reliably. The central paradigm of classical information theory is the engineering problem of the transmission of information over a noisy channel. The most fundamental results of this theory are Shannon's source coding theorem, which establishes that, on average, the number of bits needed to represent the result of an uncertain event is given by its entropy; and Shannon's noisy-channel coding theorem, which states that reliable communication is possible over noisy channels provided that the rate of communication is below a certain threshold, called the channel capacity. The channel capacity can be approached in practice by using appropriate encoding and decoding systems.

The most important channel for continuous output alphabet channel which the output can take any values is the additive white Gaussian (AWGN) channel as a special case of Gaussian channel. This channel model is the best model to fit a variety of practical communication channels.

Suppose we send a message over a channel shown in Figure 2.1 that is subjected to AWGN.

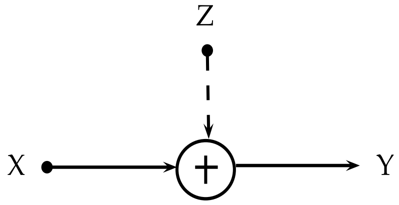


Figure 2.1: Gaussian Channel

Then the output is

$$Y_i = X_i + Z_i \tag{2.1}$$

where X_i and Y_i represent the channel input and the output, respectively and Z_i is zero-mean Gaussian random variable with $N : Z_i \sim \mathcal{N}(0, N)$. We also assume that there is a constraint on transmitting power, such that if we have an input codeword $(x_1, x_2, ..., x_n)$, we will say that the average power is constrained so that

$$\frac{1}{n}\sum_{i=1}^{n} P_i \leq P \tag{2.2}$$

Considering the probability of error for binary transmission, we can send either $-\sqrt{P}$ or $+\sqrt{P}$ over the channel, so the receiver observes the received signal and determines it according to the amplitude of the signal.

$$\begin{aligned}
P_e &= \frac{1}{2}P(Y < 0|X = \sqrt{P}) + \frac{1}{2}P(Y > 0|X = -\sqrt{P}) \\
&= \frac{1}{2}P(Z < -\sqrt{P}|X = \sqrt{P}) + \frac{1}{2}P(Z > \sqrt{P}|X = -\sqrt{P}) \\
&= P(Z > \sqrt{P}) \\
&= \int_{\sqrt{P}}^{\infty} \frac{1}{\sqrt{2\pi N}} e^{-\frac{x^2}{2N}} dx \\
&= Q(\sqrt{P/N}) = 1 - \Phi(\sqrt{P/N}) \qquad (2.3)
\end{aligned}$$

The information capacity of the Gaussian channel is

$$C = \max_{p(x):E[X^2] \leq P} I(X;Y) \qquad (2.4)$$

We can compute this as follows:

$$\begin{aligned}
I(X;Y) &= h(Y) - h(Y|X) \\
&= h(Y) - h(X + Z|X) \\
&= h(Y) - h(Z) \\
&\leq \frac{1}{2}\log 2\pi e(P + N) - \frac{1}{2}\log 2\pi eN \\
&= \frac{1}{2}\log(1 + P/N) \qquad \text{bits/transmission} \quad \diamond \qquad (2.5)
\end{aligned}$$

In order to realize such a result an (M, n) code for the Gaussian channel with power constraint P consists of the following:

1. An index set $1, 2, ..., M$
2. An encoding function $x : 1, 2, ..., M \rightarrow \mathcal{X}^n$, which maps an input index into a sequence that is n elements long, $x^n(1), x^n(2), ..., x^n(M)$, such that the average power constraints is satisfied:

$$\sum_{i=1}^{n}((x_i^n(w)))^2 \leq nP \qquad (2.6)$$

for $w = 1, 2, ..., M$.

3. A decoding function $g : \mathcal{Y}^n \to 1, 2, ..., M$.

2.2 Parallel Gaussian Channel

Before the consideration of parallel Gaussian channel, we need to mention about Karush-Kuhn-Tucker conditions. It is particularly considered in Section 2.5.

When we have k independent Gaussian channels,

$$Y_j = X_j + Z_j, \;\; j = 1, 2, ..., k. \tag{2.7}$$

where $Z_j \sim \mathcal{N}(0, N_j)$

Assume a constraint on total power extending the constraint in Section 2.1

$$E\left[\sum_{j=1}^{k} X_j^2\right] \le P. \tag{2.8}$$

The information capacity of the parallel Gaussian channel is

$$\begin{aligned} I(X_1, ..., X_k; Y_1, ..., Y_k) &= h(Y_1, ..., Y_k) - h(Y_1, ..., Y_k | X_1, ..., X_k) \\ &= h(Y_1, ..., Y_k) - h(Z_1, ..., Z_k) \\ &= h(Y_1, ..., Y_k) - \sum_{i=1}^{k} h(Z_i) \\ &\le \sum_{i=1}^{k} h(Y_i) - h(Z_i) \\ &\le \sum_i \frac{1}{2} \log(1 + P_i/N_i) \;\; \diamond \end{aligned} \tag{2.9}$$

From the equation (2.9), we can say that if the distribution of powers for each parallel channel is cleverly selected, the channel capacity is maximized. The power constraint becomes the following based on Lagrangian operation.

$$J(P_1, P_2, ..., P_k) = \sum_i \frac{1}{2} \log\left(1 + \frac{P_i}{N_i}\right) + \lambda \sum_i^k P_i \tag{2.10}$$

with a constraint that $P_i \geq 0$. Differential w.r.t. P_j to obtain

$$\frac{1}{P_j + N_j} + \lambda \geq 0. \tag{2.11}$$

with equality only if all constraints are inactive, i.e., $\lambda = 0$. After some substitutions, we obtain $P_j = \nu - N_j$.

However, we must also have $P_j \geq 0$, so we must ensure that we do not violate that if $N_j < \nu$. Thus, we let

$$P_j = (\nu - N_j)^+ \tag{2.12}$$

where

$$(x)^+ = \begin{cases} x, & x \geq 0 \\ 0, & x < 0 \end{cases} \tag{2.13}$$

and ν is chosen so that

$$\sum_{i=1}^{n} (\nu - N_i)^+ = P \tag{2.14}$$

This solution is illustrated in Figure 2.2. The vertical levels shows the power levels for the different channels and says that less power is distributed into noisier channels. This process is similar to the way in which water distributes itself in a vessel, so this process is sometimes referred to as waterfilling.

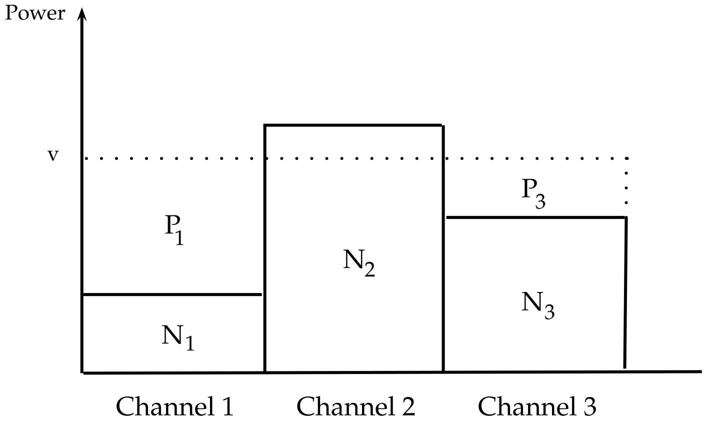


Figure 2.2: Waterfilling for Parallel Channels

2.3 Gaussian Multiple User Channels

This section shows the fundamental calculations for establishing the capacity regions of the Gaussian multiple access channel and its dual model broadcast channel, interference channel and relay channel.

2.3.1 Gaussian Multiple Access Channel

We consider m transmitters sending simultaneously their data to the common destination as shown in Figure 2.3 with the average total power mP.

Let

$$Y = \sum_{i=1}^{m} X_i + Z. \tag{2.15}$$

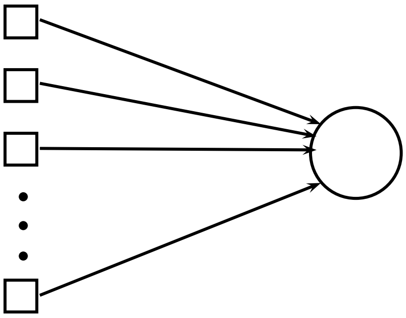


Figure 2.3: Multiple Access Channel

The achievable rate region for the Gaussian channel takes on the simple form given in the following equations

$$R_1 < \frac{1}{2}\log\left(1 + \frac{P_1}{N}\right) \tag{2.16}$$

$$R_1 + R_2 < \frac{1}{2}\log\left(1 + \frac{P_1 + P_2}{N}\right) \tag{2.17}$$

$$R_1 + R_2 + R_3 < \frac{1}{2}\log\left(1 + \frac{P_1 + P_2 + P_3}{N}\right) \tag{2.18}$$

$$\vdots$$

$$\sum_{i=1}^{m} R_i < \frac{1}{2}\log\left(1 + \frac{mP}{N}\right) \tag{2.19}$$

To achieve the rate, we need m codebooks which the ith codebook has 2^{nR_i} codewords of power P_i. Each of the independent senders arbitrarily chooses a codeword from its own codebook, then all users send these simultaneously. The destination receives the combined codewords of all ones sent with the Gaussian noise $\mathbf{Z}$, such that the received signal at time instant i for m users is:

$$Y_i = X_{1i} + X_{2i} + ... + X_{mi} + Z_i \tag{2.20}$$

The receiver decodes the received message as looking the Euclidean distance between the vector sum of the m codewords (one from each) and $\mathbf{Y}$. If $(R_1, R_2, ..., R_m)$ is in the capacity region given above, the probability of error goes to 0 as n tends to infinity.

The sum of the rates of the users $\frac{1}{2}\log(1 + \frac{mP}{N})$ goes to infinity with m. It is obviously seen that the increasing interference as the number of senders $m \to \infty$ does not limit the total received information.

2.3.2 Gaussian Broadcast Channel

In this model in Figure 2.4, we have one transmitter with power P and at least two receivers. In a two receiver case, each channel effects the transmitting signal with additive white Gaussian noise with variances N_1 and N_2. The channel model is

$$Y_1 = X + Z_1 \tag{2.21}$$

$$Y_2 = X + Z_2 \tag{2.22}$$

where Z_1 and Z_2 are arbitrarily correlated Gaussian random variables with variances N_1 and N_2, respectively. Suppose that $N_1 < N_2$, so the first channel has a better condition than the other. In other words, the receiver Y_1 is less noisy than the receiver Y_2.

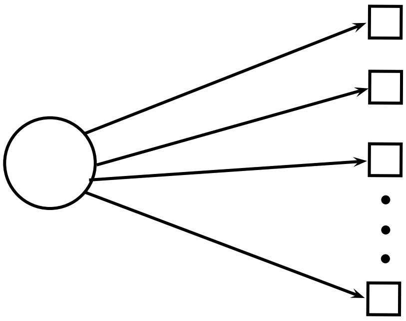


Figure 2.4: Broadcast Channel

In addition, the capacity region of a broadcast channel depends only on the conditional marginal distributions $p(y_1|x)$ and $p(y_2|x)$, i.e. $p(y_1, y_2|x) = p(y_1|x)p(y_2|x)$. This is called degraded broadcast channel. The capacity region of the Gaussian degraded broadcast channel [22, Theorem 15.6.4] is

$$R_1 \ < \ \frac{1}{2}\log\left(1 + \frac{\alpha P}{N_1}\right) \tag{2.23}$$

$$R_2 \ < \ \frac{1}{2}\log\left(1 + \frac{(1-\alpha)P}{\alpha P + N_2}\right) \tag{2.24}$$

where α is the tradeoff parameter which is chosen in $(0 \leq \alpha \leq 1)$.

The common transmitter generates two codebooks, one with power αP at rate R_1, and another codebook with power $\bar{\alpha} P$ where $\bar{\alpha} = 1 - \alpha$ at rate R_2. Indexes $w_1 \in 1, 2, ..., 2^{nR_1}$ and $w_2 \in 1, 2, ..., 2^{nR_2}$ are sent to Y_1 and Y_2, respectively. The transmitter combines these two codewords (one from each book), then sends it over the channel.

In order to decode the messages, firstly, the worse receiver Y_2 receives the second codeword with an unwanted component which is the first trasmitter's codeword. It is proven that this component can treat as a noise to Y_2. Thus, the signal-to-noise ratio at the receiver is $\bar{\alpha} P/(\alpha P + N_2)$. Then, better receiver Y_1 can decodes Y_2's codeword due to the lower channel noise, N_1. So the receiver subtracts the decoded part of the other codeword from $\mathbf{Y}_1$ and finds the codeword in the first codebook closest to $\mathbf{Y}_1 - \hat{\mathbf{X}}_2$. The resulting probability of error can be made as low as desired.

2.3.3 Gaussian Interference Channel

An interference channel consists of two senders and two receivers. Each transmitter aims to send its own message to its intended receiver, so they do not care what the other sender are transmitting through the channel. This simultaneous communication causes which one of the signal interferes with the other. This channel model is illustrated in Figure 2.5.

If we suppose the channel is symmetric $(a = b)$, we have

$$\begin{aligned} Y_1 &= X_1 + aX_2 + Z_1 && (2.25) \\ Y_2 &= X_2 + bX_1 + Z_2 = X_2 + aX_1 + Z_2 && (2.26) \end{aligned}$$

where Z_1, Z_2 are independent Gaussian random variables with $Z \sim \mathcal{N}(0, N)$.

