EOQ MODEL WITH TRANSPORTATION COST UNDER DETERMINISTIC DEMAND

CİHAN KARS

B.S. Industrial Engineering and Operation Research Işık University, 2009

Submitted to the Graduate School of Science and Engineering

in partial fulfillment of the requirements for the degree of

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Abstract

Inventory planning issues are one of the most important aspects of the companies. To have optimal stock level both for production planning and low stock holding cost with successful demand forecast is also very important. If the companies work with high stock level, it causes the increase of the unused capital and it hinders to use the money for more effective operations.

In global world, the companies purchase goods, semi-finished goods or materials from different regions of the world. They purchase the goods with variable quantities from different regions and directly affect the stock level of the companies. Furthermore it is essential to take transportation cost into account with the consumption quantity of material and the purchase price of unit material while determining inventory stock levels. There are also different payment options in purchasing of materials. The cash payment or the payment after receiving goods have also role in determining stock levels.

When we take into account the transportation options and the payment options of purchasing materials the classical economic order quantity model is inadequate. The aim of this thesis is to have a new approach of economic order quantity model by considering the transportation cost and different payment options when we have deterministic demand. By the calculation of all possible transportation and payment alternatives, it becomes possible to find the optimum options of transportation and payment. The new model was implemented for a specific material of company X, for deterministic demand and the aim is to have optimal stock level for the specific material.

The program MINITAB is used to find the appropriate method among different forecasting methods. The consumption of specific materials is evaluated and a sample group is composed to implement the new model. The all possible transportation modes, transportation durations and the historical demand data of 2010 and 2011 are collected and analyzed. Demand forecasting methods are applied on historical demand data and the results are accepted as deterministic and transportation modes and times are investigated in excel and used in problem. The process of the model is shown for the material WTR910R and the results are compared with the old system of company X.

DÜZENLİ TALEP ORTAMINDA ULAŞIM MALİYETLERİ İLE EKONOMİK SİPARİŞ MİKTARI MODELİ

Özet

Envanter planlama bir şirketin en önemli çalışma bölümlerinden biridir.Talep tahminlerinin başarılı olarak yapılması ile stok seviyelerinin uygun seviyede tutulması hem üretim planı açısından hem de stok maliyetlerinin düzenli ve düşük devam etmesi için çok önemlidir. Yüksek stok seviyesi ile çalışılması paranın gereksiz yere bağlanmasına sebep olur ve efektif olarak kullanılmasını engeller.

Globalleşen dünya ile şirketler dünyanın her yanından mamul, yarı mamul veya malzeme tedariki sağlamaktadır. Farklı bölgelerden farklı yükte mal getirtilmesi envanter planı ve maliyetlerini de önemli ölçüde etkilemektedir. Artık stok seviyelerinin hesaplanmasında malzemenin tüketim miktarı ve fiyatının yanında ulaşım maliyetlerinde hesaba katılması gereklidir. Bunun yanında tedarikçiden malzeme satın alınmasında farklı ödeme şekilleri de vardır. Peşin ödeme yapılması veya malzemenin stoka girdiği anda ödemenin yapılması da stok seviyelerinin belirlenmesinde önemli bir etkendir.

Bu noktada klasik ekonomik sipariş miktarı modeli yetersiz kalmaktadır. Bu tezin amacı düzenli talep hareketlerinde ekonomik sipariş miktarı modelinin ulaşım maliyetleri ve ödeme şeklinin de hesaba katılarak farklı açıdan değerlendirilmesidir. Farklı ödeme şekillerinde ve farklı ulaşım alternatifleri arasından en düşük toplam maliyetin bulunmasıdır. Ancak tüm alternatiflerin maliyetlerinin de bulunması sayesinde stratejik açıdan diğer seçenekleri inceleme şansıda yakalanmış olacaktır. Oluşturulan modelin uygulaması X firmasının talebi düzenli olan belirli malzemeler üzerinde yapılmıştır ve stok seviyeleri optimum seviyeye getirilmesi hedeflenmiştir.

Talep tahmin yöntemlerinin belirlenmesi için MINITAB programı kullanılmıştır. Malzemelerin tüketim hareketleri incelenerek tüketim talepleri düzenli olan malzemeler seçilerek model için örnekleme hazırlanmıştır. Bu gruba ait malzemelerin ulaşım çeşitleri, ulaşım süreleri ve 2010, 2011 yılına ait tüketim rakamları dataları toplanmış ve analiz edilmiştir. Son iki yıla ait olan talepler kullanılarak talep tahmin yöntemleri uygulanmış ve deterministtik hale getirilmiş, ulaşım çeşitleri ve ulaşım süreleri MS Excell programı kullanılarak standardize edilip problemde kullanılmıştır. Modelin süreçleri WTR910R malzemesi üzerinde uygulanıp sonuçlar incelenmiş, X firmasında devam eden sistem ile karşılaştırılmıştır.

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List of Symbols

α	Single exponential smoothing constant
β	Second smoothing constant, for trend
γ	Third smoothing constant, for seasonality
X_t	The observed value at time t
F_t	Forecast value at time t
$X_t - F_t$	Forecast error
L_t	An estimate of the level of the series at time t
b_t	An estimate of the slope of the series at time t
S_t	Seasonal component
c	Unit purchasing cost
Q	Quantity which is procured from source
K	Fixed cost of purchasing operation
r	Interest rate
λ	Demand rate
h_1	Stock holding cost
h_2	Holding cost which is caused by transportation time
w	Weight of component
φ	Startup cost of transportation
δ	Fixed import-export cost
β	Transportation cost

List of Abbreviations

MAD	Mean Absolute Deviation
MSD	Mean Standard Devation
S	Standard Deviation
S^2	Variance
ME	Mean Error
MAE	Mean Absolute Error
MSE	Mean Squared Error
PE_t	Percentage Error
MPE	Mean Percentage Error
$MAPE$	Mean Absolute Percentage Error
CIF	Cost and Freight – An option of payment
FOB	Free on Board – An option of payment
OUT	Order up-to policy
R,Q	R,Q ordering policy
s,S	s,S ordering policy
ACF	Autocorrelation Function

Chapter 1

1. Introduction

Inventory planning represents important part of the all expenses for business. On the other hand it could be said that the business has the right goods or raw material on hand to avoid stock-outs, to prevent spoilage with optimal stock level. In general, businesses have too much of their resources like capital, human power and storage space for inventory. Related to inventory planning parts of the components like raw material, semi-finished goods or finished goods must be planned correctly and with optimal value to avoid for extra costs.

The main part of the inventory planning is to predict the demand for the planning periods. Conversely demand forecast should be made very well to make a good inventory control. In the inventory there could be many components, parts and the consumption of each of them can be different. In such a situation, the managers forecast the demands of each component and part to choose the inventory control policy accordingly.

First the classification of the materials in the inventory should be identified. AHP or ABC/XYZ analysis can be implemented to determine which parts and components have high percentage of the total cost in order to find the components and parts which affect the system most. Because the category A materials could cover a high percentage of the costs and be more effective in comparison to B and C on total inventory budget. At second step demand characteristics should be clarified. Lastly an inventory control policy can be applied. In this master thesis a different approach of EOQ model with transportation cost has been applied as an inventory policy to a manufacturer company.

In chapter 2, the underlying knowledge of inventory control policies and forecasting methods are summarized and some related articles have been studied and summarized to gain different ideas and see different aspects on the theme of the thesis.

In chapter 3, the whole system has been tried to be identified and the problem of the company has been tried to be defined. The first stage is demand forecasting which we have to use in inventory models. Additionally safety stock calculation, types of inventories, ordering policies and the cost of inventories has been studied.

In chapter 4, a different approach is implemented which takes transportation cost into account which is an addition to the classical EOQ model with deterministic demand.

In chapter 5, implementation of the solution approach is made. This is a different aspect of the classic EOQ model and it is implemented for a part searched in details.

Chapter 6 is provided for the discussion of the study and further analysis.

Chapter 2

2. Underlying Knowledge and Literature Survey

2.1 Demand Forecasting For Inventory Planning

"*Forecasting is an activity to calculate or predict some future event or condition, usually as a result of rational study or analysis of pertinent data. Forecasting is widely used today in many fields, especially in industry, marketing, economy and finance. Such as in consumable product manufacturing, an accurate prediction of the future demand is very helpful in providing precise inventory, reducing transportation costs, then increasing profit* "(Spyros Makridakis,1998)

A forecasting exercise is usually carried out in order to provide an aid to decision-making and in planning future. Typically all such exercise work on premise that if we can predict the future we can modify our behavior now to be in a better position, than we otherwise would have been, when the future arrives. Applications for forecasting include:

- Inventory control - production planning: "*Forecasting the demand for a product enables us to control the stock of raw materials and finished goods, plan the production schedule, etc.*"

- Investment policy: "*Forecasting financial information such as interest rates, exchange rates, share prices, the price of gold, etc. This is an area in which no one has yet developed a reliable forecasting technique*."

- Economic policy: "*Forecasting economic information such as the growth in the economy, unemployment, the inflation rate, etc. is vital both government and business in planning for the future*."(John E. Beasley,2002)

2.2 What Is The Importance of Demand Forecasting?

Frequently there is a time lag between awareness of an impending event or need and occurrence of that event. This lead time is the main reason for planning and forecasting. If the lead time is zero or very small, there is no need for planning. If the lead time is long, and the outcome of the final event is conditional on identifiable factors, planning can perform an important role. In such situations forecasting is needed to determine when an event will occur or need arise, so that appropriate actions can be taken.

In management and administrative situations the need for planning is great because the lead time for decision making ranges from several years to a few days or hours to a few seconds. Forecasting is an important aid in effective and efficient planning. (Spyros Makridakis,1998)

Forecasting is an integral part of the decision making activities of management. The need for forecasting is increasing as management attempts to decrease its dependence on chance and becomes more scientific in dealing with its environment. A good or bad forecast can affect the entire organization. Some of the areas in which forecasting currently plays an important role are:

1- Scheduling: "*Efficient use of resources requires the scheduling of production, transportation, cash, personnel, and so on. Forecasts of the level of demand for product, material, labor, financing, or service are an essential input to such scheduling*."
2- Acquiring resources: "*The lead time for acquiring raw materials, hiring personnel, or buying machinery and equipment can vary from a few days to several years. Forecasting is required to determine future resource requirements*."
3- Determining resource requirements: "*All organizations must determine what resources they want to have in the long-term. Such decisions depend on market opportunities, environmental factors, and the internal development of financial, human, product, and technological resources. These determinations all require good forecasts and managers who can interpret the predictions and make appropriate decisions*."(Spyros Makridakis,1998)

Forecast methods may be broadly classified into qualitative and quantitative techniques. Qualitative methods are intuitive, largely educated guesses that may or may not depend on the past data. Quantitative methods use mathematical or statistical models to generate a reasonable prediction from the information of the past. Compared to qualitative methods, quantitative methods have the advantage of being supported by mathematical a statistical theory, and can be fully reproduced by any forecaster.

2.3 Forecasting Methods

There are two types of demand forecasting.

i. Qualitative Forecasting
ii. Quantitative Forecasting

In this part of the study the both forecasting analysis will be investigated.

2.3.1 Qualitative Forecasting

Qualitative forecasting methods rely on judgments about future experience collection. These techniques are often referred to as judgmental approaches. In addition to their relatively small dependence on numbers, these techniques frequently do not provide a rigorous specification of underlying assumptions.

a) Judgmental Forecasting: This technique involves having an individual or small group of people make assessments of likely future conditions. This technique can produce good estimates, especially when experienced persons are involved. The forecaster will utilize experience in conjunction with consideration of historical trends, current economic conditions, and other factors relevant to the source.

b) Consensus Forecasting: This is a variation of the judgmental approach. In this forecasting method experts meet to discuss near-term conditions in order to reach agreement about what is likely to happen.

Guajardo and Miranda (2000) provide the following list of the major weaknesses of the qualitative forecasting methods:

- anchoring events – allowing recent events to influence perceptions about future events, e.g. the city hosting a recent major convention influencing perceptions about future room taxes
- information availability – over-weighting the use of readily available information
- false correlation – forecasters incorporating information about factors that are assumed to influence revenues, but do not
- inconsistency in methods and judgments – forecasters using different strategies over time to make their judgments, making them less reliable
- selective perceptions – ignoring important information that conflicts with the forecaster’s view about causal relationships
- group think – when the dynamics of forming a consensus tends to lead individuals to reinforce each other’s view rather than maintaining independent judgments

2.3.2 Quantitative Forecasting

Quantitative forecasting methods are used when historical data on variables of interest are available-these methods are based on an analysis of historical data concerning the time series of the specific variable of interest and possibly other related time series. There are two major categories of quantitative forecasting methods. The first type uses the past trend of a particular variable to base the future forecast of the variable. As this category of forecasting methods simply uses time series on past data of the variable that is being forecasted, these techniques are called time series methods.

The second category of quantitative forecasting techniques also uses historical data. But in forecasting future values of a variable, the forecaster examines the cause-and-effect relationships of the variable with other relevant variables such as the level of consumer confidence, changes in consumer's disposable incomes, the interest rate at which consumers can finance their spending through borrowing, and the state of the economy represented by such variables as the unemployment rate. Thus, this category of forecasting techniques use past time series on many relevant variables in order to produce the forecast for the variable of interest.

Forecasting techniques falling under this category are called causal methods, as the basis of such forecasting is the cause-and-effect relationship between the variable forecasted and other time series selected to help in generating the forecasts.

(Anderson and Sweeney,1997)

Makridakis represents the steps of a forecasting study in five parts.

i. Problem definition
ii. Gathering information
iii. Preliminary analysis
iv. Choosing and fitting models
v. Using and evaluating a forecasting models

2.3.3 Forecasting Study

Makridakis's forecasting tasks are represented in five steps. These are:

Step 1: Problem definition

A forecaster has a great deal of work to do to properly define the forecasting problem, before any answers can be provided. For example, we need to know exactly what products are stored, who uses them, how long it takes to produce each item, what level of unsatisfied demand the company is prepared to bear, and so on.

Step 2: Gathering information

Mostly there are at least two kinds of information available: statistical data and accumulated judgment and expertise of key personnel. It is necessary to collect historical data of the items of interest. Historical data can be gathered from database of ERP systems of companies.

Step 3: Preliminary (exploratory) analysis

The data should be analyzed to understand what the numerical information tells us. Some simple descriptive statistics (e.g., mean, standard deviation, minimum, maximum, percentiles) of the related data is computed. Graphical analysis can also be used if there are more than one series of historical data.

Step 4: Choosing and fitting models

There are many types of quantitative forecasting models. Exponential smoothing methods, regression models, Box-Jenkins ARIMA models, Double exponential methods and Winter's method are some forecasting methods. Couple of these methods will be used to make a forecast in our inventory planning problem will be discussed in next chapters.

Step 5: Using and evaluating a forecasting model

A forecasting assignment is not complete when the model has been fitted to the known data. The performance of the model can only be properly evaluated after the data for the forecast period have become available.

2.4 Data Summary

Another way to summarize data is numerical analysis. In order to analyze a set of data commonly statistic methods are used. For a single time series or a single data set researcher use generally the mean, the standard deviation and the variance of the sample. On the other hand for a pair of random variables researcher tries to investigate the relationship between datasets. In order to find this relationship covariance and correlation are used.

Related to these explanations some expressions are used to analyze the time series. To explain these expressions in details suppose that there are **n** observations and individual observations are denoted by i.

MAD (Mean Absolute Deviation) is a statistical measure used in evaluating forecasts. MAD is the average of the absolute values of the error, or deviations of observed values from expected or forecasted values. In other words MAD is the mean or average of the absolute values of all the errors. Because MAD is based on absolute errors, it provides average magnitude of error regardless of the direction of errors. Smaller the MAD the better forecast.

X_i for i= 1, 2… **n**.