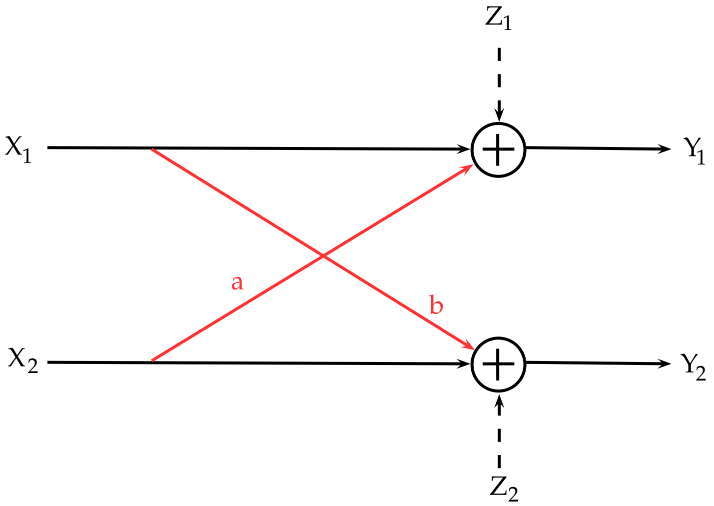


Figure 2.5: Gaussian Interference Channel

This channel has not been solved in general, even in the Gaussian case. However, it can be shown that the capacity region of this channel is the same as when the channel is exposed to strong interference, i.e., where $a > 1$ and $b > 1$ It is proven that a clear message is transmitted in strong interference channel. It is particularly detailed in Chapter 3.

Considering two codebooks, each with power of P and the rate of $\frac{1}{2}\log(1+P/N)$. Each transmitter chooses a codeword from its own codebook and sends it. If the interference a satisfies the condition of $\frac{1}{2}\log(1+a^2P/(P+N)) > \frac{1}{2}\log(1+P/N)$, the first transmitter perfectly understands the index of the second transmitter and looks through the closest codeword. The receiver then subtracts it from his waveform received. So there is a clean channel between a transmitter-receiver pair. Lastly, the receiver searches the senders codebook to find the closest codeword and declares that codeword to be the one sent.

2.3.4 Gaussian Relay Channel

In a relay channel, there are a transmitter X, an intended receiver Y and also a node X_1 which relays the transmitter's message to the sender. The Gaussian relay channel as shown in Figure 2.6 is given by

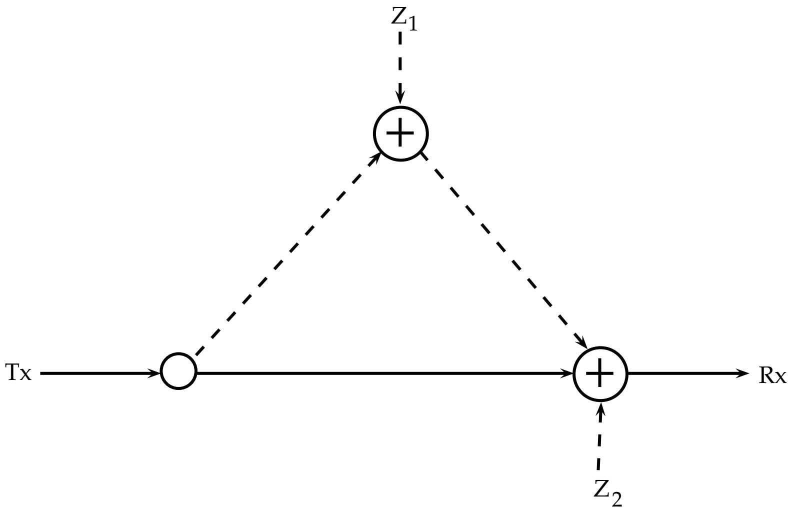


Figure 2.6: Gaussian Relay Channel

$$Y_1 = X + Z_1 \tag{2.27}$$

$$Y = X + Z_1 + X_1 + Z_2 \tag{2.28}$$

where Z_1 and Z_2 are independent zero-mean Gaussian random variables with $Z_1 \sim \mathcal{N}(0, N_1)$ and $Z_2 \sim \mathcal{N}(0, N_2)$, respectively. The encoding can be applied by the relay as following scheme

$$X_{1i} = f_i(Y_{11}, Y_{12}, ..., Y_{1i-1}). \tag{2.29}$$

Sender X has power of P and sender X_1 has power of P_1, the channel capacity is then found as

$$C = \max_{0\leq\alpha\leq 1} \min \left\{ \frac{1}{2} \log \left(1 + \frac{P + P_1 + 2\sqrt{\bar{\alpha} P P_1}}{N_1 + N_2} \right), \frac{1}{2} \log \left(1 + \frac{\alpha P}{N_1} \right) \right\} \tag{2.30}$$

The effect of the relay can be observed for such a condition in which if there is no relay in the system, the capacity becomes $\frac{1}{2}\log(1+P/(N_1+N_2))$ when α is selected as 1. When we also determine $\frac{P_1}{N_2} \geq \frac{P}{N_1}$, the capacity is $C = \frac{1}{2}\log(1+P/N_1)$. Thus, the channel appears noise-free after the relay. For large N_2, we can extract that the rate can be increased from $\frac{1}{2}\log(1+P/(N_1+N_2)) \approx 0$ to $C = \frac{1}{2}\log(1+P/N_1)$.

There are two codebooks are needed to create the opportunity for cooperation by the relay. The first codebook has 2^{nR_1} codewords of power αP and the second one has 2^{nR_0} codewords of power $\bar{\alpha}P$. A codeword from the first codebook is first sent. The relay knows the index of this codeword since $R_1 < \frac{1}{2}\log(1+\alpha P/N_1)$, but the intended receiver does not. However, the receiver has a list of possible codewords of size $2^{n(R_1-\frac{1}{2}\log(1+\alpha P/(N1+N2)))}$. Then, the transmitter and the relay wish to cooperate to resolve the receiver's uncertainty about the codeword sent previously that is on the receiver's list. Unfortunately, they cannot be sure what this list is because they do not know the received signal Y. Thus, they randomly partition the first codebook into 2^{nR_0} cells with an equal number of codewords in each cell. The relay, the receiver, and the transmitter agree on this partition. The relay and the transmitter find the cell of the partition in which the codeword from the first codebook lies and cooperatively send the codeword from the second codebook with that index, that is, X and X_1 send the same designated codeword. The relay, of course, must scale this codeword so that it meets its power constraint P_1. They now transmit their codewords simultaneously. An important point to note here is that the cooperative information sent by the relay and the transmitter is sent coherently. So the power of the sum as seen by the receiver Y is $(\sqrt{\bar{\alpha}P} + \sqrt{P_1})^2$.

However, this does not exhaust what the transmitter does in the second block. It also chooses a fresh codeword from the first codebook, adds it on paper to the cooperative codeword from the second codebook, and sends the sum over the channel.

The reception by the ultimate receiver Y in the second block involves first finding the cooperative index from the second codebook by looking for the closest codeword in the second codebook. It subtracts the codeword from the received sequence and then calculates a list of indices of size 2^{nR_0} corresponding to all codewords of the first codebook that might have been sent in the second block.

Now it is time for the intended receiver to complete computing the codeword from the first codebook sent in the first block. It takes its own list of possible codewords that might have been sent in the first block and intersects it with the cell of the partition that it has learned from the cooperative relay transmission in the second block. The rates and powers have been chosen so that it is highly probable that there is only one codeword in the intersection. This is Ys guess about the information sent in the first block.

In each new block, the transmitter and the relay cooperate to resolve the list uncertainty from the previous block. In addition, the transmitter superimposes some fresh information from its first codebook to this transmission from the second codebook and transmits the sum. The receiver is always one block behind, but for sufficiently many blocks, this does not affect its overall rate of reception.

2.4 User Cooperation

User cooperation is a cooperative multiple antenna technique in order to improve individual or total network capacities. Cooperation can be achieved by amplifying or decoding the combined signal of the relaying signal and the direct signal. Hence, it can be seen that cooperative diversity is an antenna diversity that uses distributed antennas belonging to each user in a wireless system.

In this thesis, our proposed model is based on the cooperative communication framework, we should thus consider the relaying strategies mentioned above. In our topology, the cognitive user use Decode and Forward method to relay the primary user's message to the destination. It is described in Chapter 4 in detail.

The main cooperative communication signaling methods can be categorized into three parts. There are also their combinations in literature. These are Amplify and Forward, Decode and Forward and Compress and Forward.

Amplify and Forward (AF) is the most fundamental of the relaying methods. It is often referred to as the non-regenerative relaying protocol. AF will amplify the received signal to compensate the channel loss, and then forward it to the receiver. In this technique, each user receives a noisy version of the signal transmitted by its cooperative user. The performance of this protocol suffers from the fact that it also amplifies the noise factor and requires constant estimation of channel quality. Nevertheless, AF is a simple method to analyze in literature and therefore has been very useful to understand the cooperative communication.

Decode and Forward (DF) method is any protocol where relay performs decoding and re-encoding before forwarding the data to the receiver, and is sometimes referred to as regenerative relaying protocol. The requirement for cooperation is that the relay successfully decodes the message from the source. In this technique, faulty information is not transmitted to destination, so the performance for the decoder can be decreased. Thus, the main advantage of AF over DF is that no hard decisions are needed, but on the contrary AF does not regenerate the signal.

In our model, Decode and Forward strategy is employed. There are various encoding and decoding protocols have established in literature. Most used technique is block Markov superposition encoding and backward decoding proposed by Carleial [24] and irregular block Markov superposition encoding is proposed by Cover and El-Gamal. This methodology is also investigated with decoding strategies in Chapter 4.

Compress and Forward (CF) method allows the relay to compress the received signal from the source and forward it to the destination without decoding the signal. In this technique, Wyner-Ziv coding can be used for optimal compression.

2.5 Nonlinear Optimization

In the previous sections and next chapters, parallel Gaussian channel and some Gaussian channel models are considered. To understand the idea of power allocation, we need a result from constrained optimization theory as known the

Karush-Kuhn-Tucker (KKT) conditions in order to cleverly distribute transmitting power.

In mathematics, KKT conditions are necessary for a solution in nonlinear programming to be optimal, provided that some regularity conditions are satisfied.

Suppose we minimize a convex objective function $L(x)$,

$$\min L(x) \tag{2.31}$$

subject to

$$f(x) \leq 0. \tag{2.32}$$

Then, say that the optimal value is x_0. The constraint is inactive, when we get

$$\left.\frac{\partial L}{\partial x}\right|_{x_0} = 0 \tag{2.33}$$

or, if the constraint is active, it must be the case that the objective function increases for all admissible values of x :

$$\frac{\partial L}{\partial x_{x \in A}} \geq 0 \tag{2.34}$$

where A is the set of admissible values, for which $\dfrac{\partial f}{\partial y} \leq 0$.

Thus,

$$\frac{\partial L}{\partial x} + \lambda \frac{\partial f}{\partial x} = 0 \qquad \lambda \geq 0. \tag{2.35}$$

We can create a new objective function

$$J(x, \lambda) = L(x) + \lambda f(x), \tag{2.36}$$

so the necessary conditions become

$$\frac{\partial J}{\partial x} = 0 \qquad \text{and} \qquad f(x) \leq 0 \tag{2.37}$$

where

$$\begin{array}{lll} \lambda \geq 0 & f(y) = 0 & \text{constraint is active} \\ \lambda = 0 & f(y) < 0 & \text{constraint is inactive} \end{array} \tag{2.38}$$

For a vector variable **x**, then the condition (2.35) means: $\frac{\partial L}{\partial x}$ is parallel to $\frac{\partial f}{\partial x}$ and pointing in opposite directions where $\frac{\partial L}{\partial x}$ is interpreted as the gradient.

Thus, what the condition (2.35) says that the gradient of L with respect to x at a minimum must be pointed in such a way that decrease of L can only come by violating the constraints. Otherwise, we could decrease L further. This is the essence of the Kuhn-Tucker conditions.

Chapter 3

Cognitive Radio Systems

Cognitive radio is a device to use spectrum efficiently getting assistance of its ability of sensing environmental conditions for wireless communication. This information contains presence of a user, channel gains, other users' codebooks and even their messages. Cognitive users aim to communicate with their intended or common receiver(s) underlaying, interweaving, or overlaying their own signals with the other (licensed) users' signals. In this chapter, these three techniques which do not impact the existing users' communication are summarized and overlay technique which is the directly related in the thesis is detailed and the most investigated channel model for cognitive radio systems, interference channel, is used to better understand these concepts, and also encoding/decoding strategies for this type of channel are considered to determine achievable rate region. Most of the ideas comes from user cooperation techniques which are mentioned in Chapter 2.

3.1 Cognitive Radio Communication Techniques

There are three strategies to communicate over a wireless cognitive channel as mentioned above.

3.1.1 Interweave Technique

The important feature of a cognitive radio comes from the idea which was propounded by Mitola so that it is able to sense its surrounding spectra. It is based on the filling the spectrum holes which mean frequency voids that are not in use by the licensed users. This technique is also referred to as interference avoidance technique. In this setting, the cognitive device observes the radio spectrum and determines the presence or absence of the licensed users, then it releases its own message throughout the communication channel. An illustration of channel usage is shown in Figure 3.1. However, the cognitive transmitter has to spend some of its time and power to sense the surrounding situation. Although the catching of the spectrum holes of the primary users is a critical issue both over a wide bandwidth system and in a low SNR scenarios [25] collaborating of multiple cognitive users and exchanging their own spectrum knowledge combat the limitations of resources in such this conditions [26].

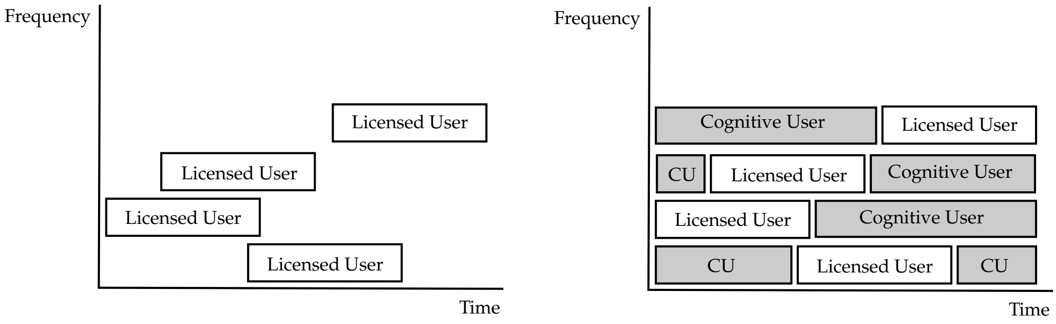


Figure 3.1: Spectrum Utilization in Interweave Technique

As illustrated fundamental cognitive channel model in Figure 3.2 with idealization where the cognitive user has the knowledge of the spectral gaps is perfect and the time spent for sensing is ignored, the rate region for primary and secondary users by time sharing fashion is

$$R_1 \leq tC(P_1) \tag{3.1}$$

$$R_2 \leq (1-t)C(P_2) \tag{3.2}$$

where $C(x) = \frac{1}{2}\log(1 + x)$. P_1, P_2 are the primary user's and secondary user's average power constraint, respectively and $0 \leq t \leq 1$. The noise variance is normalized to the unity.

3.1.2 Underlay Technique

A cognitive radio has a knowledge of interference caused by its transmitter to the receivers of all noncognitive users due to its own intelligence. In this setting, simultaneous secondary and primary transmissions can be allowed only if the interference is below some acceptable level [27] because of the fact that the primary users' data rates should not be less than the desired level. This type of interference controlling policy covers a large spectrum behavior, i.e., small networks or ultrawideband (UWB) communications. In small networks, the cognitive user has knowledge of the maximum of the interference temperature for the licensed user at receivers, the current interference level and how its own transmit power will change at the primary receiver, then the cognitive radio may control its own transmission power to satisfy the interference temperature constraint of the primary users [28] so that this technique is also called as interference control. In UWB systems, the cognitive user spreads its signal over a huge bandwidth to ensure the interference caused to the primary users is acceptable level, then it transmits its own signal to the intended receiver. However, the underlay technique is only useful for short range communications due to the nature of the UWB systems.

Suppose the channel in Figure 3.2, each receiver obviously treats the other user's message as a noise. To satisfy the primary user's desired rate, its average power is fixed to a value P_1 generating a Gaussian codebook. Then, the cognitive transmitter allows its power to a level $P_2^{\star}$ depends on the interference-temperature. So the rate region is obtained as

$$R_1 \leq C\left(\frac{P_1}{h_{21}^2 P_2^\star + 1}\right) \tag{3.3}$$

$$R_2 \leq C\left(\frac{P_2^\star}{h_{12}^2 P_1^\star + 1}\right) \tag{3.4}$$

where channel fading coefficients h_{12}, h_{21} and $P_2^\star$ lies in the range of $[0, P_2]$.

3.1.3 Overlay Technique

The overlay technique allows that the cognitive and noncognitive users transmit their own messages simultaneously over the same spectrum. However, this concurrent communication effects both the primary and secondary users because of the interference which one is that the cognitive transmitters cause to the primary receivers and one is that the primary transmitters cause to the cognitive receivers in the general scenario of cognitive radio systems. The cognitive radio has to know or learn the channel gains, primary users' codebooks and/or the messages of the licensed users.

The thesis is directly related to overlay the transmitting signal with the cognitive user's signal. Thus, this technique is detailed in the next section.

3.2 Overlay Cognitive Radio

The cognitive user transmits over the same spectrum as the primary user while the primary user's rate is not reducing. The smallest and fundamental model in this area is the interference channel which there are two primary and two cognitive users as shown in Figure 3.2.

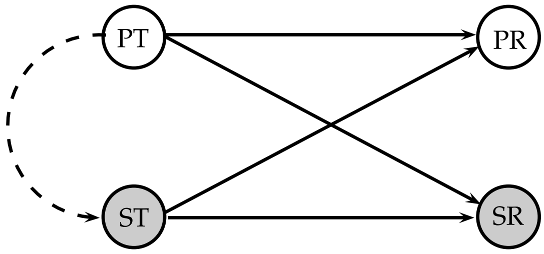


Figure 3.2: Cognitive Interference Channel

In the overlay cognitive interference channel, the cognitive user has a priori knowledge of the primary user's message. This priori knowledge can be achieved non-causally [9] or causally [13]. This type of interference channels are referred to as one-sided cooperation, the channel with asymmetric message knowledge, or degraded message sets in the literature.

It is assumed that two users share their codebooks and there is a feedback, so they also know channel gains. This means that the cognitive user knows the primary user's message W_1 and thereby its codeword X_1. Moreover, our purpose is to improve both user's data rates. To achieve that, several encoding/decoding schemes are applied by cognitive user knowing the primary user's message [5], [9], [29], [30] and [31]. An encoding scheme that the cognitive transmitter may employ to exploit its knowledge of the primary user's message is not practical because of the fact that the cognitive transmitter does not know the message of the primary transmitter at the start of their transmissions. However, this becomes very reasonable if the two transmitters are close together. Accordingly, the channel between the transmitter is better than the primary users pair and in a fraction of the transmission time, cognitive transmitter could listen to and obtain the message transmitted by the primary transmitter. In worse case, the primary message is being transmitted after a delay, and the cognitive user was able to successfully decode the first transmission. This assumption is also applicable when the primary transmitter formerly sends its message to the cognitive transmitter. Both of these scenarios might lead to partial message decoding of the primary message instead of full decoding, which also fits within the overlay network technique.

When both users do not know the message of the other, this model reduces the channel model into the interference channel or when the primary transmitter does not transmit its messages, this model reduces the channel to a broadcast channel. In a broadcast channel, the encoder knows all messages and can exploit that information to partially eliminate the interference. To achieve that, superposition coding [20] is used so that it allows sending information simultaneously to all users at higher rates than can be achieved with time-sharing. Particularly, noisier message is first encoded at a lower rate, and then better signal is superimposed on it. So the receiver can eliminate interference by decoding low-rate messages first and by subtracting their effect from the received signal. Superposition coding was shown to achieve capacity of degraded broadcast channel in [32]. If the channel is not degraded which means received signals could not be ordered successfully, interference can still be partially eliminated applying the precoding technique referred to as Gel'fand-Pinsker (GP) binning [21] and, specifically, DPC [22] in Gaussian channels.

The opposite action of a broadcast channel scenario is the multiple access channel. The capacity region of this channel is known in [33]. As explained in Chapter 2, when the interference channel which suffers strong interference, i.e., the cross-channel gains are more than unity, the optimal strategy for decoders is to decode both transmitted messages. Thus, the interference channel behaves as two MAC channels.

We now review the various encoding/decoding techniques that have been proposed for overlay cognitive networks, which are mostly derived from encoding strategies for the interference, broadcast, and multiaccess channels: Rate-splitting is the well known encoding technique for interference channels, so it is natural to consider it for overlay encoding as well. This technique was applied to overlay networks in [9], [11], and [12]. The cognitive user can eventually cooperate by superimposing the primary user's message in order to help the primary decoder.

In this way, the cognitive transmitter declares a part of its power to send the primary user's message W_1 and increase its rate R_1. On the other hand, any signal conveying W_1 is interference to the cognitive encoder's receiver. This interference is known at the cognitive transmitter since it consists of codewords used for W_1. This is where the GP binning [21] and DPC [22] in Gaussian channels can be applied.

Therefore, although the encoding techniques for overlay cognitive radio certainly borrow from existing strategies for the classical interference channel, a number of additional techniques are needed to fully exploit the knowledge of the primary user's message. The encoding strategies that have been investigated in the next sections.

3.3 Capacity of Cognitive Interference Channel

The capacity region for the channel model in Figure 3.2 is still an open problem in most cases. However, some results can be achieved under some special conditions. In strong interference, both decoders can decode the two users' messages, so each receiver behaves as the multiple access channel with common information considered in [34]. When the primary user's message W_1 is known to both encoders, it is thought as the common information for which the capacity is known [30]. Therefore, rate splitting or superposition encoding is not needed at the cognitive encoder. Accordingly, the cognitive user divides its transmit power P_2 into αP_2 and $(1-\alpha)P_2$ for cooperation and its own transmission, respectively. Then, the capacity region is bounded by the rate of

$$R_2 \leq \frac{1}{2}\log\Big(1+(1-\alpha)P_2\Big) \tag{3.5}$$

When the cognitive user and noncognitive user share a common information like in our thesis problem, the the cognitive decoder decodes both messages, there is no need for binning, as there is no interference at that decoder. Thus, rate-splitting and superposition coding achieve capacity. [35].

In weak interference case, where the cross-channel (ending to the primary receiver) gain is small, the decoder does not need to cancel interference. Interference at the cognitive decoder can be fully eliminated by dirty paper coding and does not depend on the value of between the cognitive transmitter and primary receiver. Thus, there is no need for rate-splitting at the encoders. For example, we employ an encoding protocol that exploits an asymmetric message knowledge at the transmitters in AWGN channel such that both transmitters use random Gaussian codebooks, the primary transmitter's average power of P_1 and the secondary transmitter's P_2 but into two parts: αP_2 for relaying the message of the primary user and $(1-\alpha)P_2$ for transmitting its own message using the interference-mitigating technique of dirty-paper coding [22]. This strategy may harm the primary receiver and is treated as noise at primary receiver. Thus, the rate region may be expressed as

$$\begin{aligned} R_1 &\leq C\left(\frac{\left(\sqrt{P_1}+h_{12}\sqrt{\alpha P_2}\right)^2}{h_{12}^2(1-\alpha)P_2+1}\right) && (3.6) \\ R_2 &\leq C\big((1-\alpha)P_2\big) && (3.7) \end{aligned}$$

where $0 \leq \alpha \leq 1$.

As focusing on our channel model, unknown capacity of the interference channel with one cognitive encoder is overcame using several encoding strategies that combine the above techniques [9], [11], [12]. The relative performance of these various encoding schemes depends on the channel conditions and topology. For the Gaussian channel, a comparison of the achievable rate regions for the encoding schemes proposed in [11] and [12]

The rate gains of having one cognitive encoder versus the two primary encoders in the traditional interference channel were evaluated in [9]. The encoding schemes we have described are known to be capacity-achieving under certain assumptions about the channel or specific encoding/decoding constraints. Finding additional regimes where these and other encoding schemes achieve capacity is the topic of ongoing investigation. The impact of feedback and common information has also been investigated in [14] and [15].

The practical designs based on these paradigms, various assumptions and constraints about message knowledge will affect these conclusions, and many of these issues have yet to be investigated. Also, the bigger networks are becoming popular to consider. Intuitively, some form of cooperation between these users will be required, but the best form of cooperation is unclear. Another interesting question is whether cognition is more beneficial at the transmitter or the receiver.

For more practical scenarios, work in [9] and [13] considers the causal message sharing at the cognitive radio. In this setting, the cognitive user decodes the message sent by the other encoder based on the observation from the channel introducing a delay. The paper [9] proposes a two-phase protocol where the first phase is used solely to convey the message from the primary to the cognitive encoder. The second phase is identical to the noncausal scenario with the difference of the fact that [13] handles the delay by adjusting the encoding strategies as follows: Rate-splitting of a message available to an encoder can employed at the interval of the delay. In terms of cooperation, the causal setting resembles the classic relay channel [36]: the cognitive encoder can be viewed as the relay that forwards the message of the other user after decoding it. Techniques such as block Markov superposition encoding [37] and sliding-window decoding [38] need to be used to facilitate the message encoding/decoding over two blocks, thus handling delay. However, this approach relies on the primary encoder and decoder to employ this scheme. Still, two techniques clearly extend to the causal setting. In contrast, the precoding against interference and DPC critically depend on the noncausal knowledge of the interfering message and do not extend to the cases with delay. The encoding scheme that incorporates rate-splitting and cooperation demonstrates performance losses in comparison to the noncausal system performance when the channel between two encoders is weak [13]. Also present perfect message knowledge at the cognitive encoder but only partial channel knowledge in recent works. The case where the phase of the channel between the primary transmitter and the cognitive receiver is unknown at the cognitive transmitter is considered and the main challenge in this scenario is that Costa's DPC technique [22] cannot be applied directly.

In brief, the cognitive interference channel is investigated in this chapter yielding the literature. The main types of the cognitive channels we deal with are sorted as competitive, cognitive and cooperative. In competitive channel, the secondary user has no additional side information and two users compete for the channel. In cognitive channel, the secondary transmitter has knowledge of the primary user's message and codebook. It can be also said that asymmetric cooperation is possible between the users. In cooperative channel, both users know each other's messages and symmetrically cooperate in their transmission. The channel models are illustrated in Figure 3.3 as described in [3].