Table2.1Expression of time series

$Median$	Middle observation if n odd, average of middle two observations if n even.
$MAD -\ Mean\ Absolute\ Deviation$	$\frac{1}{n}\sum \lvert Y_i - \overline{Y} \rvert$
$MSD - Mean\ Squared\ Deviation$	$\frac{1}{n}\sum (Y_i - \overline{Y})^2$
$S^2 -\ Sample\ Variance$	$\frac{1}{n-1}\sum (Y_i - \overline{Y})^2$
$S - Sample\ Standard\ Deviation$	$\sqrt{\frac{1}{n-1}\sum (Y_i - \overline{Y})^2}$

MSD is one of several measures for evaluating forecasts accuracy. It is calculated by squaring the individual forecast deviation for each period and then finding the average or mean value of the sum of squared errors. Forecast error is actual observation for a period minus the forecast for the period. The mean squared error is used because, by squaring the error values, the resulting values are all positive. "*Mean squared deviations are easier to handle mathematically, and so it is often used in statistical optimization*."(PaulSwamidass,2000)

2.6 Measurement of Forecasting Accuracy

Forecast accuracy is so important to determine the response of observations in future. In many instances, the word "accuracy" refers to "goodness of fit" which in turn refers to how well the forecasting model is able to reproduce the data that are already known. To the consumer of forecasts, it is the accuracy of the future forecast that is most important. Therefore some expressions will be given in details.

$$ME = \frac{1}{n}\sum_{k=1}^{n} e_t \qquad mean\ error \qquad 2.1$$

$$MAE = \frac{1}{n}\sum_{k=1}^{n} |e_k| \qquad mean\ absolute\ error \qquad 2.2$$

$$MSE = \frac{1}{n}\sum_{k=1}^{n} e_k^2 \qquad mean\ squared\ error \qquad 2.3$$

Suppose that X_k is the actual observation for period $k$and F_t is the forecast for the same period and $e_t = X_k - F_t$. In such an equation e_t tells one-step-forecast error. And thus if there are n observations and forecasts it means that there are n errors. Mean error calculates the mean of the difference between observations and forecasts. Equation 2.2 MAE takes its absolute value of errors and calculates the average. MSE (Equation 2.3) takes the squares of errors and gives average.

2.7 Quantitative Forecasting Methods

There are many forecasting methods like single exponential smoothing method, double exponential smoothing method, Winter's method etc. for both qualitative and qualitative forecasting. In this report there will be given some forecasting methods in details. Makridakis shows the forecasting scenario in five steps.

$Step$ 1 : $Choose\ a\ time\ series\ to\ test$

$Step$ 2 : $Choose\ a\ forecasting\ method$

$Step$ 3 : $Implement\ the\ method\ on\ data\ set$

$Step$ 4 : $Analyze\ test\ measures\ \ MAPE, MSE, etc.$

$Step$ 5 : $Appraisal\ decision$

Exponential smoothing method uses a technique that implies exponentially decreasing weights as the observations get older. Thus they are called exponential smoothing procedures. There is a variety of exponential smoothing methods. They all have in common the property that recent values are given relatively more weight in forecasting than the older observations.

- **Single Exponential Smoothing Method**

Suppose we wish to forecast the next value of our time series $X_k$which is yet to be observed. Our forecast is denoted byF_t. When the observation $X_k$becomes available, the forecast error is found to be $X_k - F_{t.}$The method of single exponential forecasting takes the forecast for the previous period and adjusts it using the forecast error. Suppose that α is a constant between 0 and 1 and the forecast for the next period is

$$F_{t+1} = F_t + \alpha(X_t - F_t) \quad Single\ exponential\ forecast$$

It can be seen that the new forecast is simply the old forecast plus an adjustment for the error that occurred in the last forecast. When α value close to 1, the new forecast has will include a substantial adjustment for the error in the previous forecast. Conversely, when α is close to 0, the new forecast will include very little adjustment. The past forecast error is used to correct the next forecast in a direction opposite to that of the error. Related to that simple exponential forecast formulation can be written in other form

$$F_{t+1} = \alpha X_t + (1-\alpha)F_t$$

Example 2.1:

We illustrate single exponential smoothing with α = 0.1for the first six months of TV sales. The results are given in table. We assume that 32 TVs were sold last month, so we initialize the procedure with F_t=32. Here are some illustrations of the computations:

$$F_{t+1} = 0.1X_t + 0.9F_t = 0.1 * 30 + 0.9 * 32 = 31.8$$

$$F_{t+1} = 31.8 \qquad e_1 = 32 - 31.8 = 0.2$$

Table 2.2 The results of single exponential smoothing method

Month	Actual Sales	Forecast	At	et
1	30	32,00	31,80	-2,00
2	32	31,80	31,82	0,20
3	30	31,82	31,64	-1,82
4	39	31,64	32,37	7,36
5	33	32,37	32,44	0,63
6	34	32,44	32,60	1,56

$$MAD = \frac{|-2| + |0.2| + |-1.82| + |7.36| + |0.63| + |1.56|}{6} = 2.26$$

(Wayne Winston,2003)

- **Double Exponential Smoothing Method**

Double exponential smoothing also known as Holt exponential smoothing is a refinement of the popular simple exponential smoothing model but adds another component which takes into account any trend in the data. Simple exponential smoothing models work best with data where there are no trend or seasonality components to the data. When data exhibits either an increasing or decreasing trend over time, simple exponential smoothing forecasts tend to lag behind observations. Double exponential smoothing is designed to address this type of data series by taking into account any trend in the data.

Double exponential smoothing still does not address seasonality. For better exponentially smoothed forecasts using data where there is expected or known to be seasonal variation in the data, use triple exponential smoothing.

The forecast for double exponential smoothing is found using two smoothing constants, α and β

$$L_t = \alpha X_t + (1-\alpha)(L_t + b_{t-1}) \qquad 2.4$$

$$b_t = \beta(L_t - L_{t-1}) + (1+\beta)b_{t-1} \qquad 2.5$$

$$F_{t+m} = L_t + b_t m \qquad 2.6$$

X_t is the observed value at time t.
L_t is an estimate of the level of the series at time t
$b_t$an estimate of the slope of the series at time t
α represents alpha, is the first smoothing constant, used to smooth the observations
β represents gamma, is the second smoothing constant, used to smooth the trend

The smoothing constants must be values in the range 0 and 1. But the problem is to choose the best values for alpha and gamma. This depends on the data series being modeled. In general, the speed at which the older responses are dampened is a function of the value of the smoothing constant. When this smoothing constant is close to 1 dampening is quick and when it is close to 0 dampening is slow and relatively less weight is given to recent observations.

Example 2.2:

Using the inventory demand data from table shows the application of double exponential smoothing to a series with trend. The smoothing parameters alpha and beta were chosen by minimizing the MSE over observation. The calculation involved can be illustrated by looking at the forecast period 23. $\alpha = 0.501$ and $\beta = 0.072$.

$$F_{23} = L_{22} + b_{22}$$

$$where\ L_{22} = 0.501 * Y_{22} + 0.499 * (L_{21} + b_{21})$$

$$= 0.501 * 242 + 0.499 * (227.33 + 5.31)$$

$$= 273.33$$

$$b_{22} = 0.0.72 * (L_{22} - L_{21}) + 0.928 * b_{21}$$

$$= 0.072 * (237.33 - 227.33) + 0.928 * (5.31)$$

$$= 5.64$$

$$Thus,\ F_{23} = 237.33 + 5.64 * (1) = 242.97$$

The same procedure can be applied for the next forecast observations.

Table 2.3 Forecast results of Example 2.2

	Period (t)	Observed Data (Y_t)	Smoothing of Data (L_t)	Smoothing of Trend (b_t)	Forecast when m=1 (Ft)	
	1	143	143,00	9,00	-	
	2	152	152,00	9,00	152,00	
	3	161	161,00	9,00	161,00	
	4	139	154,47	7,88	170,00	
	5	137	149,64	6,96	162,34	
	6	174	165,32	7,59	156,60	
	7	142	157,42	6,47	172,91	
	8	141	152,47	5,64	163,89	
	9	162	160,03	5,78	158,06	
Test Set	10	180	172,92	6,30	165,82	
	11	164	171,59	5,75	179,22	
	12	171	174,16	5,52	177,34	
	13	206	192,87	6,47	179,68	
	14	193	196,16	6,24	199,34	
	15	207	204,71	6,41	202,40	
	16	218	214,56	6,66	211,11	
	17	229	225,12	6,94	221,22	
	18	225	228,52	6,68	232,06	
	19	204	219,57	5,55	235,20	
	20	227	226,06	5,62	225,12	
	21	223	227,33	5,31	231,68	
	22	242	237,33	5,64	232,64	
	23	239	240,98	5,50	242,97	
	24	266	256,26	6,21	246,48	
	25				262,47	(m=1)
	26				268,68	(m=2)
	27				274,89	(m=3)
	28				281,09	(m=4)
	29				287,30	(m=5)
	30				293,51	(m=6)

Analysis of errors from period 10 to period 24

0,78 = Mean Error	5,45 = Mean Absolute Percentage Error
11,29 = Mean Absolute Error	0,78 = Theil's U-statistic
194,78 = Mean Square Error	

(Spyros Makridakis,1998)

- **Holt Winter's Trend and Seasonality Method**

Time series data display behavior that is seasonal. Seasonality is defined to be the tendency of the time-series data to exhibit behavior that repeats itself every L periods. The term season is used to represent the period of the time before behavior begins to repeat itself. L is therefore the season length in periods.

For example, during the month of December the sales for a particular toy may increase by 1 million dollars every year. Thus, we could add to our forecasts for every December the amount of 1 million dollars to account for this seasonal fluctuation. In this case, the seasonality is additive. Instead of increasing or decreasing by certain number, sales response can be changed with a multiplicative value. In this case the sales increase or decrease by a certain factor, the seasonality component is thus multiplicative in nature. (PrajaktaKalekar,2004)

Table 2.4 Quarterly sales data

Year	Quarter	Period	Sales (thous. of francs)	Year	Quarter	Period	Sales (thous. of francs)
1	1	1	362	4	1	1	544
	2	2	385		2	2	582
	3	3	432		3	3	681
	4	4	341		4	4	557
2	1	5	382	5	1	5	628
	2	6	409		2	6	707
	3	7	498		3	7	773
	4	8	387		4	8	592
3	1	9	473	6	1	9	627
	2	10	513		2	10	725
	3	11	582		3	11	854
	4	12	474		4	12	661

Quarterly sales data

(Spyros Makridakis,1998)

❖ ***Additive Seasonality***

The seasonal component in Holt-Winter's method may also be treated additively, although this is less common. The basic equations for Holt-Winter's additive method are as follows:

$$\begin{aligned} &Level &: \quad & L_t = \alpha(X_t + S_{t-s}) + (1-\alpha)(L_{t-1} + b_{t-1}) \\ &Trend &: \quad & b_t = \beta(L_t - L_{t-1}) + (1-\beta)b_{t-1} \\ &Seasonal &: \quad & S_t = \gamma(X_t - L_t) + (1-\gamma)S_{t-s} \\ &Forecast &: \quad & F_{t+m} = L_t + b_t m + S_{t-s+m} \end{aligned}$$

❖ ***Multiplicative Seasonality***

The basic equations for Holt-Winter's multiplicative method are as follows:

$$\begin{aligned} &Level &: \quad & L_t = \alpha\frac{X_t}{S_{t-s}} + (1-\alpha)(L_{t-1} + b_{t-1}) \\ &Trend &: \quad & b_t = \beta(L_t - L_{t-1}) + (1-\beta)b_{t-1} \end{aligned}$$

$$Seasonal \quad : \quad S_t = \gamma \frac{X_t}{L_t} + (1 - \gamma)S_{t-s}$$

$$Forecast \quad : \quad F_{t+m} = (L_t + b_t m)S_{t-s+m}$$

Where s is the length of seasonality, L_t represents the level of the series b_t denotes the trend, S_t is the seasonal component, and F_{t+m} is the forecast for m periods ahead.

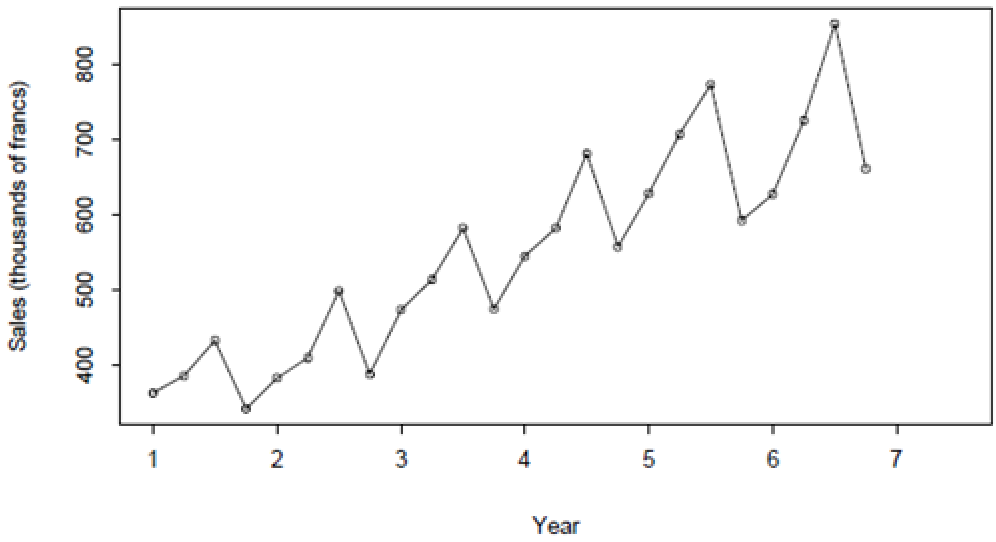

Figure 2.1 Time plot of quarterly sales data

2.8 Inventory Planning

The control and maintenance of inventories of physical goods is a problem common to all enterprises in any sector of a given economy. For example, inventories must be maintained in agriculture, industry, retail establishments and the military.

There are many reasons why organizations should maintain inventories of goods. The fundamental reason for doing so is that it is either physically impossible or economically unsound to have goods arrive in a given system precisely when demands for them occur. Without inventories customers would have to wait until their orders were filled from a source or were manufactured. In general customers will not or cannot be allowed to wait for long period of time. For this reason alone the carrying of inventories is necessary to almost all organizations that supply physical goods to customers. Also there are some other reasons for holding inventories. The price of materials can be changed by vendors according to seasonal demand. For this reason manufacturers want to carry inventories because of not to purchase the materials from high level. Also vendors may force manufacturers to purchase the raw material because of theirs production plan. The manufacturer's replenishment policies may force themselves to purchase extra material. In a joint replenishment policy, manufacturers want to collect all order to reduce transportation cost if transportation cost is higher than stock holding cost.

There are many questions that must be answered in controlling the inventory of any physical good. These questions are

- When to replenish the inventory?
- How much to order for replenishment?
- What should be the safety stock level if there is fluctuated demand movement?
- Is there any possibility to bind different orders to reduce transportation cost?
- What is the transportation type in considering payment options (CIF-FOB freight etc.)?
- What is the importance of demand forecasting studies to reduce stock holding cost?
- Classification inventory (ABC analysis – AHP method)
- Determination of due-date success and quality rate of vendors related historical data
- Minimizing lot size of orders by using equivalent materials in considering price, weight, etc.

"*To find correct answers to these questions mathematical analysis can be used to develop operation rules for controlling inventory systems. When mathematics is used for the solution of inventory problems, it is necessary to describe the system mathematically the system. Such a description is defined as mathematical model. Generally, it is never possible to represent the real world with the complete accuracy. Certain approximation and simplifications must be made when constructing a mathematical model. The main reason is that it is impossible to find out what the real world is really like. Conversely, very accurate model of the real world can become impossibly difficult to work with mathematically*." (Eliezer Naddor,1966)

2.9 Types of Inventories

When we consider inventories in the context of manufacturing and distributions, there is a natural classification scheme suggested by the value added from manufacturing or processing.

1. Raw Materials: There are the resources required in the production or processing activity of the firm.

2. Components: Components correspond to items that have not yet reached completion in the production process. Components are sometimes referred to as subassemblies.

3. Work-in-Process: WIP is an inventory either waiting in the system for processing or being processed. WIP inventories include component inventories and may include some raw materials inventories as well. The level of WIP inventories is often used as a measure of the efficiency of a production scheduling system.

4. Finished Goods: These are also known as end items, the final products of the production process. (Steven Nahmias,2008)

2.10 The Cost of Inventories

Procurement cost is the amount which must be paid to the source from which the procurement is made. The sum paid to this source simply represents the cost of the units procured.(EliezerNaddor,1966) Procurement cost can be splatted to two parts: purchasing and setup cost. The other possible costs are stock holding cost, transportation cost, loss of interest rate cost and stock out cost.

2.10.1 Purchasing and Setup Cost

- Purchasing Cost:

The costs incurred by the inventory system itself in placing an order can be divided in two classes. Those which depend on the quantity ordered and those which are independent of the quantity ordered. If there is quantity discount namely unit purchasing cost is depended the quantity ordered, average unit cost can be found as C(Q)/Q. On the other hand calculation is easier if unit purchasing cost independent on quantity order, the cost of the Q units is $C(Q) = cQ$

- Setup Cost:

These include paper and postage costs, labor costs, perhaps the cost of a telephone call to the source, or the cost of computer time needed to make any necessary computations or to update accounting records.

Also there are the costs of processing an order through the purchasing and accounting department. Additionally companies should pay transportation costs. But this will be explained in the coming section widely. If there is no quantity discount, procurement cost can be defined as function

c = unit purchasing cost ($ / unit)

K = postage cost + labor cost + telephone cost + computer time + salary of personal (Addition of these costs can be fixed - $)

Q = Quantity which is procured from source

$G(Q) = K + c * Q$

This is same cost expression which already defined in economic order quantity model.

2.10.2 Stock Holding Cost

The holding cost, also known as the carrying cost or the inventory cost, is the sum of all costs that are proportional to the amount of inventory physically on hand at any point in time. The components of the holding cost include a variety of seemingly unrelated items. Some of these are

- Cost of providing the physical space to store the items
- Taxes and insurance
- Breakage, spoilage, deterioration and obsolescence
- Opportunity cost of alternative investment

Inventory and cash are in some sense equivalent. Capital must be invested to either purchase inventory, and decreasing inventory levels results in increased capital. This capital could be invested by the company either internally, in its own operation, or externally.(StevenNahmias,2008)

In such an idea, companies must earn higher rates of return on their investments than do individuals in order to remain profitable.

2.10.3 Transportation Cost

Inventory system should pay the transportation costs, too. EOQ model does not contain transportation cost but it has an important effect on determination order quantity. Transportation costs depend on the mode of transportation mode of transportation used.

Some modes of transport

- Airways
- Seaways
- Highway
- Railways
- Pipelines

It is possible to spread transportation cost in more details. Companies should pay a fixed cost independent of weight or volume of raw materials. Insurance cost, cost of instructions to a shipper concerning the taking on and delivery of a consignment, repository cost, insurance cost are the other costs which should be evaluated when operating an order.

2.10.4 Loss of Interest Rate Cost

This type of cost is very important for companies especially if companies work with export and import operations. According to payment options there would be a loss of interest rate cost and faced with operational risk factors. The mostly used international payment options are

- ✓ Cash in Advance
- ✓ Cash against Goods
- ✓ Cash against Documents

❖ **Cash-in-Advance**

With the cash-in-advance payment method, the exporter can avoid credit risk or the risk of non-payment since payment is received prior to the transfer of ownership of the goods. Foreign buyers are often concerned that the goods may not be sent if payment is made in advance. Exporters who insist on cash-in advance as their sole method of doing business may loss out to competitors who are willing to offer more attractive payment terms. In such a payment option there will be a loss of interest rate as much as the time which is required for seaway, airway or highway shipment. But on the other hand it eliminates risk of non-payment.

For example:

A company which produces electrical devices purchases couple of components from a vendor. The location of vendor is in China. Company selected seaway shipment and cash-in advance payment. Seaway shipment takes 30 days to Turkey. If company purchases 1.000.000 $ components in total, it means that they will lose the money of interest rate.

R = interest rate (% 0,125)

C*Q = 1.000.000 $

Loss of interest rate cost = 1.000.000 * 0, 00125 = 1.250 $

There would be 1250$ cost but exporter is exposed to virtually no risk as the burden of risk is placed nearly completely on the importer. In FOB (Free-on-Board) consignment cash-in-advance payment is used because after consignment risk belongs to importer.

- **Cash against Goods**

In this payment option, companies pay after freight is realized. That means there would be no risk and no loss of interest rate cost. This option is very advantageous for importers and too risky for exporters. Although there would be no interest rate cost, companies choices this option if importer and exporter works for the first time and there is no confidence. In CIF (Cost, Insurance and Freight) consignment this payment is used because money is paid after goods are received. Commonly the terms CIF and FOB are used seaway shipment.

2.10.5 Stock-out Cost

Stock-out cost is the cost of not having sufficient stock on hand to satisfy a demand when it occurs. This cost has a different interpretation depending on whether excess demand is back-ordered or lost sales. In the back-order case it includes whatever bookkeeping and delay costs might be involved. In the lost sales case it includes the lost profit that would have been made from sale. In either case it would also include the loss-of-goodwill cost, which is measure of customer satisfaction. In reality it is very difficult to calculate and estimate loss-of-goodwill cost.

2.11 Lead Time and Safety Stock

“*Lead time is the period between a customer’s order and delivery of the final product. A small order of a pre-existing item may only have a few hours lead time, but a larger order of custom-made parts may have a lead time of weeks, months or even longer*.” (www.wisegeek.com).

Generally lead time is taken constant and independent from order quantity in ordering systems but in real life this does not seem accurate. It fluctuates depending on seasonal factors, order quantity, location etc. But for mathematical applications it is possible to assume fixed lead time or to calculate a function of lead time. Below figure can help us to investigate lead time in details.

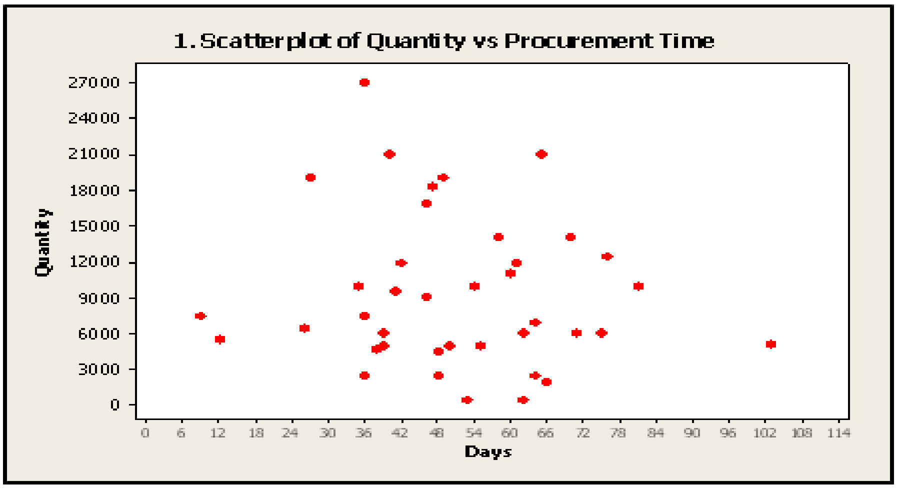

Figure 2.2 Scatter plot of Q and procurement time of “YTM882W”

A component which is coded YTM882W is bought from China and the figure shows the relationship between quantity and procurement time for a sample which is collected by different orders. In fact we think that there is a linear relationship between them. But it does not seem like it is desired. Because there are many possibilities

1. Vendor can supply the orders from its stock.
2. Vendor should produce the materials which is ordered if it has zero stock
3. Order can be supplied a part of from stock + other part of production

These possibilities may change in every order. Now the question is

Is it possible to determine a relationship between quantity and lead time?

The first option is to calculate a linear regression between quantity and procurement time, on the other hand regression should have a significant correlation. But it does not seem like that.

Second option is to work on graph one step more. In Figure 2.2extreme points which are seem exceptional is eliminated. It is understood that left exceptional points shows that vendor supplies order from stock because order is procured in a few days. Right points showed that order is supplied with production of vendor or vendor delayed due-date of orders. In Figure 2.3 vendor supplies the orders between 35 and 75 days. Now it is possible to assume that there would be an average lead time in respect of quantity.

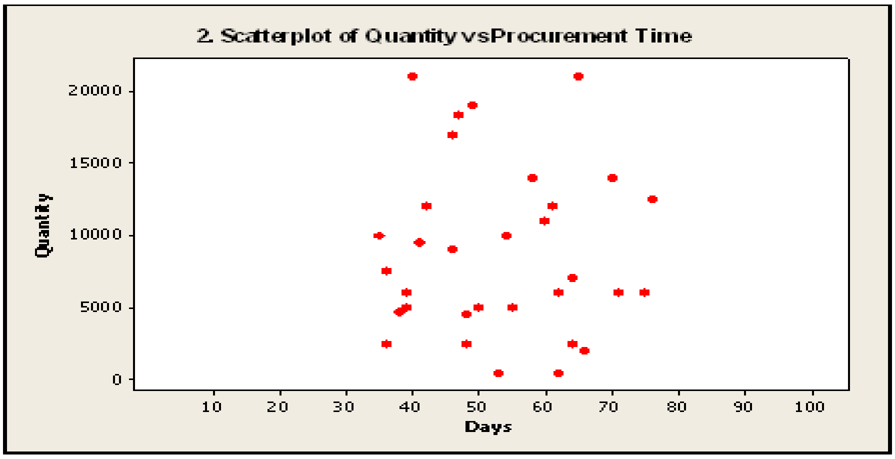

Figure 2.3 Scatter plot of Q and procurement time of "YTM882W" after elimination of exceptional observations

In fact lead time can be shorter for small orders but this fixed lead time can be useful when scheduling production and inventory.

Safety stock level is the second theme which must be investigated to have a successful inventory planning.

- **Safety Stock**

Theoretically, it should be easy to determine when to reorder a stocked item from a supplier. If it is known that customers will order ten pieces of the product each day and it means that it will take seven days to get the shipment from the vendor, the products should be reordered when there are seventy pieces on the self.

Table 2.5 The order point related to lead time

Demand Per Day	Lead Time	Usage During Lead Time
10 pieces	7 days	70 pieces

This quantity is called as order point. But the order point formula contains one more element: safety stock. "*Safety stock provides protection against running out of stock during the time it takes to replenish inventory*."(Jon Schreibfeder,1999)

Why is this protection necessary?

1. *Demand is a prediction based on past history, trend factors and/or known future usage of a product. The item's actual usage will probably be more or less than this quantity. Safety stock is needed for those occasions when actual usage exceeds forecasted demand. It is "insurance" to help ensure that you can fulfill customer requests for a product during the time necessary to replenish inventory.*
2. *The anticipated lead time is also a prediction, usually based on the lead times from the last several stock receipts. Sometimes the actual lead time will be greater than what was projected. Safety stock provides protection from stock outs when the time it takes to receive a replenishment shipment exceeds the projected lead time*.(Jon Schreibfeder,1999)

The following diagrams illustrate how safety stock is used:

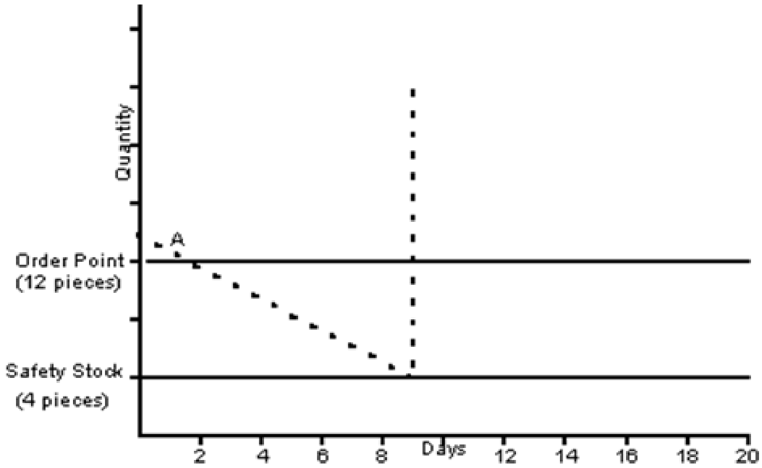

Figure 2.4 Plot of safety stock in days vs quantity

(Jon Schreibfeder,1999)

Projected Lead Time = 8 Days

Demand = 1piece/day

On Order with Vendor = 0

The dotted line in the graph represents the available quantity of item. A replenishment order is placed on the first of the month as the available quantity available reaches the order point. In this example, there is none of the product currently on the incoming replenishment orders. Therefore, at point "A" the item's available quantity equals its replenishment position

The actual usage of eight pieces during the lead time is consistent with projected demand. The shipment arrives on the 9th of the month. As the stock is receipt is processed, the available quantity on the shelf is equal to the safety stock quantity. The projection provided by the safety stock was not needed. (Jon Schreibfeder,1999)

The product again reaches the reorder point on the 11th of the following month. ("B" in the Figure 2.5)

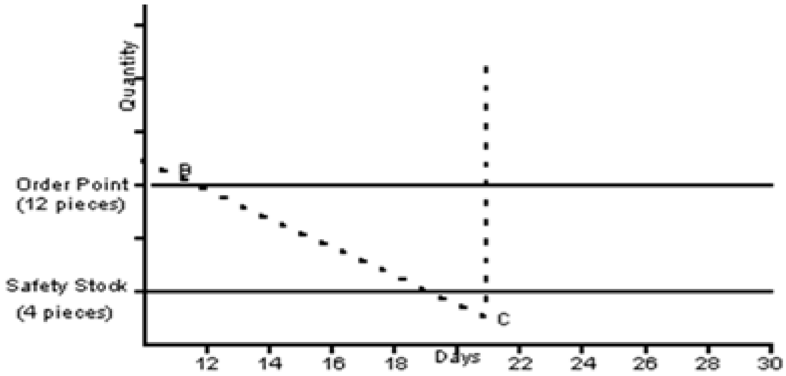

Figure 2.5 Plot of safety stock and reorder point

(Jon Schreibfeder,1999)

Projected Lead Time = 8 days and Demand = 1 piece/day

On Order with Vendor = 0

Another order is placed with the supplier. But the vendor is temporarily experiencing manufacturing problems and the shipment arrives two days late ("C" in the graph). If it were not for the safety stock, we would have run out of the product. Shortly after the shipment arrive, a customer orders 10 pieces of the product. You experience more than a week's usage in just one day. The available quantity falls to "D" in the following graph. A replenishment order is issued that day, but the available quantity is already below the order point.

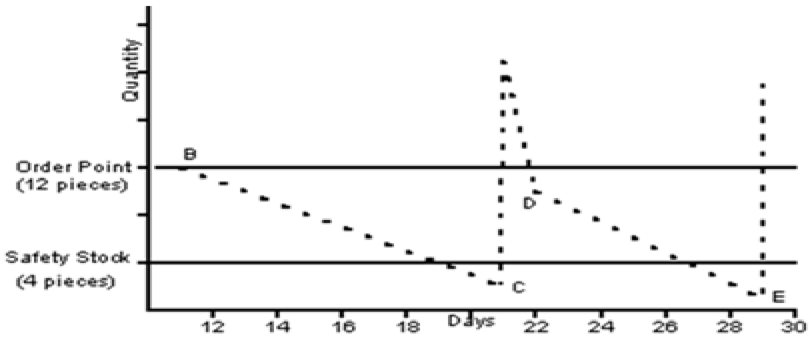

Figure 2.6 Plot of customer orders

(Jon Schreibfeder,1999)

The safety stock quantity allows the decision maker to satisfy customer demand for the product until the replenishment shipment arrives from the supplier on the 29th of the month. Again safety stock prevented a stock out.

- How much safety stock is needed?

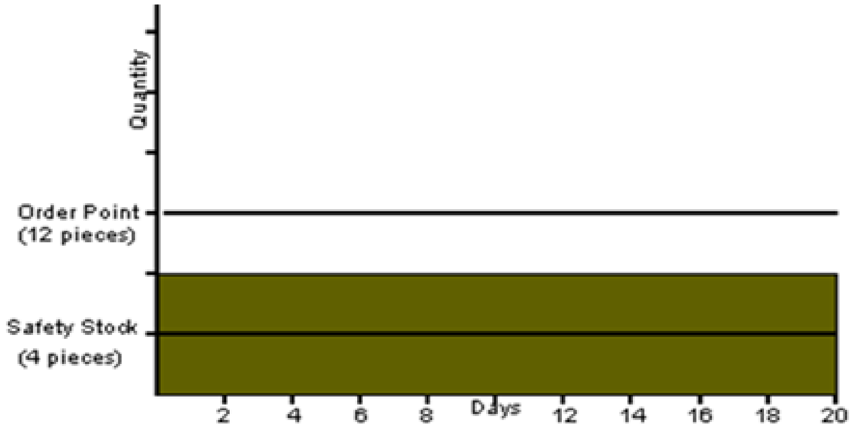

Figure 2.7 How much safety stock do we need?