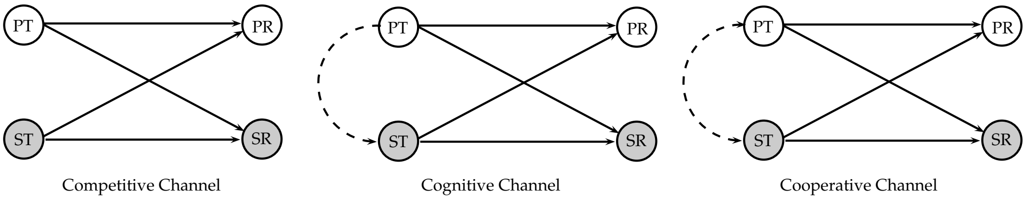


Figure 3.3: Types of Channels depending on Side Information

Chapter 4

Two-User Cooperative Cognitive Multiple Access Channel

As described in the literature survey, cognitive radio channels are generally modeled as two transmitter two receiver pairs in the case of overlay techniques. Cognitive user has a priori fully knowledge of noncognitive user's message in the most of the works. Although many elegant papers have published on causal cognitive radio networks. Nevertheless, the power control policy does not investigated because of whether it is not directly related to the system model (interference channel in the most of the works) or it is too hard to apply for the cognitive radio systems. However, some implementations of power allocation have proposed in [28] and [39] for the case of underlay cooperative cognitive channels.

The investigation of the cooperative cognitive multiple access systems are crucial because cognitive radios are not only a part of communication channels, they are also an element of a network which consists of many primary and secondary users. The biggest advantage of multiple access channel compared to the interference channel is ease of the decoding by a single common receiver in the sense of applicability. This thesis aims to develop an encoding and decoding protocol, to allocate optimal transmitting powers so that we achieve higher data rates for overlay cooperative cognitive channels.

In this thesis, several known techniques from the cooperative communications are applied into the systems which consist of cognitive radios. Accordingly, using an information theoretical approach, our main goal is to come up with encoding and decoding techniques which maximize the rate achievable by secondary user while improving the rates of primary user or keeping constant in worst case. Thus, we have to add a constraint on the primary user's achievable rate.

While developing cooperative cognitive protocols in multiple access settings, we use block Markov superposition encoding technique, which is based on decode and forward (DF) strategy with rate-splitting. For all the cooperation protocols that will be developed in this thesis, our main purpose is to characterize, and maximize the achievable rates of the cognitive user and sum of all users. Therefore, we also propose to optimize the rate expressions which shall be derived based on each encoding/decoding strategy, in terms of the transmit powers of the users.

In this chapter, we consider a two user (one is cognitive and one is noncognitive) system where the users communicate with the common receiver by using a cooperative scheme over a fading channel. The main goal is to obtain an effective way of reaching as high a capacity region as possible by using a half duplex (only one direction at a time) communication scheme [40].

4.1 System Model

There is only one common receiver which is aimed to communicate by one cognitive and one noncognitive user in a network. In the system model, the cognitive user listens the primary user's message and transmits its own message to the receiver at the same time, while the primary user sends the message. The channel model is shown in Figure 4.1

Accordance with Figure 4.1,

$$\begin{aligned} Y_0 &= \sqrt{h_{10}}X_1 + \sqrt{h_{20}}X_2 + n_0 && (4.1) \\ Y_2 &= \sqrt{h_{12}}X_1 + n_2 && (4.2) \end{aligned}$$

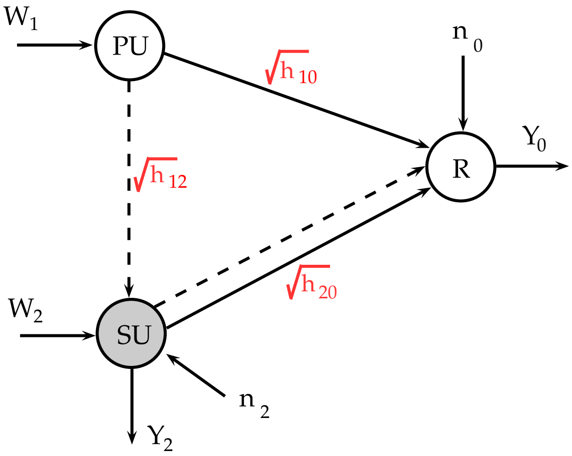


Figure 4.1: Proposed System Model

where $\sqrt{h_{ij}}$ represents the channel coefficient between nodes i and j, n_i is additive white Gaussian noise, W_1 is the message of the primary user, W_2 is cognitive user's message, Y_0 is the signal at the receiver, and Y_2 is the signal at the cognitive user. The receiver is defined as the node 0.

As illustrated in Figure 4.1, the secondary user receives an attenuated and noisy version of transmitted signal. Nevertheless, secondary user has decoding capability for its partner's transmitted signal X_1. The transmitted and received signals at each mobile nodes are encoded by superposition block Markov encoding and decoded by backward decoding [41] algorithm where the receiver does not work on decoding until all B block codewords completely received and relevant information and illustrations can be found in the next subsection of the thesis.

As understood from the equations (4.1) and (4.2), X_i is the signal transmitted by user i which is encoded by superposition Markov encoding scheme. This coding scheme, for a two user MAC with generalized feedback is illustrated in [8]. The transmitted signal X_i for the two user scheme can be expressed as below:

$$X_1 = \sqrt{P_{10}(\mathbf{h})}X_{10} + \sqrt{P_{12}(\mathbf{h})}X_{12} + \sqrt{P_{u1}(\mathbf{h})}U \tag{4.3}$$

$$X_2 = \sqrt{P_{20}(\mathbf{h})}X_{20} + \sqrt{P_{u2}(\mathbf{h})}U \tag{4.4}$$

where X_{10} and X_{20} are transmitted signals to the destination which are treated as a noise for the other user. X_{12} is transmitted signal towards the cognitive user and U is the cooperative signal which contains information from the previous blocks. $P_{10}(\mathbf{h})$, $P_{12}(\mathbf{h})$, $P_{u1}(\mathbf{h})$, $P_{20}(\mathbf{h})$ and $P_{u2}(\mathbf{h})$ are the allocated power values for subsignals of X_1 and X_2, which are functions of the channel state vector $\mathbf{h} = [h_{10}, h_{12}, h_{20}]$. Beside this, the power allocations are defined with the following constraints

$$\begin{aligned} P_1(\mathbf{h}) &= P_{10}(\mathbf{h} + P_{12}(\mathbf{h}) + P_{u1}(\mathbf{h}) && (4.5)\\ P_2(\mathbf{h}) &= P_{20}(\mathbf{h}) + P_{u2}(\mathbf{h}) && (4.6)\\ E\Big[P_i(\mathbf{h})\Big] &\leq \bar{P}_i \qquad \text{where} \quad i \in 1, 2 && (4.7) \end{aligned}$$

where $\bar{P}_i$ is average power of user i.

The subsignal X_{10}, X_{12} and U can be expressed as below

$$\begin{aligned} X_{10} &= \sqrt{P_{10}} X_{10}(W_{10}(i), W_{12}(i-1)) && (4.8)\\ X_{12} &= \sqrt{P_{12}} X_{12}(W_{12}(i), W_{12}(i-1)) && (4.9)\\ U &= \sqrt{P_{u1}}(W_{12}(i-1)) \qquad \text{where} \quad i \in 1, 2, ..., B && (4.10) \end{aligned}$$

where i and $i-1$ indicate the current block and previous block, respectively. W_{10}, W_{20} are the parts of information intended to the destination and W_{12} is the part of information intended to the cognitive transmitter and the destination, indirectly. The signal X_1 does not only depend on new information $W_{10}(i)$ and $W_{12}(i)$, but it is also depends on the previous block $W_{12}(i-1)$ which are helpful to generate cooperative signal.

The cooperative cognitive channel model was explained in the equations (4.1) and (4.2). As described before, the communication structure is based on superposition Markov encoding and backward decoding scheme.

The transmitted signal of primary user can be modeled as below without power allocation

$$X_1 = X_{10} + X_{12} + U \tag{4.11}$$

where X_{10} is employed for transmitting of W_{10} directly to the destination at rate R_{10}, X_{12} is employed for transmitting of W_{12} to the cognitive user at rate R_{12} and U is employed for transmitting cooperative signal $W_{12}(i-1)$ to the receiver. The cognitive user works to send its signal as

$$X_2 = X_{20} + U \tag{4.12}$$

Here, X_{20} is employed for transmitting of W_{20} directly to the receiver at rate R_{20} and U is employed for transmitting cooperative signal to the receiver. Separately, the power allocation must satisfy the equations (4.5) and (4.6).

Based on block Markov encoding and backward decoding scheme, we can generate the achievable rate region as similar to [8]. R_1 and R_2 denote the rates of primary and cognitive users. They have subcomponents such as R_{10}, R_{12}, R_{20}.

As describing the decoding procedure, the destination must wait until all B blocks are received. In the last block of code sequence $W_{10}(B), W_{12}(B), W_{20}(B)$ contain no new information and they can be set $W_{10}(B), W_{12}(B), W_{20}(B) = (0, 0, 0)$. Then the decoding algorithm starts decoding from the B'th block to first block. In the case of transmitting no new information at the last block, the total information rate reduces by a coefficient of $(B-1)/B$. However, the reduction of information ratio can be undervalued at large B. In contrary to the encoding scheme, in the backward decoding the destination wants to decode message by help of $W(i+1)$. This decoding strategy can be clarified in Figure 4.3.

By using properties of the mutual information, the achievable rates of the primary and the secondary users can be described as

$$\begin{aligned} R_1 &\leq I(X_1; Y|X_2) \quad (4.13) \\ R_2 &\leq I(X_2; Y|X_1) \quad (4.14) \\ R_{sum} = R_1 + R_2 &\leq I(X_1, X_2; Y) \quad (4.15) \end{aligned}$$

where $R_1 = R_{10} + R_{12}$ and $R_2 = R_{20}$ for generality.

As explained before, the achievable rate of the primary user to the receiver can be expressed as

$$R_{10} \leq E\left\{ \log \left[1 + \frac{h_{10}P_{10}(\mathbf{h})}{\sigma_0^2} \right] \right\} \quad (4.16)$$

Nevertheless, in order to find R_{12}, which is the rate of primary user to the cognitive user, we first look at block 1 illustrated in Figure 4.3. Here, we set $W_{12}(0) = 0$. These signals are known by both users. The first signal $X_1(1)$ contains $W_{12}(0)$ which are known, $W_{12}(1)$ which is of interest to cognitive user $W_{10}(1)$ which is not attempted to be decoded by the cognitive user. Cognitive user will treat X_{10} as noise. The rate R_{12} between primary and secondary users can then be found by the following equation

$$R_{12} \leq E\left\{ \log \left[1 + \frac{h_{12}P_{12}(\mathbf{h})}{h_{12}P_{10}(\mathbf{h}) + \sigma_2^2} \right] \right\} \quad (4.17)$$

where P_{10} is treated as a noise component at the cognitive side so it must be in denominator of logarithm. R_{20} is in similar fashion and can be expressed as

$$R_{20} \leq E\left\{ \log \left[1 + \frac{h_{20}P_{20}(\mathbf{h})}{\sigma_0^2} \right] \right\} \quad (4.18)$$

And the sum of the rates at the destination, R_{10}, R_{20} are bounded by

$$R_{10} + R_{20} \leq E\left\{ \log\left[1 + \frac{h_{10}P_{10}(\mathbf{h}) + h_{20}P_{20}(\mathbf{h})}{\sigma_0^2}\right]\right\} \tag{4.19}$$

The decoding starts from the last block where it has no new information. In the last block, the decoder wants to decode $W_{12}(B-1)$ since $W_{10}(B)$, $W_{20}(B)$, $W_{12}(B)$ are set to zero. The following rate constraint needs to be satisfied at the receiver

$$R_1 + R_2 \leq E\left\{ \log\left[1 + \frac{h_{10}P_1(\mathbf{h}) + h_{20}P_2(\mathbf{h}) + 2\sqrt{h_{10}h_{20}P_{u1}P_{u2}}}{\sigma_0^2}\right]\right\} \tag{4.20}$$

where P_1 and P_2 are total power which are described in the equations (4.7).

The term $2\sqrt{h_{10}h_{20}P_{u1}P_{u2}}$ in the equation (4.20) comes from the coherent addition of cooperative signals U at the receiver. In the next step, the decoder wants to decode $W_{10}(B-1), W_{20}(B-1), W_{12}(B-1)$.

However, the sum rate in the side of destination is dominated by the equation (4.21) and can be expressed as proved in [42]

$$R_{10} + R_{20} + R_{12} \leq E\left\{ \log\left[1 + \frac{h_{10}P_1(\mathbf{h}) + h_{20}P_2(\mathbf{h}) + 2\sqrt{h_{10}h_{20}P_{u1}P_{u2}}}{\sigma_0^2}\right]\right\} \tag{4.21}$$

There is also a constraint to meet the primary user's optimal achievable rate which is

$$R_1^\star \geq E\left\{ \log\left[1 + \frac{h_{10}P_1^\star(\mathbf{h})}{\sigma_0^2}\right]\right\} \tag{4.22}$$

where $P_1^\star$ is the optimum power level to guarantee not to underlay the desired data rates for licensed user under the condition of

$$E[P_1^\star(\mathbf{h})] \;=\; P_1 \tag{4.23}$$

Besides, in consideration of given power allocation structures and constraints in the equations (4.5) and (4.23), this rate in the equations (4.21) and (4.22) can be dominated by sum of individual rates. Thus, sum of R_1 and R_2, which are denoted by $R_{10} + R_{12}$ is bounded by

$$\begin{aligned} R_1 + R_2 \;\leq\; & \min\left\{ E\left\{ \log\left[1 + \frac{h_{10}P_1(\mathbf{h}) + h_{20}P_2(\mathbf{h}) + 2\sqrt{h_{10}h_{20}P_{u1}P_{u2}}}{\sigma_0^2}\right]\right\},\right. \\ & \left. E\left\{ \log\left[1 + \frac{h_{10}P_{10}(\mathbf{h}) + h_{20}P_{20}(\mathbf{h})}{\sigma_0^2}\right] + \log\left[1 + \frac{h_{12}P_{12}(\mathbf{h})}{h_{12}P_{10}(\mathbf{h}) + \sigma_0^2}\right]\right\}\right\} \end{aligned} \tag{4.24}$$

4.1.1 Block Markov Superposition Coding

Various Markov encoding techniques are proposed for specific channels, such as multiple access channel with feedback, relay channels. In this section, relay channel, similarly our channel model of interest which the cognitive user is as a relay is examined. Transmission using block Markov coding operates over a number of blocks. In the first block, a codeword is sent from the first codebook and the relay knows the index of this codeword, but the intended receiver does not because of the fact that the channel between the transmitter and the relay is better. However, the receiver has a list of possible codewords. In the next block, the users randomly divide the first codebook into equal sized cells. The transmitters and the receiver agree on what this partition is. The relay and the transmitter find the cell of the partition in which the codeword from the first codebook lies and cooperatively send the codeword from the second codebook with that index. The transmitter also chooses a fresh codeword from its codebook, adds it to the cooperative codeword from its second codebook, and sends this sum over the channel. The reception by the receiver in the second block involves first finding the cooperative index from the second codebook by looking the closest codeword and subtracts it, then calculates a list of indices corresponding to all transmitted words from the first codebook in that second block. The receiver now computes the codeword from the first codebook sent in the first block. In each new block, the transmitter and the relay cooperate to resolve the list uncertainty from the previous block. In addition, the transmitter adds some fresh information from its codebook to its transmission from the second codebook and transmit the sum. This encoding and decoding process is referred as to regular encoding and backward decoding.

To summarize, in each block, with the exception of the first or the last block, a new message is sent. However, the codeword to send at each block depends on not only fresh information but also past information from one or more previous blocks. Therefore the name is Markov encoding. The information from previous blocks can be refined information for the previous message or cooperative information for other users. At the receiver side, the channel output at each block is related with messages from previous blocks, so various decoding schemes have also been proposed.

In our scheme for the channel model 4.1, PU sends the codeword $\mathbf{x}_1(w_{B-1}, w_B)$ at block B. SU estimates $\hat{\hat{w}}_{B-1}$ from the previous block $B-1$, transmits then $\mathbf{x}_2(\hat{\hat{w}}_{B-1})$. An illustration of block Markov encoding is given in Figure 4.2 according to the proposed model.

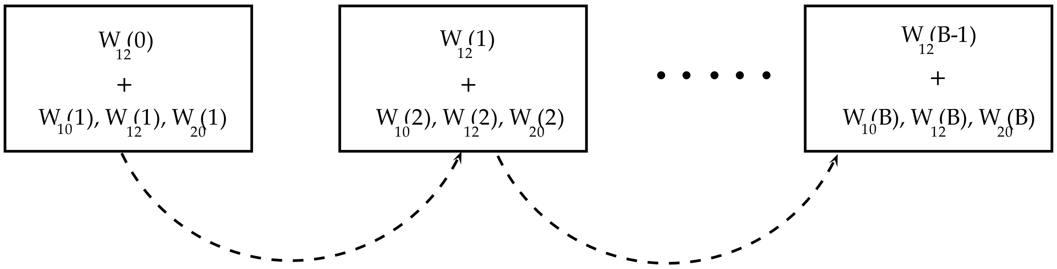


Figure 4.2: An Illustration of Superposition Markov Encoding

At the end of the block B, given $\hat{\hat{w}}_{B-1}$, SU estimates $\hat{\hat{w}}_B = w$ if and only if there exists unique w such that $(\mathbf{x}_2(\hat{\hat{w}}_{B-1}), \mathbf{x}_1(w, \hat{\hat{w}}_{B-1}), \mathbf{y}_2(B))$ is jointly typical. After retransmitting the information from the SU, the receiver starts decoding from the last block B and proceeds backwards. At block B, assume $\hat{w}_B$ is known following decoding of block $B+1$, the decoder estimates $\hat{w}_{B-1} = w$ if and only if there exists a unique w such that $(\mathbf{x}_1(\hat{w}_B, w), \mathbf{x}_2(w), \mathbf{y}_3(B))$ is jointly typical. Backward decoding is illustrated as follows in Figure 4.3

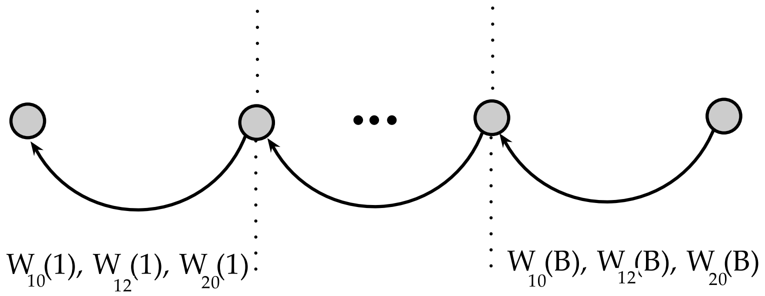


Figure 4.3: An Illustration of Backward Decoding

4.2 Optimization of Sum Rate of Two-User Cooperative Cognitive Multiple Access Channel

There are two main problem of interest which the first is one of maximizing the sum rate of the system and the other one of aimimg to optimize the cognitive radio user's achievable rate by getting assistance of the fully knowledge of the channel state information at both the users and the receiver. Therefore, our problems are based on power allocation showing as in the equations from (4.5) to (4.7). Before we give our proposed practical scenario for the first case, we summarize the problem associating the Lagrangian multipliers $\gamma_1, \gamma_2, \gamma_3, \gamma_4 > 0$ to the inequality constraints, $\lambda_1, \lambda_2 > 0$ to the power constraints, and $\mu_i(\mathbf{h}) \geq 0, i = 1, ..., 5$ to the non-negativity constraints, and noting that the power constraints need to be satisfied by equality, we obtain the KKT conditions, which are necessary and sufficient for optimality.

The optimization problem for the maximizing of the sum rate of the system with satisfying the primary user's quality of service is

$$\max_{p(\mathbf{h})} \quad R_1 + R_2$$

$$\textbf{s.t.}$$

$$R_1 < E\left\{ \log\left[1 + \frac{h_{10}P_{10}(\mathbf{h})}{\sigma_0^2}\right] + \log\left[1 + \frac{h_{12}P_{12}(\mathbf{h})}{h_{12}P_{10}(\mathbf{h}) + \sigma_0^2}\right]\right\} \tag{4.25}$$

$$R_2 < E\left\{ \log\left[1 + \frac{h_{20}P_{20}(\mathbf{h})}{\sigma_0^2}\right]\right\} \tag{4.26}$$

$$R_1 + R_2 < E\left\{ \log\left[1 + \frac{h_{10}P_1(\mathbf{h}) + h_{20}P_2(\mathbf{h}) + 2\sqrt{h_{10}h_{20}P_{u1}(\mathbf{h})P_{u2}(\mathbf{h})}}{\sigma_0^2}\right]\right\} \tag{4.27}$$

$$R_1 \geq E\left\{ \log\left[1 + \frac{h_{10}P_1^{\star}(\mathbf{h})}{\sigma_0^2}\right]\right\} \tag{4.28}$$

$$P_1 = E\big[P_{10}(\mathbf{h}) + P_{12}(\mathbf{h}) + P_{u1}(\mathbf{h})\big] \tag{4.29}$$

$$P_2 = E\big[P_{20}(\mathbf{h}) + P_{u2}(\mathbf{h})\big] \tag{4.30}$$

$$P_{10}(\mathbf{h}), P_{12}(\mathbf{h}), P_{u1}(\mathbf{h}), P_{20}(\mathbf{h}), P_{u2}(\mathbf{h}) \geq 0 \tag{4.31}$$

For simplicity of the derivations, we assume that the fading distributions are such that all realizations of the fading values satisfy which the cooperative link has better channel conditions compared to the direct links from both transmitters to the receiver, $s_{12} > s_{10}$ where the channel fading coefficients are normalized by the noise powers as $s_{ij} = h_{ij}/\sigma_j^2$. This can be thought as particular case is of practical interest since the cooperating transmitters are likely to be closely located with less number of scatterers and obstructions when compared to their paths to the receiver. Since, $P_{10}(\mathbf{h}) = 0$ and the problem of maximizing the cognitive user's rate reduces to

$$\begin{aligned}
\mathcal{L}_1 \;=\;& R_1 + R_2 + \gamma_1\Big\{E\Big[\log\Big(1 + s_{12}P_{12}(\mathbf{h})\Big)\Big] - R_1\Big\} \\
&+\gamma_2\Big\{E\Big[\log\Big(1 + s_{20}P_{20}(\mathbf{h})\Big)\Big] - R_2\Big\} \\
&+\gamma_3\Big\{E\Big[\log\Big(A\Big)\Big] - R_1 - R_2\Big\} + \gamma_4\Big\{R_1 - B^{\star}\Big\} \\
&+\lambda_1\Big\{P_1 - E\Big[P_{12}(\mathbf{h}) + P_{u1}(\mathbf{h})\Big]\Big\} + \lambda_2\Big\{P_2 - E\Big[P_{20}(\mathbf{h}) + P_{u2}(\mathbf{h})\Big]\Big\} \\
&+\mu_2 P_{12}(\mathbf{h}) + \mu_3 P_{u1}(\mathbf{h}) + \mu_4 P_{20}(\mathbf{h}) + \mu_5 P_{u2}(\mathbf{h}) \qquad (4.32)
\end{aligned}$$

This optimization problem actually includes the next one of optimization of the cognitive user's data rate. The constraint which is weighted by γ_4 checks whether the optimal rate for the primary user is satisfied or not. If this equation is satisfied with inequality, then we can say that the constraint for γ_4 is inactive.

Considering the case which the rate of the primary user by the power control policy is equal to the optimal value, then the γ_4 becomes active, and R_1 can be replaced with $B^{\star}$ in the rest of the equations. Thus, the optimization problem is transformed into the problem of maximizing the rates achievable of the cognitive user.

Taking the partial derivations of $\mathcal{L}_1$ with respect to the power components of the primary and the secondary users, we handled out the expressions from (4.33) to (4.36). Note that for optimality, for any given $\mathbf{h}$, the components of powers should be non-negative.

$$\begin{aligned}
\gamma_1 \frac{s_{12}}{1+s_{12}P_{12}(\mathbf{h})} + \gamma_3 \frac{s_{10}}{A} &\leq \lambda_1 && (4.33)\\
\gamma_2 \frac{s_{20}}{1+s_{20}P_{20}(\mathbf{h})} + \gamma_3 \frac{s_{20}}{A} &\leq \lambda_2 && (4.34)\\
\gamma_3 \frac{s_{10}\sqrt{P_{u1}(\mathbf{h})} + \sqrt{s_{10}s_{20}P_{u2}(\mathbf{h})}}{A\sqrt{P_{u1}(\mathbf{h})}} &\leq \lambda_1 && (4.35)\\
\gamma_3 \frac{s_{20}\sqrt{P_{u2}(\mathbf{h})} + \sqrt{s_{10}s_{20}P_{u1}(\mathbf{h})}}{A\sqrt{P_{u2}(\mathbf{h})}} &\leq \lambda_2 && (4.36)
\end{aligned}$$

where $\gamma_2+\gamma_3 = 1$ and $\gamma_1+\gamma_3 = 1+\gamma_4$ and for this case, we also zero γ_4. Thus, we extract that $\gamma_1 = \gamma_2$. From the equations (4.35) and (4.36). If $P_{u1}(\mathbf{h}), P_{u2}(\mathbf{h}) > 0$, then the equations (4.35) and (4.36) is satisfied with equality and getting ratio of these two equations we also found a linear relationship between $P_{u1}(\mathbf{h})$ and $P_{u2}(\mathbf{h})$ by dividing the equations (4.35) to (4.36) such that

$$\lambda_1^2 s_{20} P_{u1}(\mathbf{h}) = \lambda_2^2 s_{10} P_{u2}(\mathbf{h}) \tag{4.37}$$

Therefore, the powers allocated for the transmission of the message components are given by

$$\begin{aligned}
P_{12}(\mathbf{h}) &= \left(\frac{\gamma_2(\lambda_2 s_{10} + \lambda_1 s_{20})}{\lambda_1^2 s_{20}} - \frac{1}{s_{12}}\right)^+ && (4.38)\\
P_{20}(\mathbf{h}) &= \left(\frac{\gamma_2(\lambda_2 s_{10} + \lambda_1 s_{20})}{\lambda_2^2 s_{20}} - \frac{1}{s_{20}}\right)^+ && (4.39)\\
P_{u1}(\mathbf{h}) &= \frac{\dfrac{1-\gamma_2\left(s_{10}+\frac{\lambda_1}{\lambda_2}s_{20}\right)}{\lambda_1} - \Big[1+s_{10}P_{12}(\mathbf{h})+s_{20}P_{20}(\mathbf{h})\Big]}{\left(s_{10}+\frac{\lambda_1}{\lambda_2}s_{20}\right)^2} s_{10} && (4.40)\\
P_{u2}(\mathbf{h}) &= \frac{\dfrac{1-\gamma_2\left(s_{20}+\frac{\lambda_2}{\lambda_1}s_{10}\right)}{\lambda_2} - \Big[1+s_{10}P_{12}(\mathbf{h})+s_{20}P_{20}(\mathbf{h})\Big]}{\left(s_{20}+\frac{\lambda_2}{\lambda_1}s_{10}\right)^2} s_{20} && (4.41)
\end{aligned}$$

Next, we include the case when the power allocated to transmit cooperative messages, i.e., the equations (4.40), (4.41) result with negativity. In this case, we should set $P_{u1}(\mathbf{h}) = P_{u2}(\mathbf{h}) = 0$ in our analysis for optimality. Then, the power levels of $P_{12}(\mathbf{h})$ and $P_{20}(\mathbf{h})$ become as in the roots of equations (4.42) and (4.43) by replacing P_{u1} and P_{u2} terms including in the sum rate with coherent addition, A as zero.