(Jon Schreibfeder,1999)

When a replenishment shipment arrives, the available quantity is usually somewhere in the shaded area of the graph. Notice that the safety stock quantity is in the middle of the shaded area. Half of the time some or all of the safety stock will be used before the replenishment shipment arrives. The other 50% of the stock receipts will arrive before any of the safety stock is used. On average, the full safety stock quantity is always on the shelf when the replenishment shipment arrives. It is, on average, "non-moving" inventory. A distributor puts inventory in her warehouse to sell it to customers. Profits from these sales are necessary to pay the distributors expenses and provide a return on her investment. With this thought in mind, it seems as though it would not be a good idea for a distributor to intentionally have non-moving inventory in stock. On the other hand, keep in mind the goal of effective inventory management:

"Effective inventory management allows a distributor to meet or exceed his (or her) customers' expectations of product availability with the amount of each item that will maximize the distributor's net profit." "Safety stock is, in reality, an expense of doing business. But it is necessary to ensure good customer service. To maximize profits, we must carefully control all expenses, including safety stock. Therefore, we want to achieve our customer service goals with the least possible amount of safety stock."(Jon Schreibfeder,1999)

- A Better Way of Maintaining Safety Stock

Remember that the purpose of safety stock is to protect customer service from unusual customer demand during the lead time or delays in receiving a replenishment shipment. Why not base the amount of safety stock maintained for each item on the variations in demand and lead time? The greater the variation in demand and/or lead time, the more safety stock will be maintained for the item. This is referred to as the "average deviation method."We'll consider the variation or deviation in demand as the difference between the forecast demand of a product in a month and the actual usage in the past three months.

Table 2.6 Deviation between actual usage and forecast demand

	Forecast Demand	Actual Usage	Deviation
January	50	60	10
February	76	80	4
March	80	70	-10

The demand forecast was 50 pieces and actual usage was 60 pieces, resulting in a deviation or difference of 10 pieces. In February, the demand forecast was 76 pieces and actual usage was 80 pieces, which produced a deviation of four pieces. The average deviation is: (10 + 4) ÷ 2 = 7 pieces per month

The deviation for March in which demand exceed usage, is not considered in our calculation of safety stock. Because, if the prediction of what customers wants to exceed actual sales, we certainly do not want to add more safety stock to inventory. We probably have more than enough on the shelf already.(Jon Schreibfeder,1999). Next we have to calculate the average deviation of the products lead time. In calculation this amount, we'll just look at the last three stock receipts from the primary source of supply. A lot of things can occur over extended periods of time that will affect the lead time for an item. For example:

- The vendor can add or shut down production lines.
- Freight carriers can use different routes.
- The availability of the raw materials needed to make the product may change.

The three most recent stock receipts for the item along with the anticipated lead time for the product when the purchase order associated with the stock receipt was entered:

Table 2.7 Deviation between actual lead time and anticipated lead time

Date of Receipt	Anticipated Lead Time	Actual Lead Time	Deviation
June 15th	10 days	17 days	7 days
April 20th	8 days	13 days	5 days
February 2nd	8 days	6 days	2 days

As with our analysis of demand and usage, we will not consider any stock receipt whose actual lead time was less than its anticipated lead time – in other words, any time we received the item early. The average lead time deviation of the remaining two stock receipts is six days:

(7 + 5) ÷ 2 = 6 days

The six days is multiplied by the current anticipated demand per day to determine the anticipated usage during the six-day period. Demand per day is calculated by dividing the current monthly demand by the number of work days in the month. For example, say the current monthly demand is 90 pieces and the current month has 18 work days. The demand per day is five pieces; multiplied by the six-day deviation equals 30 pieces. The 30 pieces is added to the demand deviation to determine the total safety stock for the item:

30 + 7 = 37 pieces

As a final step in determining the safety stock quantity, we'll multiply the average deviation by a deviation multiple. The deviation multiple used is dependent on the customer service level we want to provide to our customers. Customer service level is defined as the percentage of line items for stocked products shipped complete by the promise date. The higher the multiple, the more safety stock we'll maintain, and the higher the customer service level. Please refer to our other articles for a complete discussion of the customer service level.

Generally we've found that the following multiples provide the corresponding level of customer service:

If the goal is a 95% customer service level, the average is multiplied deviation by a multiple of two (37 x 2 = 74 pieces). Using a higher deviation increases the amount of non-moving inventory on your shelf. In our example, the difference between a safety stock quantity derived using a deviation multiple of two or three is an additional 37 pieces.(Jon Schreibfeder,1999)

This is a more involved way of calculating safety stock than the conventional methods previously discussed. But it reflects the variations in market conditions, and therefore better predicts if a particular product needs more or less safety stock.

2.12 Inventory Control Policies

Inventory control policies are based on two models. The first model is continuous review and second control policy is periodic review model.

An inventory control system can be designed so that the inventory position is monitored continuously. If the inventory level is low, an order should be triggered. In such a case the policy is called as continuous review model.

The second policy is an alternative policy of the continuous review model. Under periodic review model the inventory position is considered only at a certain given points in time. Namely the intervals between these reviews are constant.

There are two types of inventory control policy in general but both of them have disadvantages and advantages. In continuous review model, company works with low safety stock. In other words this policy will reduce the safety stock. When using a continues review system the inventory position when ordering should guard against demand during lead time L.(Sven Axsäter,2006)

Periodic review has advantages if the company orders different items. Although modern information technology has reduced the costs for inspections considerably, periodic review will also reduce the costs for the inventory control system. This is especially true for items with high demand. For items with low demand it does not cost much more to use continuous review. The advantages of continuous review are also usually larger for such item. In practice it is therefore common to use continues review for items with low demand, and periodic review for items with higher demand.

2.13 Ordering Policy

According to the inventory control policy companies should use an ordering system. The purpose of an ordering policy is to control production or distribution in such a way that supply is matched to demand, inventory levels are maintained within acceptable levels and capacity requirements are kept to a minimum. The two most common ordering policies in connection with inventory control are often denoted (R, Q) policy and (s, S) policy. Additionally order up to policy and base stock policy will be explained in this study.

2.13.1 (R,Q) Policy

When the inventory level is below or in the reorder point R, a batch quantity of size Q is ordered. Additionally inventory level is too low, more than one batch can be ordered to reach over R. If demand is continuous or one unit at a time it is always hit the reorder point exactly in case of continuous review.

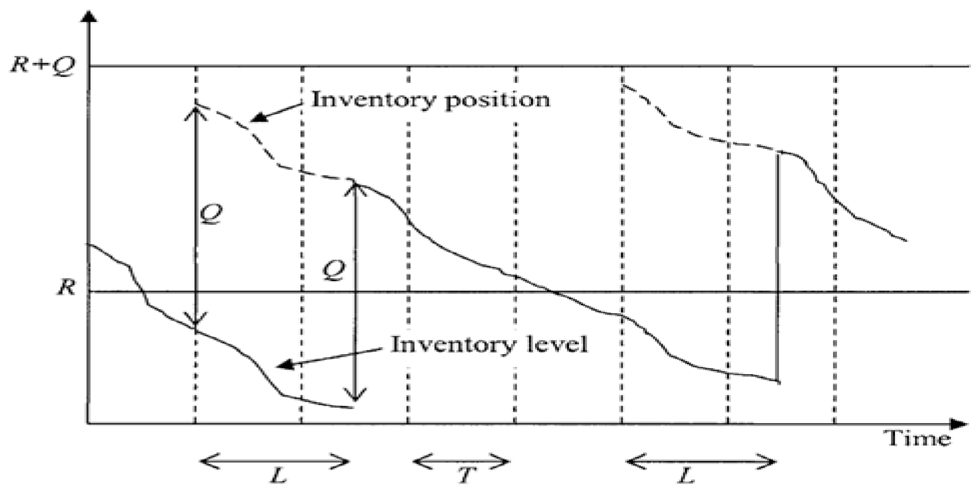

Figure 2.8 (R,Q) policy with periodic review. Continuous demand

(Sven Axsäter,2006)

In case of periodic review or if the triggering demand is for more than one unit, it will often occur that the inventory level is below R when ordering. In this case it would not be reached the position $R+Q$after ordering. Figure 2.8 illustrates this for a periodic review (R,Q) policy. Note that the development of the inventory level is completely independent of the lead time L.

Japanese ordering system kanban is very similar to R,Q policy. With a kanban policy there are N containers, each containing Q units of the item and with a card on the bottom. When a container becomes empty, the card which is called kanban is used as an order for Q units.

2.13.2 (s,S) Policy

This policy is similar to a R,Q policy. The reorder point is denoted by s. Maximum inventory level is denoted by S. When the inventory level drops below s, an order should be placed up to the maximum level S. In compare to R,Qpolicy multiples of a given batch is not be ordered. If it is always hit the reorder point exactly the two policies are equivalent provided $S=R$ and $S=R+Q$. But it is not always hit the reorder point exactly, the equivalence does not hold. Figure 2.9 illustrates such a situation with periodic review.

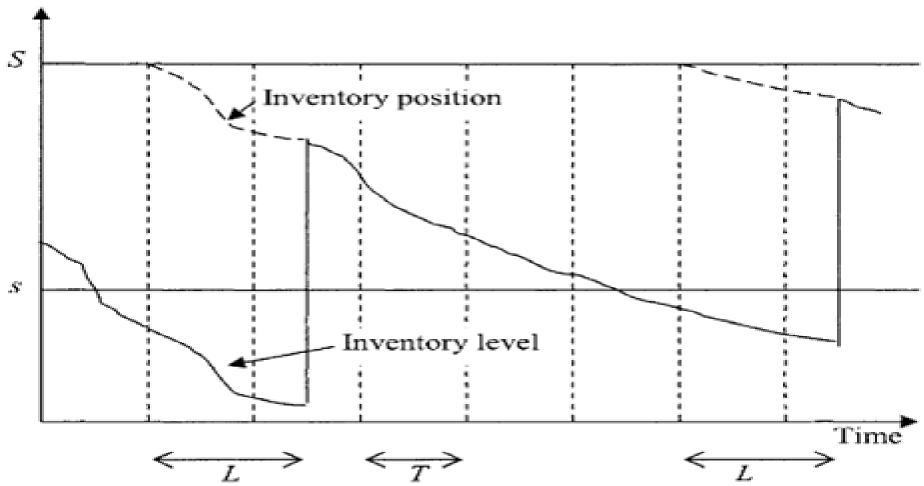

Figure 2.9 (s,S) policy with periodic review. Continuous demand

(Sven Axsäter,2006)

In case of periodic review, it may happen with either policy that we do not reach the reorder point at a certain inspection. In that case no order is triggered. (Sven Axsäter,2006)

2.13.3 Order up-to Policy

The order up to (OUT) policy is a standard ordering algorithm in many MRP systems that are used to achieve the customer service, inventory and capacity trade-off. This policy is often set by the company to coordinate orders for multiple items from the same supplier, where setup costs may be reasonably ignored. Conceptually, order up to policy is very easy to understand. Periodically, we review our inventory position and place an order to bring the inventory level “up-to” a defined level. (Frank Chen and Stephen Disney,2003)

2.13.4 Base Stock Policy

Under this policy company set the reorder level equal to the maximum inventory and orders are placed after each withdrawal from the inventory. Consequently, the sum of the amount of inventory on hand an amount is being ordered. The maximum inventory level is referred to as the base stock level. (CDI, Inventory Management,1998)

2.14 EOQ Inventory Model with Deterministic Demand

- ❖ Model Development

  The model based on the following assumptions.

1. Demand arrives continuously at a constant and known rate of λ units per year. (User can change the unit time – month, week, day) Arrival of demand at a continuous rate implies that the optimal order quantity may be non-integer.
2. Whenever an order is placed, a fixed cost K is incurred. Each unit of inventory costs $/ to stock per year per dollar invested in inventory. Therefore, if a unit's purchasing cost is c, it will cost r*c to stock one unit of that item for a year.
3. The order arrives τ years after the placement of the order. It is assumed that τ is deterministic and known.
4. Backorders are not allowed.
5. All the model parameters are unchanging over time.
6. The length of the planning horizon is infinite.
7. All the demand is satisfied on time.

The steps which creates the model:

Step1:

The cost expression should be created. Total demand per year is λ and the total purchasing cost for one year is c*λ.

Step 2:

The number of orders placed per year is equal to λ/Q. Therefore the total annual average cost of placing orders is K*λ/Q.

Step 3:

The holding cost should be included to cost function. Firstly average inventory per cycle would be calculated. Since each cycle is identical to any other cycle, the average inventory per year is the same as the average inventory per cycle. The holding cost is equal to the average inventory per year times the cost of holding one unit of inventory for one year.

Therefore, the cost function of classical economic order quantity model is

$$\min Z(Q)_{Q>0} = c * \lambda + \frac{K * \lambda}{Q} + \frac{Q * h}{2} \qquad 2.7$$

Related to this cost expression it can be paraphrased that the higher value of the fixed cost K, the fewer the number of orders that should be placed every year. This means that the quantity ordered per order will be high. Second if the holding cost rate is high, placing orders more frequently is economical since inventory will on average be lower. A higher frequency of order placement leads to lower amounts ordered per order. Therefore, our intuition tells us that the optimal order quantity should increase as the fixed ordering cost increases and decrease as the holding cost rate increases. (Amar Sapra and John Muckstadt,2008)

2.14.1 Optimal Quantity

To compute the optimal order quantity, we take the first derivate of $Z(Q)$Equation 2.20 with respect to Q and set it equal to zero:

$$\frac{dZ}{dQ} = 0 - \frac{K\lambda}{Q^2} + \frac{h}{2} = 0$$

or

$$Q^* = \sqrt{\frac{2 * K * \lambda}{h}} \qquad 2.8$$

Q^* is the optimal order quantity. The derivation of the purchasing cost c*λ is zero since it is independent of Q. (Amar Sapra and John Muckstadt,2008)

2.14.2 Reorder Point and Reorder Interval

In the economic order quantity model the demand rate and lead time are known with certainty. Therefore, an order is placed such that the inventory arrives exactly when it is needed. This means that if the inventory is going to be depleted at time t and lead time is τ, then an order should be placed at time $t - \tau$. If the order is placed before time $t - \tau$, then the order will arrive at time t. On the other hand, delaying the placement of an order so that it arrives after time t is not permissible since a backorder will occur.

There are two cases depending upon whether the lead time is less than or greater than the reorder interval. That is $\tau \leq T$ or $\tau \geq T$.

Since the on-hand inventory at time an order arrives is zero, the inventory at time $t-\tau$ should be equal to the total demand realized during the time interval $(t-\tau, t]$ which is equal to $\lambda*\tau$. Therefore the reorder point when $\tau \leq T$ is equal to

$r^* = \lambda*\tau$

Namely whenever the inventory drops to the level $\lambda*\tau$, an order must be placed. On the other hand when $\tau \geq T$, the reorder point is equal to

$r^* = \lambda*\tau_1$

where τ_1 is the remainder when τ is divided by T. that is , $\tau = m*T + \tau_1$, where m is a positive integer.

The time between the placements to two successive orders, T, is equal to the time between the receipts of two successive order deliveries, since the lead time is a known constant. Since order is received when the inventory level is zero, the quantity received. Therefore if the optimal reorder interval is denoted by T*, $Q^* = \lambda*T$. E.g., (Amar Sapra and John Muckstadt,2008)

Hence

$$T^* = \frac{Q^*}{\lambda} = \sqrt{\frac{2 * K}{\lambda * h}}$$

2.14.3 EOQ Model with Backorder Allowed

It is assumed that backorders are not allowed. Backorders can make sense in a deterministic model. Therefore it shall be allowed backorders and introduce a certain penalty cost proportional to waiting time for the customer.

B_1 = shortage penalty cost per unit and time unit.

The backorder penalty costs per time unit are $b_1$times the average number of backorders, since each unit backordered during one time unit means that some customer is waiting during that same time. The backorder cost considered has a structure that is very similar to the holding cost. The only difference is that the backorder cost is charged when the inventory level is negative.