For $P_{12}(\mathbf{h})$ and $P_{20}(\mathbf{h})$ when $P_{u1}(\mathbf{h}) = P_{u2}(\mathbf{h}) = 0$

$$a_1 P_{12}(\mathbf{h})^2 + b_1 P_{12}(\mathbf{h}) + c_1 = 0 \tag{4.42}$$

$$a_2 P_{20}(\mathbf{h})^2 + b_2 P_{20}(\mathbf{h}) + c_2 = 0 \tag{4.43}$$

The solution for the equation (4.42) is

$$\begin{aligned} a_1 &= \lambda_1 s_{10} s_{12} && (4.44)\\ b_1 &= \lambda_1 \Big(s_{10} + s_{12} + s_{12} s_{20} P_{20}(\mathbf{h})\Big) - s_{10} s_{12} && (4.45)\\ c_1 &= \lambda_1 \Big(1 + s_{20} P_{20}(\mathbf{h})\Big) - \gamma_2 \Big(s_{12} + s_{12} s_{20} P_{20}(\mathbf{h}) - s_{10}\Big) - s_{10} && (4.46) \end{aligned}$$

The solution for Equation (4.43) is

$$\begin{aligned} a_2 &= \lambda_2 s_{20}^2 && (4.47)\\ b_2 &= \lambda_2 \Big(2 s_{20} + s_{10} s_{20} P_{12}(\mathbf{h})\Big) - s_{20}^2 && (4.48)\\ c_2 &= \lambda_2 \Big(1 + s_{10} P_{12}(\mathbf{h})\Big) - \gamma_2 \Big(s_{10} s_{20} P_{12}(\mathbf{h})\Big) - s_{20} && (4.49) \end{aligned}$$

4.2.1 Iterative Algorithm

Our algorithm searches the optimal Lagrangian multipliers iteratively. We have to find out optimal γ_2, λ_1 and λ_2 values. We force that each user's transmitting power is satisfied with the equality to average total power with the amount of error of 10^{-4}. Program firstly scans the optimal combination of λ_1 and λ_2 considering the satisfaction of the power. Our calculations which are shown in the analytical part in the thesis. We calculate the water level for each user and then determine the powers of the sub-messages using the equation (4.38), (4.39), (4.40) and (4.41) when $P_{u1}(\mathbf{h})$ and $P_{u2}(\mathbf{h})$ are positive, otherwise we assign the new power values of $P_{12}(\mathbf{h})$ and $P_{20}(\mathbf{h})$ using the equation (4.42) and (4.43) and setting $P_{u1}(\mathbf{h}) = P_{u1}(\mathbf{h}) = 0$ for those index. Thus, we iteratively look for the suboptimal λ values. After finding the temporary optimal values, we have to check whether the summation of the achievable rates of the users reaches the its maximal values. The rate constraint gets the maximal value when the equations of (4.25)+(4.26) and (4.27) are the equal with an acceptable error.

Algorithm 1 Optimization of Sum Rate with Multidimensional Search

```
while $\gamma_2$ does not satisfy the equation (4.25)+(4.26)=(4.27) do
  All power values $\leftarrow 0$;
  Initiate $\lambda_1$;
  while All components of powers do not converge as in the equations (4.29) and (4.29) do
    while $\lambda_1$ where controls (4.29) is not optimal do
      Assume $P_{U1} > 0$ calculate $v_1, P_{12}, P_{U1}$ and assign zero where $P_{U1} < 0$;
      while $P_{U1} \neq 0$ do
        Recalculate $P_{12}$ using (4.42) for those index and find where $P_{U1} = 0$;
      end while
      if First user transmitting power is satisfied its total average power then
        Modify $\lambda_1$ (if the result is less, decrease $\lambda_1$;
      end if
    end while
    Use new suboptimal $\lambda_1$ and initiate $\lambda_2$;
    while $\lambda_2$ is not optimal as in (4.30) do
      Calculate $V_2, P_{20}, P_{U2}$ and find where $P_{U2} < 0$;
      while $P_{U2} \leq 0$ do
        Recalculate $P_{20}$ using (4.43) for those index and find where $P_{U2} = 0$;
      end while
      if Cognitive user transmitting power is satisfied its total average power then
        Modify $\lambda_2$;
      end if
    end while
  end while
  Calculate $R_1$ and $R_2$ using (4.25), (4.26) and (4.27);
  if $R_1 - R_2 <$ Threshold then
    Decrease $\gamma_2$;
  else
    Increase $\gamma_2$;
  end if
end while
```

4.2.2 Simulation Results

In this section, we provide some numerical examples to illustrate the performance of the power allocation and cooperative cognitive scheme. We solve the optimization problem by two different approaches; maximization of the sum rate and maximization of the cognitive user's rate.

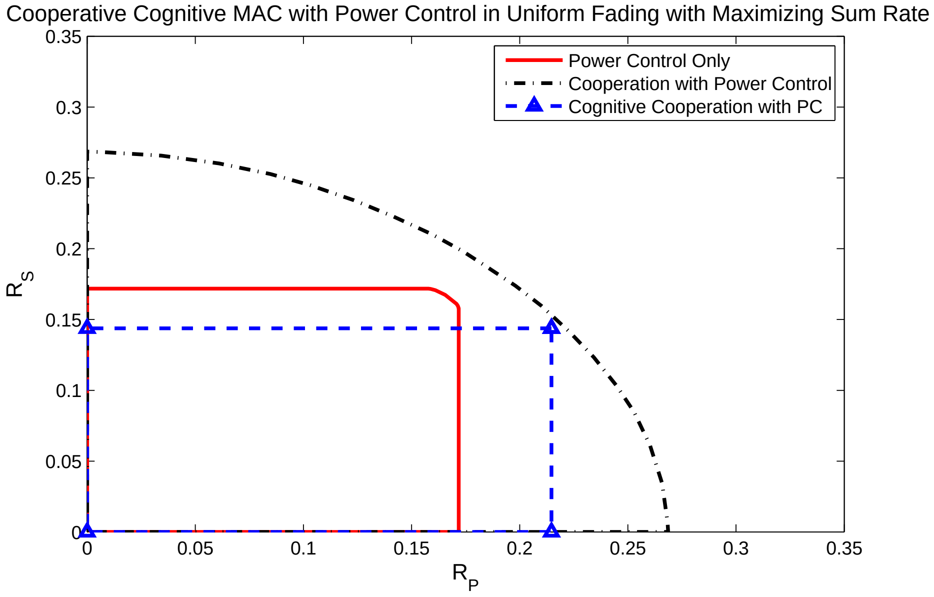


Figure 4.4: Rates achievable with the power control and user cooperation for uniform fading

Figure 4.4 illustrates the achievable rate region we obtain for a system with the average total power of 1 for each user subject to uniform fading. The links from the primary user and cognitive user to the destination takes values from the set of $\{0.025, 0.050, ..., 0.25\}$. The link between the transmitters is also uniform and takes values from the set of $\{0.26, 0.27, ..., 0.35\}$. As we discussed in the analytical part of this chapter, the fading coefficient of the cooperation link is always better than the direct links and we set the primary user's transmitting power to the zero, $P_{10}(\mathbf{h}) = 0$. Therefore, the power allocation scheme is actually the optimal power allocation policy for the block Markov superposition encoding scheme.

User cooperation and power control scheme significantly improves the rates achievable with(out) the power control as seen in Figure 4.4. For the rate pairs close to the sum rate, the primary user takes advantages of its cooperative link while it is sending its own message because of the fact that the link is always better than the direct one. Cognitive user decides which path is more profitable to the sum rate of the system and makes a decision on whether it helps to cooperate to the primary user or it sends its own message over the channel to the destination. Secondary user is able to reach the rate of 0.144 symbols/transmission when the primary user exhausts its change in order to maximize the sum rate of 0.358 symbols/transmission. The other interesting result is where the maximal point of the sum rate is close to the case of the cooperation with power control.

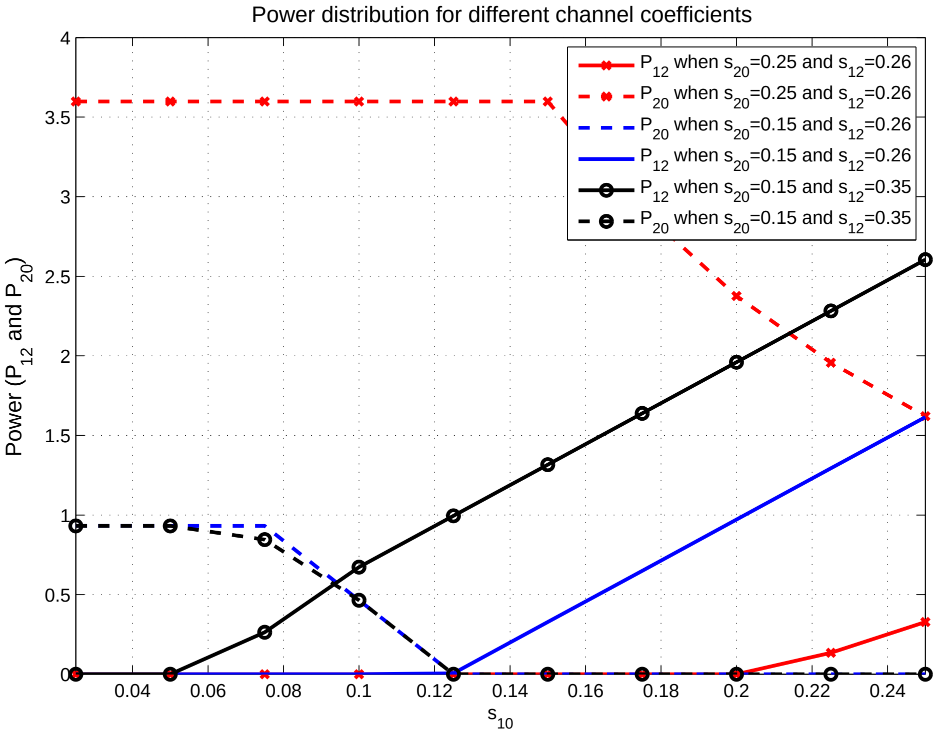


Figure 4.5: Power distribution for different channel coefficients of s_{12} and s_{20}

The power distributions of the primary and the secondary users for different channel fading coefficients are investigated to make sense of the results of the sum rate region. In the first case where fixed s_{12} and s_{20}, the transmitting powers behave as in Figure 4.5. To optimize the sum rate of the system, the higher the direct connection of the primary user s_{10} is, the more power should be allocated to send the cooperative codeword of X_{12}. However, if the link from the secondary user to the destination, s_{20} has a active part of maximizing the sum rate so that the power allocated to P_{20} can be more reasonable if this link is better. In our example, when $s_{20} = 0.15$, P_{12} is increasing early compared to the case of $s_{20} = 0.25$ or when s_{12} is increasing and s_{20} is constant, the power should be increased to transmit more number of symbols to the destination.

In addition to the power distribution of the common link, the power allocated to the direct path of the secondary user is described. We know of that s_{12} is always better than the other channel conditions. If we want to increase the sum rate, P_{20} should be moved up to the case of when s_{10} reaches to the desired level because of that s_{12} is bigger. Moreover, when the link of the direct link from the secondary user to the destination is stabilized and s_{12} moved from 0.25 to 0.35, the level of P_{20} starts to decrease early.

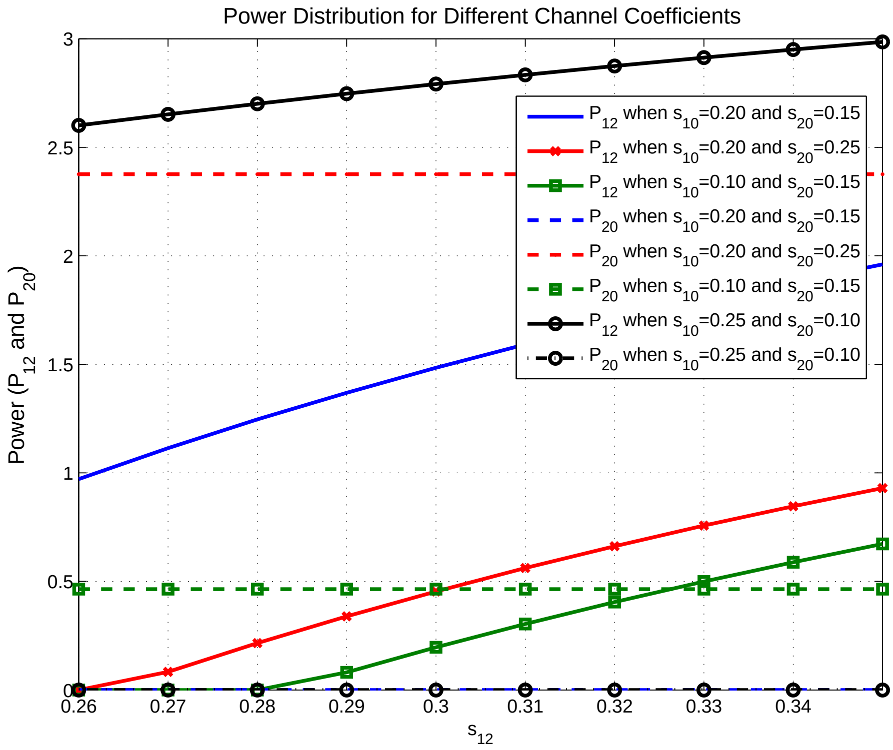


Figure 4.6: Power distribution for different channel coefficients of s_{10} and s_{20}

In Figure 4.6, the power distribution is given with respect to s_{12} with fixing the direct links, s_{10} and s_{20}. It is obviously that the power allocated to the cooperative path is related to s_{12}. Thus, increasing the coefficient means distributing more power of P_{12}. However, the message sent to the cognitive user from the primary user is decoded by the receiver with dependency on the situation of the relationship between the direct links of the primary user and the cognitive user, s_{10} and s_{20}. If s_{10} is getting better compared to s_{20}, then the more power of P_{12} should be assigned. On contrary, it increases with less amount of power. In other case of looking the distribution of P_{20}, the common information between the users does not depend on s_{12} as formulating in the analytical part of this chapter, so the lines with zero slope is reasonable under the condition of $P_2 = P_{20} + P_{U2}$. It differs accordance with the channel fading coefficient of the link from the cognitive user to the destination. Considering the black dashed line in Figure 4.6, s_{20} is approximately worst and s_{10} and s_{12} is nearly two or three times higher than s_{20}. To optimize the sum rate, P_{20} is initialized and its total power is transferred to the common information.

4.3 Optimization of Rate of Cognitive User of Two-User Cooperative Cognitive Multiple Access Channel

As mentioned in the previous section, we have two main problem and our interest in this section is to optimize the cognitive radio user's achievable rate by getting assistance of the fully knowledge of the channel state information at both the users and the receiver under the condition of the fact that the primary user's achievable rate does not decrease the optimal level. We investigate the power allocation policy showing as in the equations from (4.5) to (4.7) by solving Lagrangian optimization problem. We obtain the KKT conditions, which are necessary and sufficient for optimality.

The problem which aims the maximization of the rate achievable of the cognitive user can be expressed as

$$\max_{p(\mathbf{h})} \quad R_2$$

$$\textbf{s.t.}$$

$$R_1 < E\left\{ \log\left[1+\frac{h_{10}P_{10}(\mathbf{h})}{\sigma_0^2}\right] + \log\left[1+\frac{h_{12}P_{12}(\mathbf{h})}{h_{12}P_{10}(\mathbf{h})+\sigma_0^2}\right]\right\} \tag{4.50}$$

$$R_2 < E\left\{ \log\left[1+\frac{h_{20}P_{20}(\mathbf{h})}{\sigma_0^2}\right]\right\} \tag{4.51}$$

$$R_1+R_2 < E\left\{ \log\left[1+\frac{h_{10}P_1(\mathbf{h})+h_{20}P_2(\mathbf{h})+2\sqrt{h_{10}h_{20}P_{u1}(\mathbf{h})P_{u2}(\mathbf{h})}}{\sigma_0^2}\right]\right\} \tag{4.52}$$

$$R_1 = E\left\{ \log\left[1+\frac{h_{10}P_1^\star(\mathbf{h})}{\sigma_0^2}\right]\right\} \tag{4.53}$$

$$P_1 = E\big[P_{10}(\mathbf{h})+P_{12}(\mathbf{h})+P_{u1}(\mathbf{h})\big] \tag{4.54}$$

$$P_2 = E\big[P_{20}(\mathbf{h})+P_{u2}(\mathbf{h})\big] \tag{4.55}$$

$$P_{10}(\mathbf{h}), P_{12}(\mathbf{h}), P_{u1}(\mathbf{h}), P_{20}(\mathbf{h}), P_{u2}(\mathbf{h}) \geq 0 \tag{4.56}$$

Assumptions for the first case are also valid for this problem. In other words, all realizations of the fading values satisfy $s_{12} > s_{10}$ where the channel coefficients are normalized by the noise powers as $s_{ij} = h_{ij}/\sigma_j^2$ and $P_{10}(\mathbf{h}) = 0$ and the problem of maximizing the achievable rate of the cognitive user becomes as

$$\begin{aligned}\mathcal{L}_2 = {} & R_2 + \gamma_1\Big\{E\Big[\log\Big(1+s_{12}P_{12}(\mathbf{h})\Big)\Big] - B^\star\Big\} \\ & +\gamma_2\Big\{E\Big[\log\Big(1+s_{20}P_{20}(\mathbf{h})\Big)\Big] - R_2\Big\} + \gamma_3\Big\{E\Big[\log\Big(A\Big)\Big] - B^\star - R_2\Big\} \\ & +\lambda_1\Big\{P_1 - E\Big[P_{12}(\mathbf{h})+P_{u1}(\mathbf{h})\Big]\Big\} + \lambda_2\Big\{P_2 - E\Big[P_{20}(\mathbf{h})+P_{u2}(\mathbf{h})\Big]\Big\} \\ & +\mu_2P_{12}(\mathbf{h}) + \mu_3P_{u1}(\mathbf{h}) + \mu_4P_{20}(\mathbf{h}) + \mu_5P_{u2}(\mathbf{h})\end{aligned} \tag{4.57}$$

Taking the partial derivations of $\mathcal{L}_2$ with respect to the power components of the primary and the secondary users, we found out the expressions from (4.58) to (4.61).

$$\begin{aligned}
\gamma_1 \frac{s_{12}}{1+s_{12}P_{12}(\mathbf{h})} + \gamma_3 \frac{s_{10}}{A} &\leq \lambda_1 && (4.58)\\
\gamma_2 \frac{s_{20}}{1+s_{20}P_{20}(\mathbf{h})} + \gamma_3 \frac{s_{20}}{A} &\leq \lambda_2 && (4.59)\\
\gamma_3 \frac{s_{10}\sqrt{P_{u1}(\mathbf{h})} + \sqrt{s_{10}s_{20}P_{u2}(\mathbf{h})}}{A\sqrt{P_{u1}(\mathbf{h})}} &\leq \lambda_1 && (4.60)\\
\gamma_3 \frac{s_{20}\sqrt{P_{u2}(\mathbf{h})} + \sqrt{s_{10}s_{20}P_{u1}(\mathbf{h})}}{A\sqrt{P_{u2}(\mathbf{h})}} &\leq \lambda_2 && (4.61)
\end{aligned}$$

where $\gamma_2 + \gamma_3 = 1$. From the same procedure at the above case, the linear relationship between $P_{u1}(\mathbf{h})$ and $P_{u2}(\mathbf{h})$ exists such that

$$\lambda_1^2 s_{20} P_{u1}(\mathbf{h}) = \lambda_2^2 s_{10} P_{u2}(\mathbf{h}) \qquad (4.62)$$

If and only if the equations (4.60) and (4.61) is satisfied with equality when $P_{u1}(\mathbf{h}), P_{u2}(\mathbf{h}) > 0$. Therefore, the powers allocated for the transmission of the message components are given by

$$
\begin{aligned}
P_{12}(\mathbf{h}) &= \left(\frac{\gamma_1(\lambda_2 s_{10} + \lambda_1 s_{20})}{\lambda_1^2 s_{20}} - \frac{1}{s_{12}}\right)^+ && (4.63)\\
P_{20}(\mathbf{h}) &= \left(\frac{\gamma_2(\lambda_2 s_{10} + \lambda_1 s_{20})}{\lambda_2^2 s_{20}} - \frac{1}{s_{20}}\right)^+ && (4.64)\\
P_{u1}(\mathbf{h}) &= \frac{\dfrac{1-\gamma_2\left(s_{10}+\frac{\lambda_1}{\lambda_2}s_{20}\right)}{\lambda_1} - \Big[1 + s_{10}P_{12}(\mathbf{h}) + s_{20}P_{20}(\mathbf{h})\Big]}{\left(s_{10}+\frac{\lambda_1}{\lambda_2}s_{20}\right)^2} s_{10} && (4.65)\\
P_{u2}(\mathbf{h}) &= \frac{\dfrac{1-\gamma_2\left(s_{20}+\frac{\lambda_2}{\lambda_1}s_{10}\right)}{\lambda_2} - \Big[1 + s_{10}P_{12}(\mathbf{h}) + s_{20}P_{20}(\mathbf{h})\Big]}{\left(s_{20}+\frac{\lambda_2}{\lambda_1}s_{10}\right)^2} s_{20} && (4.66)
\end{aligned}
$$

Next, we have to again include the case when the power allocated to transmit cooperative messages, i.e., the equations (4.65), (4.66) result with negativity. In this case, we should set $P_{u1}(\mathbf{h}) = P_{u2}(\mathbf{h}) = 0$ in our analysis for optimality. Then, the power levels of $P_{12}(\mathbf{h})$ and $P_{20}(\mathbf{h})$ become as in the roots of the quadratic equations (4.67) and (4.68) by replacing P_{u1} and P_{u2} terms including in the sum rate with coherent addition, A as zero.

For $P_{12}(\mathbf{h})$ and $P_{20}(\mathbf{h})$ when $P_{u1}(\mathbf{h}) = P_{u2}(\mathbf{h}) = 0$

$$
\begin{aligned}
a_3 P_{12}(\mathbf{h})^2 + b_3 P_{12}(\mathbf{h}) + c_3 = 0 && (4.67)\\
a_4 P_{20}(\mathbf{h})^2 + b_4 P_{20}(\mathbf{h}) + c_4 = 0 && (4.68)
\end{aligned}
$$

The solution for the equation (4.67) is

$$
\begin{aligned}
a_3 &= \lambda_1 s_{10} s_{12} && (4.69)\\
b_3 &= \lambda_1\Big(s_{10} + s_{12} + s_{12}s_{20}P_{20}(\mathbf{h})\Big) - (\gamma_1 - \gamma_2 + 1)\Big(s_{10}s_{12}\Big) && (4.70)\\
c_3 &= \lambda_1\Big(1 + s_{20}P_{20}(\mathbf{h})\Big) - \gamma_1\Big(s_{12} + s_{12}s_{20}P_{20}(\mathbf{h})\Big) - (1-\gamma_2)s_{10} && (4.71)
\end{aligned}
$$

The solution for the equation (4.68) is

$$a_4 = \lambda_2 s_{20}^2 \tag{4.72}$$

$$b_4 = \lambda_2\Big(2s_{20} + s_{10}s_{20}P_{12}(\mathbf{h})\Big) - s_{20}^2 \tag{4.73}$$

$$c_4 = \lambda_2\Big(1 + s_{10}P_{12}(\mathbf{h})\Big) - \gamma_2\Big(s_{10}s_{20}P_{12}(\mathbf{h})\Big) - s_{20} \tag{4.74}$$

4.3.1 Iterative Algorithm

Our second algorithm differs from the first one with the additional constraint which controls the primary user's optimal achievable rate. We have to search optimal γ_1, γ_2, λ_1 and λ_2 values iteratively. Our iterative program firstly scans the optimal combination of λ_1 and λ_2 according to the specific Lagrangian multipliers for the rate constraints, γ_1 and γ_2. We calculate the water level for each user and then determine the powers of the sub-messages using the equation (4.63), (4.64), (4.65) and (4.66) when $P_{u1}(\mathbf{h})$ and $P_{u2}(\mathbf{h})$ are positive. We assign the new power values of $P_{12}(\mathbf{h})$ and $P_{20}(\mathbf{h})$ using the equation (4.67) and (4.68) when those power values are negative and we set $P_{u1}(\mathbf{h}) = P_{u1}(\mathbf{h}) = 0$ for those index. After finding the suboptimal λ values, we have to firstly check the primary user's quality of services scanning γ_1, then we search γ_2 value for the maximum of the achievable rate of the cognitive user.