The average number of backorders equals the average customer waiting time per unit of time, the average number of backorders divided by the demand d is the average waiting time per unit demanded. This relationship is valid in much more general stochastic settings. Similarly, the average inventory on hand divided by d is the average waiting time for a unit to be demanded. Allowing backorders means that some units are delivered in stock after they have been demanded. (Sven Axsäter,2006)

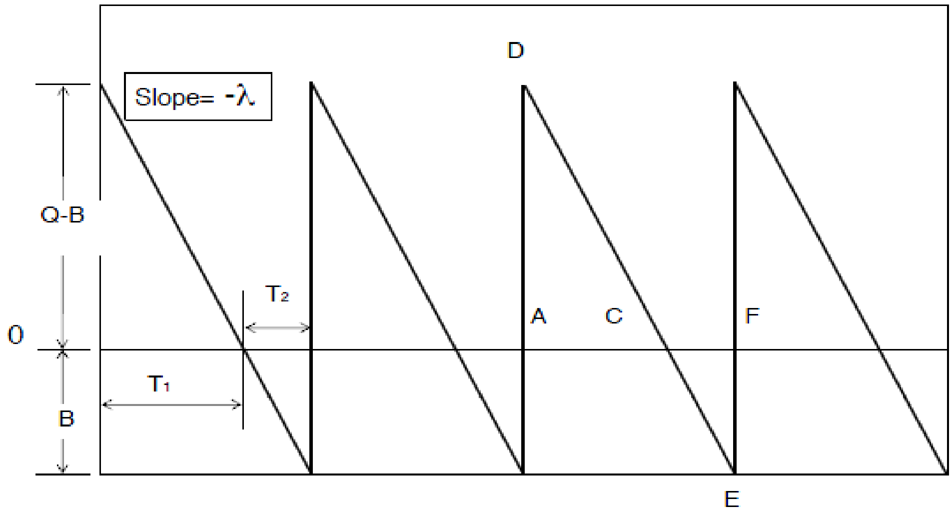

Figure 2.10 Change in inventory over time for the EOQ model

There are two decisions to be made.

1. How much to order whenever an order is placed?
2. How large the maximum backlog level should be in each cycle?

If the order quantity is Q, B is used as maximum amount of backlog allowed. When an order arrives, the entire backordered inventory satisfies demand in the first sub cycle. Since this on-hand inventory decreases at rate λ and becomes zero in T_1years. Second sub cycle, the number of backorders increases from 0 to B at rate λ over period of length T_2 years.

$$Q - B = \lambda * T_1$$

$$B = \lambda * T_2$$

$$Q = \lambda * (T_1 + T_2) = \lambda * T$$

The cost expression for the purchasing cost and annual fixed ordering cost remain the same as for the EOQ model. But holding cost is different. First the average inventory per cycle should be calculated and multiply the result by holding cost h to get the annual holding cost. The average inventory per cycle is equal to

$$\frac{Area\ of\ triangle\ ADC}{T} = \frac{\frac{(Q-B)*T_1}{2}}{T}\ substitute\ for\ T_1 and\ T$$

$$Average\ Inventory\ per\ Cycle = \frac{\frac{(Q-B)^2}{\lambda}}{2*\frac{Q}{\lambda}} = \frac{(Q-B)^2}{2*Q}$$

In the second step average number of backorders per year should be calculated. Π shows the cost of backordering a unit for one year. To get the average backorder cost per year, the average backorder quantities per year is multiplied by the backorder cost rate. The average number of backorders per cycle is equal to the area of triangle CEF and divided by the length of the cycle T:

$$\frac{Area\ of\ triangle\ CEF}{T} = \frac{\frac{B*T_2}{2}}{T} = \frac{\frac{B^2}{2*\lambda}}{\frac{Q}{\lambda}} = \frac{B^2}{2*Q}$$

$$substitute\ for\ T_2 = \frac{B}{\lambda} and\ T = \frac{Q}{\lambda}$$

All parts of the cost expression are completed and it can be combined the purchasing cost, fixed cost, holding cost and backordering cost.

$\pi =$ shortage penalty cost per unit and time unit

$$Z(B,Q) = c\lambda + \frac{K*\lambda}{Q} + h*\frac{(Q-B)^2}{2*Q} + \pi*\frac{B^2}{2*B}$$

If the fixed order cost K increases, fewer orders will be placed, which will increase the order quantity. An increase in the holding cost on the maximum possible number of units backordered should be as follows: the higher the backorder cost, the lower the maximum desirable number of backorders.

To obtain the optimal solution, the first partial derivatives of $Z(B,Q)$is taken in with respect to Q and B and set them equal to zero.

$$Q^* = \sqrt{\frac{2*K*\lambda}{h}} * \sqrt{\frac{h+\pi}{\pi}}$$

$$Q^* = Q_e\sqrt{\frac{h+\pi}{\pi}}$$

where Q_e is the optimal solution to the EOQ model

$$B^* = Q^* * \frac{h}{h+\pi}\sqrt{\frac{2*K*\lambda*h}{(h+\pi)*\pi}} \qquad 2.9$$

(Amar Sapra and John Muckstadt,2008)

2.14.4 EOQ Model with Quantity Discount

In real life it is known that unit price can be change related the ordered quantity. If it is purchased more than it is planned, unit purchase price can be lower. On the other hand seller can put an interval in which unit price is different. In such a case optimal order quantity will be different. In classical economic order quantity model it is assumed that there is no quantity discount. But in this section the assumption can be relaxed to see the details and to have more accurate model.

Imagine that

$c = price\ per\ unit\ for\ Q < Q_0 - the\ normal\ price$

$c' = price\ per\ unit\ for\ Q \geq Q_0 - where\ c' < c$

It is assumed that we get the whole order at the lower price if $Q \geq Q_0$. It is also possible to handle the case when there is a discount only for units above the breakpoint Q_0. (W.C Benton and SeungwookPark,1996)

The holding cost will depend on the unit cost and it is assumed that it has the following structure:

$h = h_0 + r * c \qquad for\ Q < Q_0$

$h' = h_0 + r * c' \qquad for\ Q \geq Q_0$

It assumed that the holding costs consist of two parts, a capital cost obtained by applying the interest rate r on the unit cost and other out-of-pocket holding costs h_0 independent of the price per unit. Purchasing costs depend on the order quantity they must be included in the objective function used to determine Q.

$$Z(Q) = c*\lambda + (h_0 + r*c)*\frac{Q}{2} + K*\frac{\lambda}{Q} \qquad for\ Q < Q_0$$

$$Z(Q) = c'*\lambda + (h_0 + r*c')*\frac{Q}{2} + K*\frac{\lambda}{Q} \qquad for\ Q \geq Q_0$$

1. First it is considered without the constraint $Q \geq Q_0$. It is obtained

$$Q' = \sqrt{\frac{2 * K * \lambda}{h_0 + r * c'}}$$

and

$$Z' = \sqrt{2 * K * \lambda * (h_0 + r * c')} + \lambda * c'$$

The first derivative of equation 2.27 is taken to find the optimal order quantity. The same procedure was implemented for equation 2.20. Because of that the steps of taking derivative are passed.

2. If $Q' < Q_0$ it is needed to optimize the cost function

$$Q'' = \sqrt{\frac{2 * K * \lambda}{h_0 + r * c}}$$

and

$$Z' = \sqrt{2 * K * \lambda * (h_0 + r * c)} + \lambda * c$$

Since c>c' we know that Q'' is smaller than Q' so we have Q''<Q'<Q_0. Clearly second step provide the lowest possible cost without a discount. Because of convexity of 1 and that Q' <Q_0 it is known that the lowest cost without a discount is

$$Z(Q_0) = \lambda * c' + \frac{Q_0}{2} * (h_0 + r * v') + \frac{\lambda}{Q_0} * K$$

(Amar Sapra and John Muckstadt,2008)

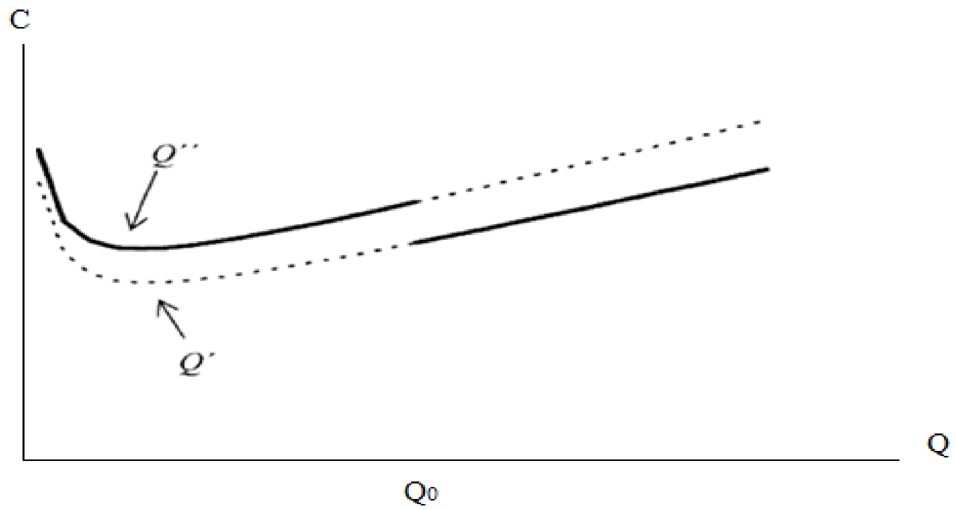

Figure 2.11 Costs of different values of Q

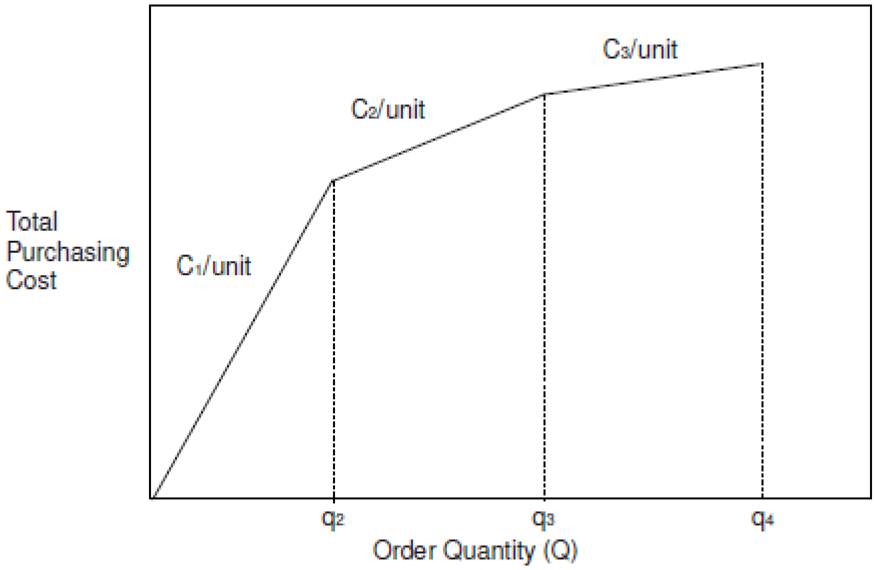

Figure 2.12 Total purchasing cost for incremental quantity discount

(Amar Sapra and John Muckstadt,2008)

2.14.5 Customer Based Service Level

An important function of logistic management is to maintain an appropriate level of safety stock. When insufficient safety stock is held, the firm fails to meet its customer service objective. Conversely inventory carrying costs are too high when excessive safety stock is maintained. (Walter Zinn and Howard Marmorstein,1990)

The base point is to specify the customer service level. The fill rate service level or customer service level measures the expected percentage of overall demand quantity. This fill rate determines the safety stock calculation. %100 success of service of the products to the customer causes high safety stock. It denotes the minimum demand satisfaction ratio that can be expected on average. This level is based on the overall quantity of customer demand that is met by stock on hand.

When demand patterns are high the normal distribution function is used to determine the needed safety stock. When these patterns are valid for the normal distribution safety stock level can be calculated as a known customer service level and lead time.

$$SS = Z * \sigma * \sqrt{L} \qquad 2.10$$

σ is the standard deviation of the demand population, L is the constant lead time of the material and z is the standard normal deviate and its value can be found on the basis of the service level requirement from standard normal distribution table.(Richard Tersine,1985)

2.15 Literature Survey about EOQ Model

Inventory systems in which the unit variable procurement cost depends on the quantity of an order are referred to as systems with quantity discounts. Hwang, Moon and Shinn studied in their article the optimal ordering quantity when all units' quantity discounts are available on the purchasing price and freight cost simultaneously. Their article is close to Lee's article but Lee formulated the classic EOQ model with setup cost and freight cost where freight cost has a quantity discount.

Development of the model:

It is possible to say that the assumption of Hwang's model same as the classic EOQ model except for ordering cost and purchasing price terms.

i. Demand rate is constant and continuous.
ii. No shortages are allowed
iii. All unit discounts are available on the purchasing price.
iv. The buyer pays the freight cost for the transportation of the quantity purchased where the freight cost has a quantity discount.

D : Annual demand rate

Q : Order Quantity

M_i : i^{th}price break quantity I = 1,2,3, ….., m

P_i : Unit purchasing price for Q, $M_{i-1} < Q \leq M_i$

N_j: j^{th}freight cost break quantity j = 1,2,3, ….., n

F_j : Freight cost for Q, $N_{j-1} < Q \leq N_j$

R : Inventory holding cost

A : Fixed ordering cost per each order

Related to these parameters they want to calculate total ordering cost. The sum ordering cost, average holding cost and purchasing cost will give the total ordering cost.

$$TC_{ij}(Q) = \frac{(A + F_i)D}{Q} + \frac{RP_iQ}{2} + P_iD$$

As EOQ model investigates the optimal ordering quantity, the problem now is to find the optimal ordering quantity Q* where 0 < Q which minimizes the$TC(Q)$.

$$Q_{ij} = \sqrt{2D(A + F_i)/RP_i}$$

The new model can be expressed with five different properties. Now some important observations will be presented regarding the properties of the optimal solution.

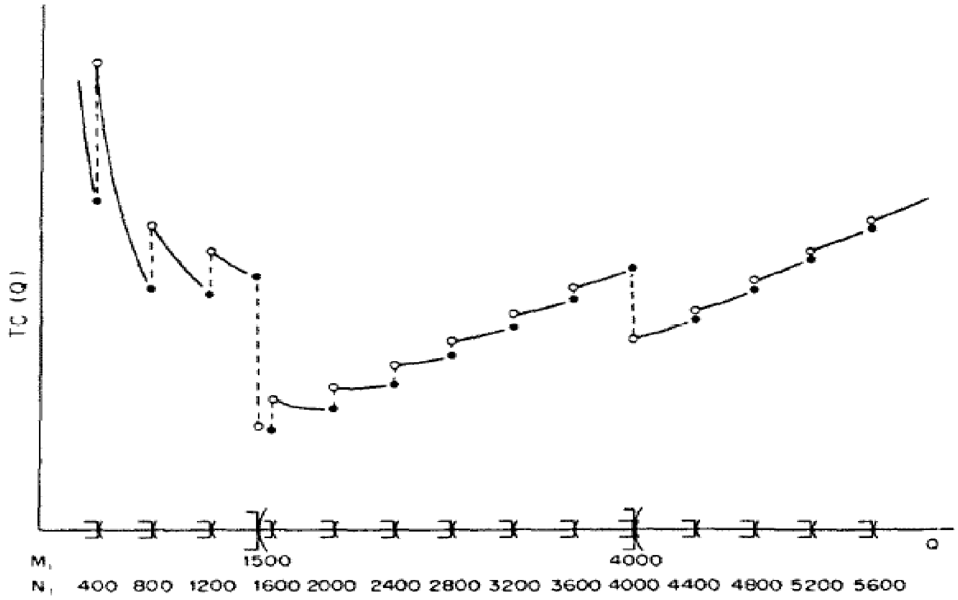

Figure 2.13 Total Cost TC(Q) versus Quantity Q

(Hark Hwang, Dug Hee Moon, SeongWhan Shinn,1990**)**

In this model the each closed interval ith price break quantity and jth freight cost break are applied to the model. And then Hwang proposes the lowest total cost as the optimal result. Hwang and Moon studied on a classical EOQ model for the case of quantity discounts when both all units price discounts and freight cost discount are available.

Qiu-Hong Zhao, Shou-Yang Wang, K-K Lai and Guo-Ping Xia studied on a model created an algorithm of an inventory problem with the consideration of transportation cost. According to the determination of Zhao and Wang's article transportation cost is calculated together with the production cost or with the ordering cost in the traditional economic order quantity. In real life transportation cost contains fixed costs and variable costs. The fixed costs are like parking fare or rewards to the driver. Variable costs depend on the consumption of oil or transportation type. So transportation cost is proportional to the quantity delivered and fixed costs. From this perspective they created and presented a modified economic ordering quantity model for a single supplier-retailer system in which the transportation costs is calculated based on the practical operation.

❖ Variables of the model

β the demand quantity per unit time

y ordering quantity of products

$\frac{y}{\beta}$ time periods

p capacity of the vehicle

f fixed cost of a vehicle

c variable transportation cost per trip

U working duration per day

t traveling time along each strip

K the cost of preparing an order

h unit inventory cost per unit time

s unit production cost

m number of vehicles used for delivering y

n total trips of these vehicles

b represents the maximum trips each vehicle is able to complete in a working day

$TCU_0(y)$ total cost per unit time

Related to these expressions they presented the model as follows:

$$Minimize\ TCU_0 = \frac{K}{y/\beta} + \frac{s*y}{y/\beta} + \frac{n*c}{y/\beta} + \frac{m*f}{y/\beta} + \frac{h}{2}$$

$$subject\ to\ (n-1)p < y \leq np$$

$$b \leq U/t$$

$$md \geq n$$

$$m, n, b\ are\ integers$$

First constraint specifies the number of trips each vehicle for delivering quantity y. Second and third constraints are about maximum trips each vehicle. Zhao modifies the model again after m = g(n);

$$Minimize\ TCU_0 = \frac{K}{y/\beta} + \frac{s*y}{y/\beta} + \frac{n*c}{y/\beta} + \frac{f*g(n)}{y/\beta} + \frac{h}{2} \qquad 2.11$$

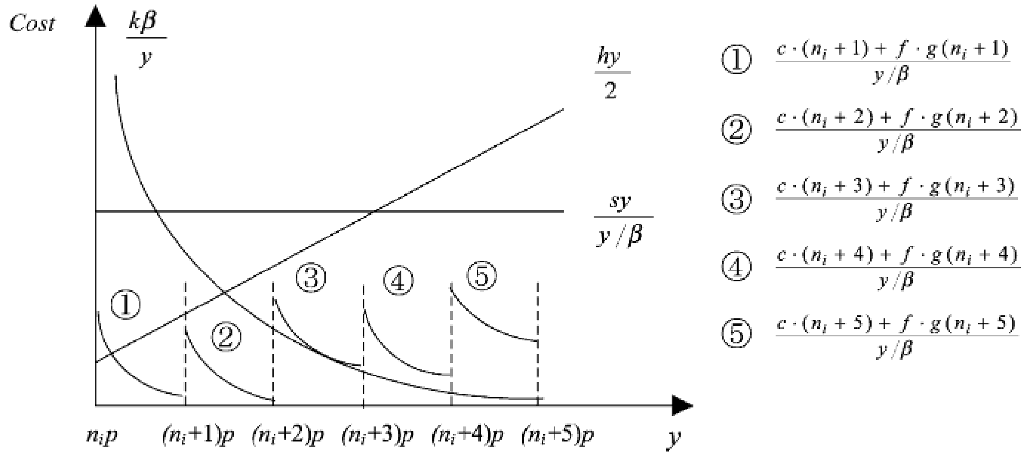

Figure 2.14 The relationship of each item in TCU with y

(Qiu-Hong Zhao, Shou-Yang Wang, K-K Lai, Guo-Ping Xia,2004**)**

In Figure 2.14it is seen that TCU is not continuous function, it cannot be differentiated during the whole interval.

Zhao takes the first derivative of TCU because of it is obvious that when y > 0 function $TCU_n(y)$ is continuous.

Optimal order quantity is

$$y_n^* = \sqrt{\frac{2*\beta*[K + c*n + f*g(n)]}{h}} \qquad 2.12$$

Qiu-Hong Zhao, Shou-Yang Wang, K-K Lai and Guo-Ping Xia tried to address a new approach of inventory model which integrates the inventory with transportation cost. Related to that they added multiple use of vehicles, number of trips and other transportation values to the classical EOQ model.

Chapter 3

3. System Identification and Problem Definition

3.1 General Information about Company X

Company X is a leading company which manufactures electronic devices like LCD TVs and monitors in Turkey. Also they supply and manufacture these devices for other companies which do not have manufacturing plant in this region. The company purchases the components which are used in assembly line, from Far East countries and Europe region of the world. Company designs the electronic devices and its boards and manufactures other plastic parts of the device in their factory. And these parts are assembled in assembly lines to finish the product.

3.2 The Planning of the Material Requirements

The engineers who work in production planning department stated that, the company X does not have a systematic and cost effective material requirement plan. Each month MRP is launched and necessary materials are shown in the system like in Table 3.1. So engineers can see the production capacity and related material requirements for the next months. But the real requirements and the output of MRP does not overlap because of different reasons.

Table 3.8 Material requirements planning

Material	Stock	Sas	on Road	March	April	May	June	July	Agust	September	October
VDC910R	10,575	23,750	1,500	11,546	11,380	7,025	3,567	0	0	0	0
VEV910R	20,098	0	0	4,945	5,639	6,824	2,529	2,640	0	3,495	3,495
VHR910R	105	130	0	0	100	797	3,508	0	0	5,985	5,985
XTS910R	4	3,145	0	6	200	906	1,361	0	0	3,806	3,806
YLT910R	26,144	24,250	7,100	13,723	22,781	14,171	10,356	4,559	554	14,523	14,523
YSK910R	6,549	14,344	0	4,587	8,211	5,356	3,602	2,268	0	5,596	5,596
YTA910R	24,447	49,000	24,000	15,381	27,089	28,992	20,267	15,334	6,170	25,810	25,810

Imagine that the current date is February and MRP is launched for the next months. The above dataset show a part of program output. Each component is coded in the system like VDC910R. These components are important parts which are necessary for mounting on the printed circuit boards. The future demand of component VCD910R can be observed in the Table 3.1. In March 11.564 items of VCD910R are needed. On the other hand 10.575 items are in stock. Also 1.500 items are on road and they are transported to company X plant. Sas means 23.750 items are ordered for the next months.

Company X wants to get orders approximately on the third week of the month. Because 1 week is optimal time to prepare the material for the next month. On the other hand there will be a buffer time to change the production plan if the material does not accepted by quality department. But this detail can be assumed absent when the model is developed. They desire to work with low stock level in order to reduce the inventory holding cost. The ordered components will be used in the coming months. It is observed that while the company X orders materials they do consider stock level and seasonality. The materials are ordered in each month just enough for the coming month.

The firm wants to optimize the stock movements and order cycle to reduce transportation cost, holding cost and they desire to get required stock quantity for the assembly line. So the main problem is to find the freight frequency and order quantity with a minimum cost for each component and supplier. The high freight frequency shows us that company X does not have a good demand forecast. Because the better demand plan means the low frequency.

For instance:

February MRP for May is 3615 pieces.

March MRP for May is 9477 pieces

As it is stated above this month is February. In MRP output it is seen the 3615 items are need for May, for VCD910R. They order this quantity and the components arrive in May. The next month March MRP results show the engineer the requirement quantity is increased and it is now 9.477 items. They need 5862 more items from this component. They order again. Maybe in April this quantity will be higher and they should order again. Now the possibilities are the each due-date can be different and the each transportation type can be different. The first freight can be seaway; second freight can be airway transportation option. It depends on time related to transportation duration. And they should work on import operation for each order. On the other hand there would be freight cost for each order size.

The desired solution is to reduce the freight frequency and to order material in optimal quantity. The advantages of this operation is

- The lower freight cost
    - The less freight means less money is given for import

- The lower operation time for import works
    - In production planning department there are many engineers who works and follows the import operation. Systematic material requirements planning gives chance to engineers to work for other studies in their department.

- The lower transportation cost
    - Each ordered part can cause a different transportation cost. Suppose that company X purchases a type component from a country and there is only airway option to get the material. Different due-dates namely orders will be cause extra cost. Solution is that they should joint order with other component orders or all same component order should be joint.

- The lower cost with joint order operations (This will decrease the complexity due-date determination)
    - If freight frequency will be higher engineers try to combine different type of orders to reduce the cost. This would cause complexity because suppliers have own work system and they couldn’t accept this option.

- Efficient stock management
    - A better material requirement plan would create more efficient stock.

3.3Transportation and Purchasing Process of the Company

As it is mentioned in the previous part, the necessity quantity of the material which is important to satisfy the demand is determined. But the problem is that the materials have long lead time. Because of the long lead time of the requirement of the parts the engineers cannot wait until the real demand of the products to be occurred. The first option is to send the demand forecast of the material to vendors. In other words they send beforehand the estimated quantity which will be purchased from suppliers. So suppliers can plan their production related to demand forecast. But it does not mean that the quantity have to be purchased. Second option is to wait till demands are certain. This time the demand can be certain but suppliers are not prepared to supply the demanded quantity on time.

Company X uses second option if the material has not got long lead time and is not variant. But commonly the first option is used. In first case they make the demand forecast and secondly these forecast are sent to the vendors. So suppliers have chance to prepare and produce the materials which are expected to be sold, for the coming periods.

Another method which is used by the company is splitting the required material to different transportation modes because of varying transportation cost. When using different transportation mode they use the second option and they send the demand forecasts to the vendors.

The desired model is shown in Figure 3.1. Imagine that the company is working on the material RTE104W. The lead time of the material is 30 days independent of ordered quantity. That means you can place order in any amount of the material but you should place the order 30 days in advance.

August	September	October	November	December
Demand Forecast	Manufacturing Time	Transportation Time of Seaway		50,000

Figure 3.15 Purchasing Process of Company

The plant of the vendor is in China. If company X selects seaway transportation, 60 days are needed as transit time.

In summary in August company X should determine the amount of order which is needed for December. To figure out the demand in December they use demand forecasting methods and the historical demand data are analyzed. So imagine that they need 50.000 units for RTE104W. This study can continue in all August working time. But at the end of the August the order should be sent to the vendor. In September vendor manufactures the materials. At the end of September the manufacturing of the materials are completed and loaded to ship for transportation. Two months October and November are needed for seaway freight transportation and In December the materials are received by company X and they begin to use in assembly line or rework line. For this procedure the main point is to have a good demand forecast accuracy because company X wish to make only one freight for the needed material. As it is mentioned earlier company X does not have a good ordering system. The system they use is depicted in Figure 3.2.

Period			October			November			
	August	September	1...	10...	20...	1...	10...	20...	December
1	Demand Forecast	Manufacturing Time	Transportation Time of Seaway						35,000
2	Demand Forecast	Demand Forecast			Manufacturing Time			Transportation Time of Highway	45,000
2	Demand Forecast	Demand Forecast				Manufacturing Time			50,000

Figure 3.16 Purchasing Process with Low Forecasting Accuracy

As it is stated in August company X should forecast the needed quantity of material RTE104W. Let's analyze the above table. In the first period company X forecasts the demand of December. According to the forecast suppose they found 35.000 unit and they place the order. Seaway transportation is always the cheapest transportation mode in comparison to highway and airway. But in September company X revise the forecast and the new demand quantity of December is now 45.000. At this time vendor can again produce the extra order of 10.000 but there is no time to use seaway transportation mode. The company X is obligated to use alternative transportation modes. Suppose that they select highway transportation for extra 10.000 units. This was the second freight. Until December company X still continue to control and revise the demand forecast of December. Imagine again that at the last days of October the company revised the forecast and found out that they need 50.000 units in December. They will get 35.000 by seaway freight, additionally 10.000 units by highway transportation. There is still 5.000 units gap. The vendor accepts the orders absolutely 30 days ago in advance. The vendor can supply but for transportation time there is not enough time to use seaway or highway modes. The weight of the material does not matter and company X should use airway freight if they want use extra 5.000 units. This is the third freight.

The company should pay extra and extra freight cost if they do not have a good forecast. Extra freight cost does not mean only the payment for transportation. They should pay extra cost for importation cost and fixed cost.

3.4The Analysis of Transportation Time

Another important point is the transportation modes and their transportation duration of these modes. These features are very important in calculating reorder point quantity and have larges effect on total cost. In global world different materials, cars, foods, especially oil are transferred from any point in world to any other point. As everybody knows the cheapest transportation mode is the seaway shipment. Then comes the highway transportation and the most expensive transportation mode is airway transportation. But total cost depends also on the weight of the material and transportation duration of the material. Imagine that seaway shipment is from Indonesia to Turkey is 90 days. Total purchasing cost 1.000.000$ and the total weight of the material is not heavy. If the payment is done cash to the vendor that means company X should loses a lot of interest rate cost because of the long transportation time. But airway transportation time is 5 days. In cash payment, company X loses only the loss of interest rate cost of these 5 days. Now the question is

- What is the optimal order quantity?
- How can I compare total cost of different transportation modes?

Seaway shipment is cheaper than airway freight. But on the other hand seaway shipment needs 90 days of transportation time rather than 5 days of transportation time of airway freight. In seaway freight company X loses extra 85 days of interest rate cost. Additionally if the material is not heavy, is the airway freight being cheaper under this criterion? Another question is, if there is no high cost difference between seaway shipment and airway shipment company X may select airway freight. Although by accepting the airway shipment the company loses money. They may gain 85 days stock holding cost. For the system to run better the system has,

- to have better demand forecast or certain orders
- to have better production plan
- to prepare production
- to use purchasing price for more effective operation in 85 days

These studies should be made in order to have optimal order quantity and to choose optimal transportation mode.

Company X uses three different transportation modes. In this study all transportation alternatives and transportation duration are analyzed to have standard transportation time. For instance the material YSK910R is purchased from GOBC Co which is located in China. All historical data are analyzed and it is observed that two different transportation modes are used in the past. The Figure 3.3 shows the mean transportation time of seaway from China to Turkey is 29.94 days. Standard deviation is 2.68. Material requirement planners may think that 2.68 days are not important because the reason could be that 2.68 days deviation may occurred due to holidays or low export operations. They think that it could be accepted the duration of seaway shipment is 30 days.

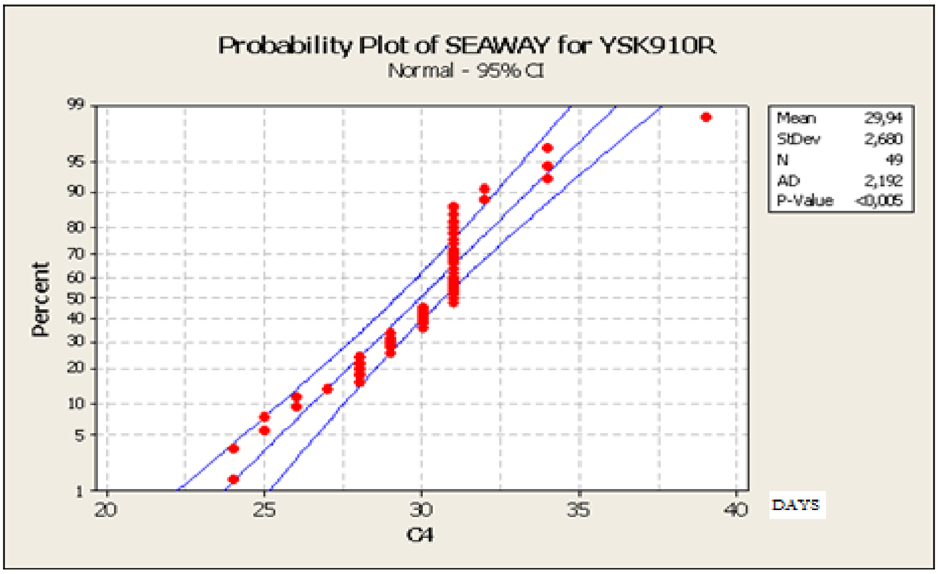

Figure 3.17 Probability Plot of Seaway Transportation of YSK910R

probability of airway freight is also investigated. The historical data is used again to plot this graph. The mean of airway freight is 1.5 days. The standard deviation is 0.94. Before loading the materials logistic firms propose the duration as 2 days. From China to Turkey transportation duration can be taken as 3 days according to material planners of company X.

3.5 The Analysis of Lead Time

Commonly lead time is represented as the latency between the initiation and execution of a process. In manufacturing it is very difficult to determine real lead time of manufacturing a material, semi-finished goods or finished goods. Because sometimes companies supply the order from stock or sometimes half of the order is supplied from stock and the rest is manufactured until the delivery time varies. In such a case it is very difficult to calculate the lead time of unit material.

In real life it is not possible to calculate precisely the lead time of a unit material. For this reason vendors specify a special date. They use these time a buffer to produce the orders. Related to the product that is manufactured a time period is determined. For instance they specify a rule and enforce customer times they accept the orders 2 months or 2 months in advance and independent of order quantity. This is called as planning time window.