Algorithm 2 Optimization of Secondary User's Rate

```
Initiate $\gamma_1$ and $\gamma_2$;
while $\gamma_2$ does not satisfy that (4.52)-(4.53)=(4.51) do
  Reset $\gamma_1$;
  while $\gamma_1$ does not satisfy the equation (4.50)¡(4.53) do
    All power values $\leftarrow 0$;
    Initiate $\lambda_1$;
    while Powers are not optimal as in the equations (4.54) and (4.55) do
      Reset $\lambda_1$;
      while $\lambda_1$ where controls (4.54) is not optimal do
        Calculate water-level $V_1$ and power components of the primary user $P_{12}, P_{U1}$ using (4.63) and (4.65);
        Find where $P_{U1} < 0$;
        while $P_{U1} \leq 0$ do
          Recalculate $P_{12}$ for that index set using the equation (4.67);
        end while
        if Primary user's power is satisfied with its average power then
          Modify $\lambda_1$ (if the result is less than required, decrease $\lambda_1$);
        end if
      end while
      Use new suboptimal $\lambda_1$ to calculate $\lambda_2$;
      Initiate $\lambda_2$;
      while $\lambda_2$ is not optimal in (4.55) do
        Calculate water level $V_2$ and power components of the cognitive user $P_{20}, P_{U2}$ (4.64) and (4.66);
        Assign $P_{U2} = 0$ where $P_{U2} < 0$;
        while $P_{U2}$ is equal to 0 do
          Recalculate $P_{20}$ for those index using (4.68);
        end while
        if Secondary user's power is satisfied with its average power then
          Modify $\lambda_2$;
        end if
      end while
    end while
    Calculate the rate of primary user, $R_3$ and $R_4$ using (4.50) and (4.53);
    if $R_3 - R_4$ ¡ Threshold then
      Decrease $\gamma_1$;
    else
      Increase $\gamma_1$;
    end if
  end while
  Calculate the rates of the cognitive user, $R_1$ and $R_2$ using (4.51) and (4.52)-(4.53);
  if $R_1 - R_2$ ¡ Threshold then
    Modify $\gamma_2$;
  end if
end while
```

4.3.2 Simulation Results

The second approach to the optimization problem is to maximize the rates achievable by the cognitive user taking into account of the fact that the primary user's data rates should not be below the level of the optimal value providing by the power control policy only. We analyzed and optimized the maximization problem, and found out achievable rate pairs, then we performed a convex hull operation over these points and we observed Figure 4.7. We obtain for a system with the average total power of 1, and a channel is exposed to uniform fading. The links between the primary user and the receiver and also the cognitive user and the destination take values from the set of $\{0.025, 0.050, ..., 0.25\}$ with probability of 1/10. The link between the transmitters is also uniform and takes values from the set of $\{0.26, 0.27, ..., 0.35\}$. The assumption of the channel condition is valid for this situation, so that the fading coefficient of the cooperation link is always better than the direct links, so $P_{10}(\mathbf{h}) = 0$.

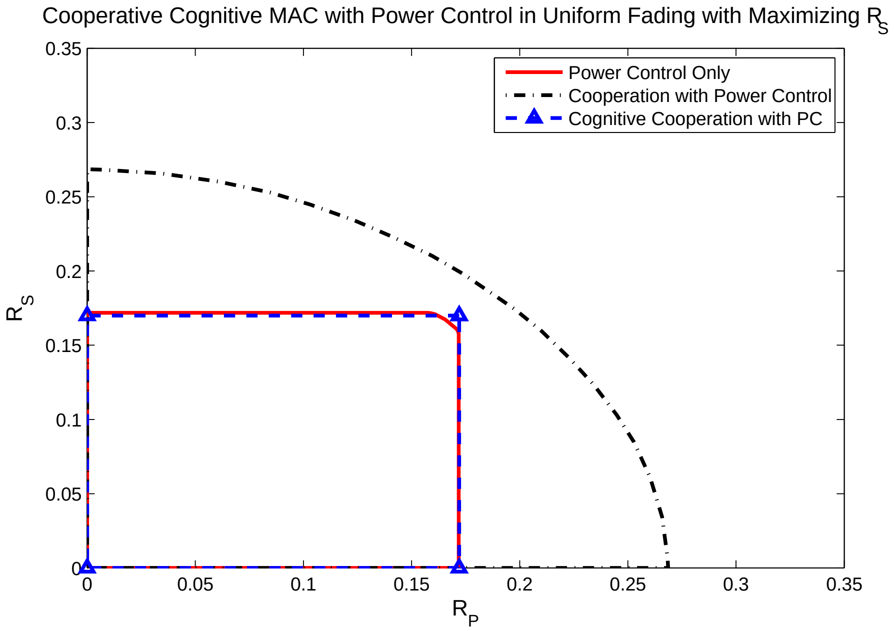


Figure 4.7: Rates achievable of cognitive user with power control and user cooperation for uniform fading

The rate achievable pairs of the two-sided cooperation in a multiple access channel is shown with the dashed black line in Figure 4.7 and the rate pairs with power control and without cooperation for the same channel is illustrated with red line. We can observe that the case of two-sided cooperation lies over the situation of the one-sided cooperation in the cognitive MAC channel as expected. It is seen that the cooperation in our case improves both the sum rate and the cognitive user rate achievable with the case of the fact that primary user's rate is fixed. The cognitive user's rate is 0.158 symbols/transmission where the rate of the primary user's is limited to 0.172 symbols/transmission in order to be guaranteed to transmit its own message reliable considering the optimization of the secondary user's data rates.

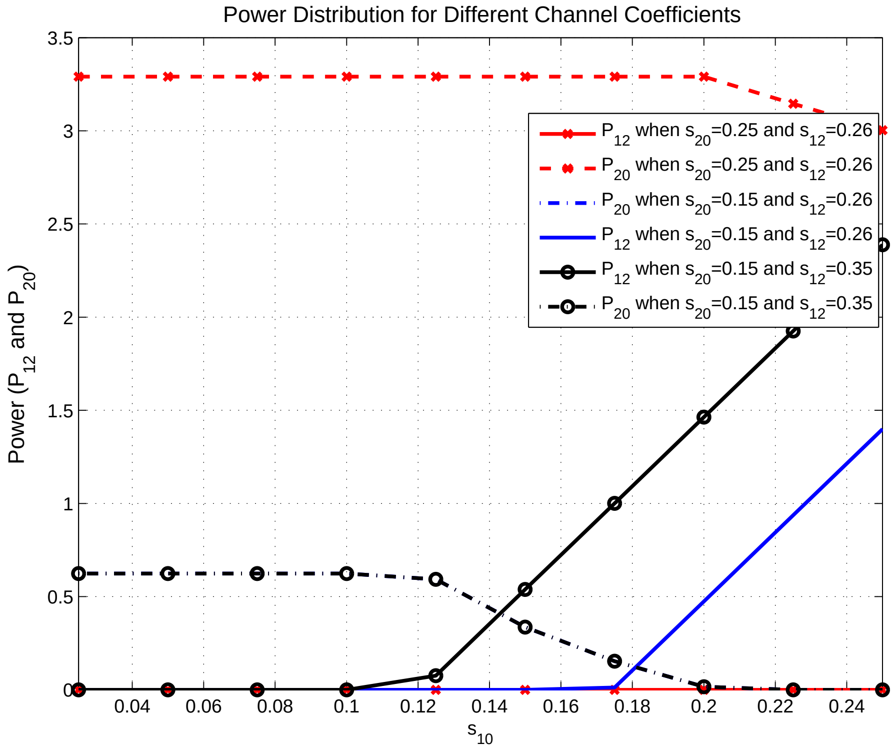


Figure 4.8: Power distribution for different channel coefficients of s_{12} and s_{20}

Figure 4.8 shows the power distributions of the one for the cooperative link and the one for the direct path of the secondary user with respect to the different channel conditions of s_{10} with some fixed values of s_{12} and s_{20}. This figure is illustrated to be able to interpret successfully what differs the results compared to the case of optimization of the sum rate of the system. In this case, our aim is to improve the rate of the cognitive user as much as possible with the care of the primary user's rate constraint. Thus, if the link from the cognitive user to the destination has the best value as possible, its power is allocated to its own link without effecting any cause to the primary user's rate, so that the direct link of the primary user, s_{10} is getting better, the power of P_{20} is enrolled to decrease. Besides, we have to consider the cooperative link condition. It is helpful to increase the primary user's achievable rate, so the increasing s_{12} contributes the primary user and the power of P_{12} gets bigger with s_{10} as shown in Figure 4.8. We have to again compare the two figures. When s_{12} reaches its maximal value, 0.35, and s_{20} is decreased from 0.25 to 0.15 with looking at the yellow lines, then we can see that more power of P_{20} is allocated to the case of optimizing the cognitive user's rate for the lower s_{10}. Another interesting result is that the blue dashed line and black dashed line overlap due to the fact that P_{U2} does not depend on s_{12}, so it does not change and so P_{20}.

In Figure 4.9, the power distribution is given with respect to s_{12} with fixing the links coefficients of the primary users, s_{10} and s_{12}.

It can be easily figured out that the message sent by the cognitive user is decoded at the receiver successfully is directly related to the link from the secondary user to the destination. Therefore, the bigger the channel fading coefficient of s_{20} is, the more power is allocated to that part of the message over that path. However, there is an other constraint to limit us which is the primary user's rate for its quality of service requirement. This requirement is provided by the channel condition of s_{10} and s_{12}. When s_{10} is selected as lower level, then the power of P_{12} becomes smaller. As an example in Figure 4.9, if $s_{10} = 0.10$, then the power allocated for the cooperative message of the first user, P_{U1} should be increased and the cognitive user should also increase its own cooperative message power, P_{U2} to improve the sum rate of the system. After the channel of the direct link of the secondary user gets better (which is bigger than 0.10), it starts to use its own direct link to transmit the message, in other words, the power of P_{20} is increasing. Moreover, we should analyze the effects of the common link between transmitters to the power distribution. We already know that increasing of s_{12} also provides

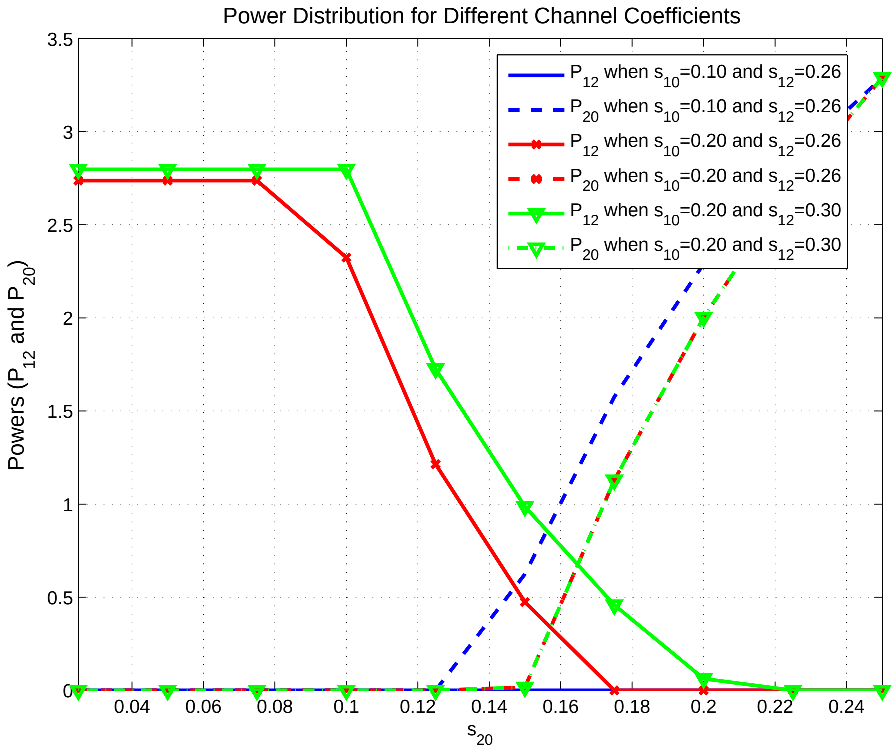


Figure 4.9: Power distribution for different channel coefficients of s_{10} and s_{12}

to get higher the P_{12}. Remember that the power for the transmitting of the cooperative message, P_{U2} is independent with s_{12}. Any change of s_{12} does not effect P_{U2}, so the power allocated to transmit its own fresh information, P_{20} as illustrated with dashed green and red lines overlapped.

Conclusion

In this thesis, we proposed a coding and decoding strategy for a wireless communication network, in which licensed and unlicensed (cognitive) users simultaneously transmit their own messages towards the common destination, such that the cognitive user relays the primary user's message while transmitting its own message. We have solved the problem of optimal power allocation for a fading cooperative cognitive multiple access channel. Transmitters can adapt the encoding/decoding strategies by allocating the transmitting powers which maximize the achievable rates by block Markov superposition coding, so that the powers for the signal components which are the one that is destined to the receiver, the one for the cognitive user, and the one for cooperation are allocated according to the channel condition. Using techniques from convex minimization (or equivalently concave maximization), we obtained the analytical structure of the optimal power allocation policies. The power control policies, which are jointly optimal with block Markov coding, were then obtained through simulations. The sum rate of the system with power control and one-directional cooperation improves the one with power control only and it is close to the one with bidirectional cooperative systems with power control. The results also show that the rate achievable by the cognitive user slightly increases considering the constraint of the primary user's optimal rate.

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Curriculum Vitae

Murat İşleyen was born in 09 August 1985, in İstanbul. He received his B.S. degree in Electronics Engineering in 2008 from Işık University. He worked as a teaching assistant at the Department of Electronics Engineering of Işık University from 2008 to 2011. His research interests include wireless communication, information theory, and cooperative communication strategies.