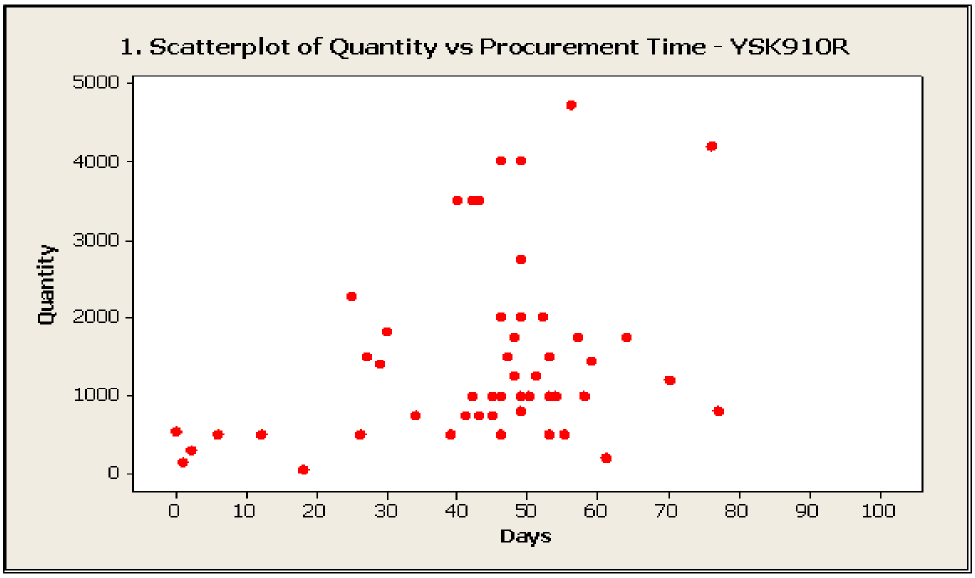

Figure 3.18 Scatter plot of Quantity vs. Procurement Time of YSK910R

The Figure 3.4 shows the procurement time of the material YSK910R which is used in assembly line of a manufacture company. This material is purchased from Korea. Y-axis shows the quantity and X-axis shows the time between placing the order and the order leave the plant of the vendor. Transportation time is not added to the lead time. Shortly this time is the manufacturing time of vendor.

The dotted points show the days in that how much quantity are supplied. This plot is not enough to calculate lead time of the material and a material planner cannot place order in considering of this plot. Dots are scattered and there is no correlation between them. For this reason material planners have difficulty to determine the lead time of the material. Then for a good planning the questions to be answered are

- When the order should be placed?
- What is reorder point?

If the details of a delivery are investigated it can be found out that some orders are supplied from stock of the vendor. For instance suppose approximately 500 units are supplied by the vendor in 3 days. It means that the vendor had the material in stock and they prepared the order in 3 days and sent to company X. But another time the same amount is prepared, in 26 days and some other time, in 50 days. There could be lots of reason for different procurement times.

- Vendor supplied the order from stock
- Vendor supplied the half order from stock , the rest are manufactured after received order
- Vendor manufactured total order after receiving the order
- Vendor did not begin immediately to produce the order. They determine a starting time of the production date related to their production plan

But company X should have standard planning time window to place the order independent order quantity.

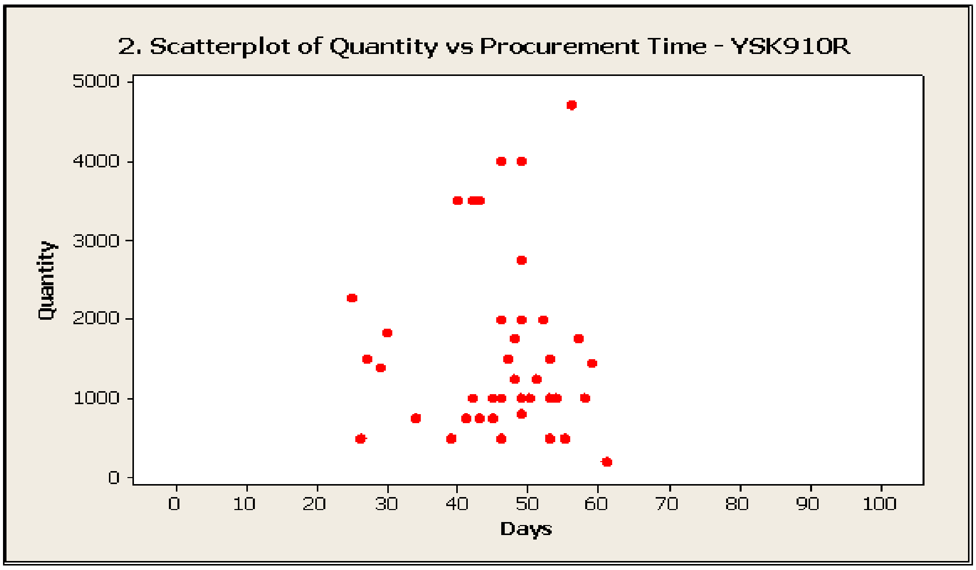

Figure 3.19 Scatter plot of Quantity vs. Procurement Time of YSK910R after some observations are eliminated

In Figure 3.6 some of special points are eliminated which represents supplying the order with ready stock or manufacturing. This graph is more meaningful than the graph given in Figure 3.5. It can be commented that manufacturing time of vendor is between 30-60 days independent of order quantity. If the lead time is analyzed in details it is seen that manufacturing time is actually between 40 and 55 days. The outliner points can be analyzed deeply why these points are away from the other points.

The best strategy is to identify all points. To find out a general lead time value depending on the observations of the sample this is shown in Figure 3.5 the moving average method to find average lead time of the material.

$n = total\ number\ of\ observations$

$q = the\ quantity\ which\ is\ ordered$

$d = the\ time\ between\ placing\ order\ and\ loaded\ to\ transportation\ vehicle$

$i = observation\ number$

$$Lead\ Time = \frac{\sum_{i=1}^{n} q_i * d_i}{\sum_{i=1}^{n} q_i} \qquad 3.1$$

After implementing the moving average method to the data it is got the as 48.3 day, as shown in Figure 3.6. The transportation time is not included to the lead time which is calculated in Equation 3.1.

In the previous section and this section of the report the main focus of the study is the transportation time and transportation modes and lead time which are identified and analyzed.

The time 48.3 days is deterministic for material YSK910R. But total lead time of the product is the total of manufacturing time of vendor and transportation time. Manufacturing time of vendor is accepted as deterministic and it is 48.3. But transportation time is variable depending on the transportation mode. That means there are three different lead times depending on the transportation mode.

These three different lead times are important when calculating reorder point. First it had to be specified optimal order quantity and transportation mode. Then reorder point can be calculated.

3.6 A New Ordering System

In the previous part of the thesis it is explained that how the material requirement planning of company X is made. Then transportation and purchasing process is identified. The manufacturing lead time of vendors are accepted deterministic. The different transportation times and modes are identified.

As it is mentioned before there are two payment options that are FOB and CIF. That means a cash payment before receiving the goods and the payment after receiving goods.

And there are three different transportation options which are airway, highway and seaway. That means there are six different combinations of transportation and payment. In this case classical economic order quantity model is not satisfactory for this case because in classical order quantity model lead time is zero and the quantity is received whenever it is ordered. Additionally there is a no criterion about payment option.

Company X wants to use a new ordering system with combining transportation time and transportation mode. Related to that economic order quantity model it can be used for the materials which have deterministic demand. Besides demand forecast should be used to work on deterministic demand. In order to find the optimum ordering system total cost of different transportation modes should be compared in each other by considering transportation time and weight of the material.

Chapter 4

4. A Different Approach: EOQ Model with Transportation Cost

4.1 Model Formulation

In general inventory models, costs of such issues are usually accounted according to the following assumptions: The production cost is proportional to the quantity of produced products. The ordering cost, which refers to the cost for preparation of purchasing materials, is independent of the quantity ordered. The inventory cost (shortage cost) is proportional to the quantity of products stored (out of stock) as well as the duration for which these items are stored (stock out).

When materials are delivered from the supplier to the customer, which in this case is the company X which manufactures LCD TVs transportation costs are incurred. In the traditional economic order quantity (EOQ) model, the transportation cost is included to the cost function together with the production cost, or with the ordering cost. However, in a real life logistic system, the transportation cost of a vehicle includes both of the fixed cost and the variable cost. The fixed cost, which is considered to be a constant sum is each period, refers to some necessary expenses such as parking fare and rewards to the driver. As to the variable cost, it depends mainly on the oil consumed; it is unreasonable to assume that the transportation cost is proportional to the quantity delivered or is a constant sum. For example, the optimal ordering quantity Q* gained according to the general EOQ formula may be partly loaded by the vehicles and the cost of the logistic system may not be the lowest.(Qio-Hong Zhao, Shou-Yang Wang, K.-K.Lai, Guo-Ping Xia.,2004)

Traditional economic order quantity model just refers an optimal order quantity without considering any different possibility of transportation. Transportation mode and transportation duration are not clear. Standard EOQ model accepts that whenever the inventory needs material, the goods are received and inventory level goes from zero to order quantity Q. But in real life there are lots of different possibilities.

Imagine that there are 2 different transportation modes which are airway and seaway. And there are two different payment options. First one is cash payment and other one is payment after receiving goods. That means there are four different options to supply the material. Traditional EOQ model does not cover these alternatives. In our model we could find a reasonable order quantity, cheapest transportation option and acceptable transportation time. But on the other hand we have chance to compare these results with other alternatives. In global business life it is not possible to measure worth of time. Seaway can be cheaper than airway option but seaway shipment duration would be ten times longer than airway transportation. If the total cost deviation is close between airway freight and seaway shipment, decision maker would chose airway in spite of losing money. On the other hand there is chance to compare of total cost between cash payment and payment after receiving. The main point of the model is to find the minimum total cost of ordering system with transportation while exploring the other possible results.

4.1.1 Components of the Model

Setup Cost

K = Setup Cost ($ / per order)

Purchasing Cost

λ = Demand rate (unit / month)

c = purchasing cost ($ / unit)

Q = order quantity (unit)

T = cycle length(Q / λ)(month)

h_1 = holding cost held per unit time ($ / month*unit)

Transportation Cost

w = weight of component(kg / unit)

φ = startup cost of transportation ($)

δ = fixed export-import cost ($)

ß = transportation cost of a logistic firm ($ / kg)

t = transportation lead time (month)

The ordering cost K ($), which refers to the charge for preparing of production, is independent of the quantity ordered. λ(unit/month) is denoted as total demand in a specific period. c is the unit purchasing cost of material. Ordered quantity is shown by Q. T (month) is the cycle length that how many times we place order in this period. Q/λ give us the cycle length. Holding cost is the most important part of the model. There are two types of holding cost. The first one is the h_1 holding cost per unit held per unit time. Average inventory level will give us the holding cost by multiplication per unit price. The second holding cost is the cost of loss interest rate in transportation time.

Seaway transportation needs approximately 30 days to procure the materials to Turkey from Far East countries. If the company works with FOB payment option, it means that the payment should be done 30 days ago in advance of receiving the goods. In such a situation account department of companies should include the loss of interest to the total purchasing cost. This cost is included as h_2($ / month*unit)in our model. W is the key component of the model. It is the weight of the material. Heavy materials will have an important role in shipment. Startup cost is denoted as φ ($). There are other costs like the cost of instructions to a shipper concerning the taking on and delivery of a consignment and customs cost. It is fixed and denoted as δ ($). Logistic firms charge transportation cost per unit weight and it is shown as ß ($/kg).

4.2 The Total Cost Function and Optimal Order Quantity

General EOQ model has two parts. First part is purchasing cost per cycle and second part is holding cost of average inventory.

Since the demand in a month is λ the purchasing cost is for a month is $c * \lambda$

The number of order placed per month is λ / Q

Therefore the monthly average cost of placing order is $K * \lambda / Q$

And the average inventory per cycle is

$$\frac{\frac{Q*T}{2}}{T} = \frac{Q}{2}$$

Since the average inventory per month is the same average inventory in a cycle, the holding cost is equal to the average inventory per month times the cost of holding one unit of inventory for one month

$$h_1 * \frac{Q}{2}$$

The total of purchasing, ordering and inventory holding cost per month is

$$G(Q) = c * \lambda + K * \frac{\lambda}{Q} + h_1 \frac{Q}{2} \qquad 4.1$$

In this model we try to combine transportation cost and freight mode to general economic order quantity model. There are different types of transportation like seaway, airway, highway, railway or pipelines. Each transportation type has own cost characteristics.

On the other hand there are two types of payment options which will be considered in the study. These are FOB (Free on Board) and CIF (Cost, Insurance and Freight).In FOB companies pay the goods before transportation or in CIF payment will be realized after receiving goods. In FOB there will be lead time which leads to holding cost and this has an important effect on cost.

Table 4.9 Transportation options

Transportation Type		
Seaway	Airway	Highway
CIF	CIF	CIF
FOB	FOB	FOB

The cost of the materials is evaluated sometimes considering weight of material or some other times considering the volume of the material. If the material has high volume although is not heavy, firm pays the transportation cost related to volume of component. In such a situation it is difficult to calculate for the transportation cost for each material, because there is no information the volume of materials. We just know the weight of components and we have information how much weight a container can carry. A 20 Dc container can carry between 15 and 40 ton.

To have a more accurate economic order quantity model these type of costs namely transportation cost should be added to the model. If order quantity has no effect on unit transportation cost, transportation cost function can be shown as

$w = unit\ weight\ or\ unit\ volume\ of\ raw\ material\ (kg/unit)$
$Q = ordered\ quantity\ (\ unit\ )$
$\varphi = fixed\ transportation\ cost\ (\ \$\ )$
$\beta = unit\ transportation\ cost\ of\ logistic\ firm\ (\$/kg)$
$\delta = fixed\ importation\ cost\ (\ \$\ )$
$T = cycle\ length\ \ (\ month\ or\ year)$

Transportation cost:

$$F(Q) = \frac{\beta * w * Q + \varphi + \delta}{T} \qquad 4.2$$

Logistic firms calculate cost of transportation depending on volume or weight of raw material.
Thus transportation cost is

$$F(Q) = ß * w * Q + \varphi + \delta$$

But this function should be divided by T to find the cost is occurred per cycle.

$$F(Q) = \frac{ß * w * Q + \varphi + \delta}{T} \qquad 4.3$$

On the other hand we must add the holding cost namely loss of interest rate cost which is coming transportation time. The more transportation time is required the more holding cost must be faced. In Figure 4.1, the gray area shows the transportation time of a seaway shipment. Additionally FOB payment causes transportation holding cost.

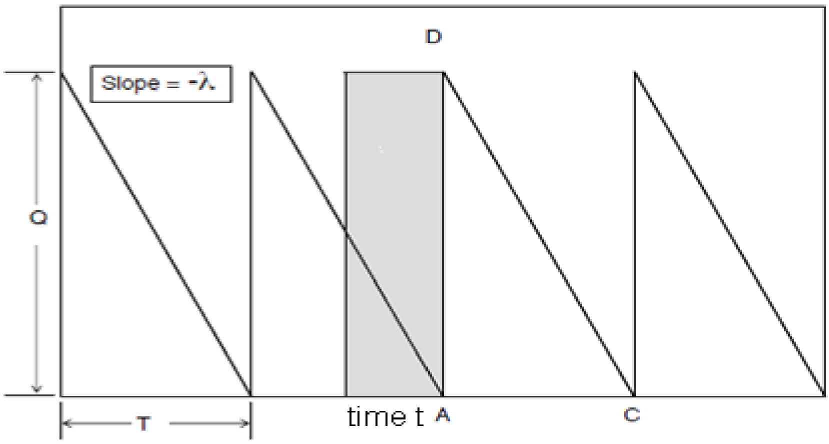

Figure 4.20 Average stock level to calculate stock holding cost

Since each cycle identical to any other cycle, the inventory per year/month/days is the same as the inventory per cycle.

$$\frac{Area\ of\ gray\ rectangle}{Length\ of\ the\ cycle} = \frac{Q * t}{T} = \frac{Q * t}{\frac{Q}{\lambda}} = t * \lambda$$

The transportation holding cost is equal to the average inventory per month times the cost of holding one unit of inventory for one month.

$$h_2 * \frac{Q * t}{T}$$

The transportation cost is $(ß * w * Q + \varphi + \delta)$for each mode.

The number of transportation should be the same as the number of placing orders per month λ / Q

If each transportation cost is multiplied with the number of transportations in a month the total cost of transportation is obtained

$$F(Q) = (ß * w * Q + \varphi + \delta) * \frac{\lambda}{Q} \qquad 4.4$$

To this cost the transportation time holding cost should be added. The total transportation cost becomes

$$F(Q) = (ß * w * Q + \varphi + \delta) * \frac{\lambda}{Q} + h_2 * \frac{Q * t}{T} \quad 4.5$$

Adding the purchasing, ordering, inventory holding cost G(Q) with the transportation cost, transportation holding cost F(Q) we find the total cost function as

$$TC(Q) = G(Q) + F(Q) \quad 4.6$$

$$TC(Q) = c * \lambda + K * \frac{\lambda}{Q} + h_1 \frac{Q}{2} + (ß * w * Q + \varphi + \delta) * \frac{\lambda}{Q} + h_2 * \frac{Q * t}{T}$$

Rewriting the function as

$$TC(Q) = c * \lambda + K * \frac{\lambda}{Q} + h_1 \frac{Q}{2} + (ß * w * Q) * \frac{\lambda}{Q} + (\varphi + \delta) * \frac{\lambda}{Q} + h_2 * \frac{Q * t}{T} \quad 4.7$$

Inserting T the reorder interval time with

$$T = \frac{Q}{\lambda}$$

$$TC(Q) = c * \lambda + K * \frac{\lambda}{Q} + h_1 \frac{Q}{2} + (ß * w * Q) * \frac{\lambda}{Q} + (\varphi + \delta) * \frac{\lambda}{Q} + h_2 * t * \lambda$$

The variables h_1 and h_2 are used for the ease of calculation. In reality they have the same values. Therefore we can use the variable h for both of h_1 and h_2.Then the formula takes the form

$$TC(Q) = c * \lambda + K * \frac{\lambda}{Q} + h \frac{Q}{2} + (ß * w * \lambda) + (\varphi + \delta) * \frac{\lambda}{Q} + h * t * \lambda$$

To find the optimal quantity we take the first derivative of the total cost function with respect to Q and set it to zero

$$TC'(Q) = -\frac{K * \lambda}{Q^2} + \frac{h}{2} - (\varphi + \delta) * \frac{\lambda}{Q^2} = 0$$

$$TC'(Q) = -\frac{2 * K * \lambda}{2Q^2} + \frac{h * Q^2}{2Q^2} - (\varphi + \delta) * \frac{2\lambda}{2Q^2} = 0$$

$$TC'(Q) = -\frac{2 * K * \lambda}{2Q^2} + \frac{h * Q^2}{2Q^2} - \frac{2 * \lambda * \varphi}{2Q^2} - \frac{2 * \lambda * \delta}{2Q^2} = 0$$

$$-2 * K * \lambda + h * Q^2 - 2 * \lambda * \varphi - 2 * \lambda * \delta = 0$$

$$h * Q^2 = 2 * K * \lambda + 2 * \lambda * \varphi + 2 * \lambda * \delta$$

$$h * Q^2 = 2 * \lambda * (K + \varphi + \delta)$$

The optimal order quantity is obtained as

$$Q^* = \sqrt{\frac{2 * \lambda * (K + \varphi + \delta)}{h}} \qquad 4.8$$

The reorder interval is

$$T^* = \frac{Q^*}{\lambda}$$

The first derivative of the cost function T(Q) with respect to Q is

$$TC'(Q) = -\frac{K * \lambda}{Q^2} + \frac{h}{2} - (\varphi + \delta) * \frac{\lambda}{Q^2} = 0$$

The second derivative of the total cost function is positive

$$TC''(Q) = +\frac{2 * K * \lambda}{Q^3} + (\varphi + \delta) * \frac{2 * \lambda}{Q^3} \geq 0$$

Therefore we can say that the total cost function T(Q) is a convex function. Hence we can state that Q* is optimal.

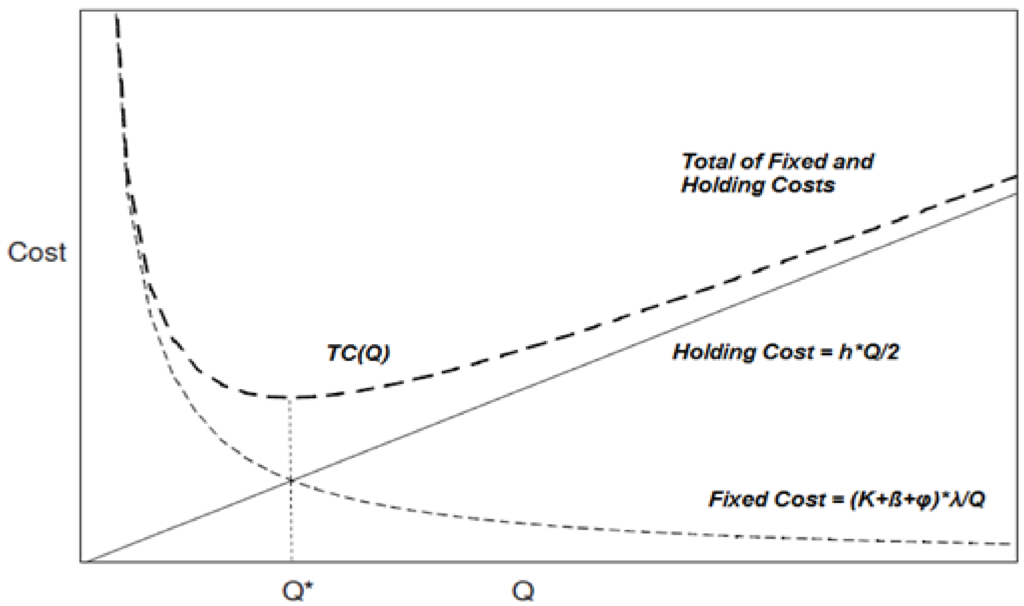

Figure 4.21 All costs in same figure

4.2.1 Minimum Cost under Different Transportation Option

As it is mentioned in the section “2.10.3 Transportation Cost” there are different transportation modes and different payment options. But while we are finding optimum order quantity we include loss of interest rate denoted h_2 and fixed cost K, fixed transportation cost φ and fixed importation cost σ. But unit transportation cost β is different for each transportation mode. That means there are different total costs for every transportation mode. Additionally there are two payment options. Totally there will be different total costs.

We assume that the company uses three different transportation types. These are highway, airway and seaway. And there are two payment options that are CIF and FOB. As a result there are 6 total cost functions. The below figure shows the options of total cost function.

Table 4.10 The alternative cost options

N	Total Cost Function	Trasportation Type	Payment Option
1	TC(Q)	Seaway	CIF
2	TC(Q)	Seaway	FOB
3	TC(Q)	Airway	CIF
4	TC(Q)	Airway	FOB
5	TC(Q)	Highway	CIF
6	TC(Q)	Highway	FOB

In such a condition decision maker should select the option which gives the minimum total cost.

$$TC(Q) = minimum(TC_1(Q),\ TC_2(Q),\ TC_3(Q), TC_4(Q), TC_5(Q), TC_6(Q))$$

This possibility gives the chance if the company has option to select the payment option when ordering. Sometimes payment option could be fixed CIF or FOB then the cost option to choose will be reduced to half. The question is if the all possibilities are free to choose what would be option to choose so that does the minimum total cost minimum?

There are 3 possibilities for transportation type.

1. Highway(transportation cost of logistic firm 0.45 \$/kg)
2. Airway (transportation cost of logistic firm 0.99 \$/kg)
3. Seaway (transportation cost of logistic firm 0.11 \$/kg)

$$TC(Q)_{FOB} = \begin{cases} TC(Q)_1 \quad if \ \ ß = 0.99\$ \quad (\,Airway\ option\,) \\ TC(Q)_2 \quad if \ \ ß = 0.45\ \$ \quad (\,Highway\ option\,) \\ TC(Q)_3 \quad if \ \ ß = 0.11\ \$ \quad (\,Seaway\ option\,) \end{cases}$$

The transportation time shows changes regarding the location of vendor and transportation type. Therefore we analyze t (transportation time) depending on the problem. In CIF payment option transportation time has no sense because payment is done when the goods are received.

$TC(Q)_{CIF}$ have another 3 options for which the manufacturer does not transportation holding cost because shipment payment is realized when the goods are received.

$$TC(Q)_{CIF} = \begin{cases} TC(Q)_4 \quad if \;\; ß = 0.99\,\$ \quad (\,Airway\ option\,) \\ TC(Q)_5 \quad if \;\; ß = 0.45\$ \quad (\,Highway\ option\,) \\ TC(Q)_6 \quad if \;\; ß = 0.11\$ \quad (\,Seaway\ option\,) \end{cases}$$

The minimum total cost can be found after all options are calculated.

$$TC(Q)_{min} = min\{\, TC(Q)_1, TC(Q)_2,\, TC(Q)_3,\, TC(Q)_4, TC(Q)_5, TC(Q)_6\}$$

In the next chapter the calculation of all possibilities are given on an example.

4.3 Reorder Point

The same reorder point formula can be used in the new approach of economic order quantity model with transportation cost. There are again two possibilities.

- If time interval is greater than lead time (T > τ)

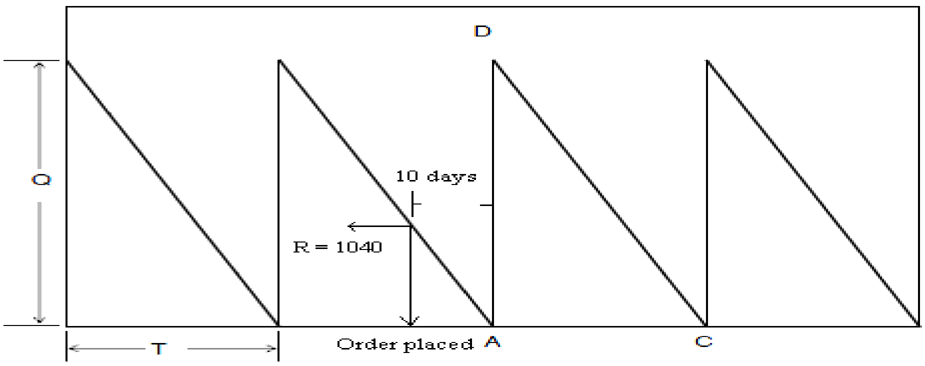

Figure 4.22 Reorder point if time interval is greater than lead time

The reorder point formula which is represented in 2.14.2 is r = λ*τ.

Example: $\lambda = 3120$ unit/month andlead time $\tau = 10$ days $= 0.33$ month

$$r^* = 3120 \frac{\text{unit}}{\text{month}} * 0.33 \text{ month} = 1.040 \text{ unit}$$

In this example it is assumed that airway transportation is used. This $\tau = 10$ days represent the lead time which is the time placing an order to vendor and receiving them to inventory. According to the total cost function TC(Q) we have obtained if we should have to use seaway transportation mode we should use another lead time value for the sea way transportation. That means company X should place the new order when stock level is 1040. The important point is here is the lead time, the time placing an order.

- If lead time is greater than time interval (T < τ)

Determining the reorder point is more difficult when lead time exceeds a cycle. But in basically the same procedure of T > τ can be used.

Example:

After calculation of optimal order, suppose economic order quantity of a material is found as 10800units. Demand rate is 4150 unit / month. Then

$$\mathrm{T} = \frac{\mathrm{Q}}{\lambda} = \frac{10800\, unit}{4150\, \frac{unit}{\mathrm{month}}} = 2.6 \text{ month}$$

Time interval is 2.6 months and lead time is 6 months of this material. Then T < τ.

$$\frac{\tau}{\lambda} = \frac{6 \text{ month}}{2.6 \text{ month}} = 2.31\, cycle$$

This means that there are exactly 2.31 cycles. In other words the order should be placed 2.31 cycle before the order has to be received. It is also exactly the same as placing the order 0.31 cycles before. The level of on-hand inventory is the same whether it is at point 2.31 or 0.31 cycle before the arrival of an order. In this case 0.31 cycles is 0.8 month. Now it is easier to find reorder point

$$R = \lambda * \tau = 4150\, unit/\text{month} \; * \; 0.31 \text{ month} \; = 3.320 \text{ unit}$$

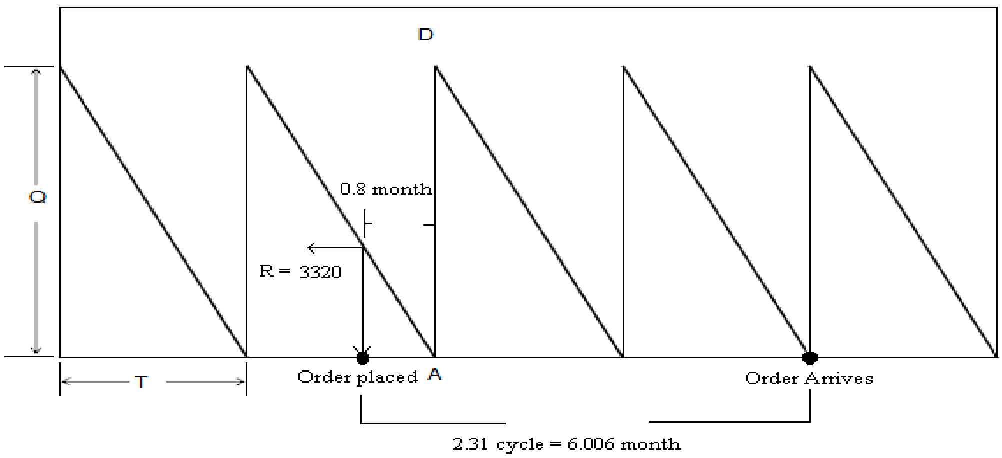

Figure 4.23 Reorder point if lead time is greater than time interval

4.4 Minimum Cost is Always Optimal Result?

When this question is asked to a decision maker, in high probability, he or she says that all results should be compared with different aspects, not only as a cost. Then how can I decide the transportation mode? Just comparing the total cost of six different ordering types?

Another aim of this study is to see deviation of transportation holding cost on the total cost. Imagine that the materials are purchased from a country and transportation time is 40 days by seaway. Vendor accepts only FOB payment. The model is applied to the problem and minimum cost is seaway transportation. Total cost 510.000$. Another result is airway transportation and it takes 513.000$. Transportation duration is 3 days but variable transportation cost is high in compare seaway shipment. But the deviation of transportation time 37 days. As a numerical calculation seaway transportation presents less cost but

May the decision maker risk to lose 3000$?

Answer is not always no. Because there are many different criterion that are not calculated and added to this model. If the person accepts to lose 3000$, he will gain 37 days on the other hand? What are the advantages of having 37 days?

- He/she pays total money after 37 days later. Approximately 500.000$ can be operated in different investments.
- He/she plans the production with clarified demands. Companies do not work always according to forecast conversely with certain orders.
- He/she would decide to place new orders to dispatch previous orders. That means when the orders are jointed together the export cost will decrease because of fixed costs.

Generally these factors are difficult to calculate as a cost. The main point of this approach is to find the minimum cost option but on the other hand to see the other possibilities. Imagine that a material is purchased from another far country and this material is not heavy additionally it is very expensive. Then other results could be selected in spite of losing money.

Chapter 5

5. Solution Approach and Results

5.1 The Effect of Transportation Cost on Company X's Budget

As we all know, logistic is an operation which manages flow of raw materials or any goods between the points of origin to the destination point in order to supply the requirements of customers. In details when the logistic operation is splatted into two side

1- External logistic
2- Internal logistic (micro logistic)

In literature there are many surveys which investigate the management of inventory from different aspects. But logistic cost is commonly is the last part to join the optimization although it has a significant effect in inventory management.

Company X is a leading manufacturer which produces LCD TV and different electronic devices. Some of materials that are used in manufacturing are supplied from internal sources. But company X purchases big part of inventory from external sources even like far away countries like Japan, China, Hong Kong etc.

As a start of a modeling inventory, demand of materials should be well forecasted. It is very important for production planning, to manage the capital, purchasing activities and for management of logistic operations. If the manufacturer works with high lead times, logistic activities become more important.

Company X has serious problems in demand forecasting and logistic operations. Due to increasing demand of some materials company X should supply the materials with urgent transportation option. Generally the materials should be transferred via airway with high freight cost. On the other hand if the material is not heavy and unit purchasing price is high, airway transportation can be a better option in comparison to the freight cost of the other transportation options. Also airway transportation time less than others and it would be an advantage in planning of production and material more effective. As another chain of this problem company X cannot have optimal stock level for each material. Because of that each material cannot be controlled under calculated stock level and therefore company should have some big inventory for specific materials.

Finally the aim of this study is to figure out an ordering system to the company X. There are two payment options which are CIF and FOB. And there are 3 transportation ways that are airway, seaway and highway. The point of this analysis is that to find the minimum cost by taking into account the other ordering possibilities and their costs. The results will be compared with existing system.

Company X holds over 3000 different materials in inventory. Before the implementation of the new approach to these materials, it would be more effective to classify the materials by using ABC and AHP method. In general the ABC model the components are evaluated related to usage ratio and unit cost of material. But in AHP model user can add different evaluation criteria like quality, delivery performance of suppliers, weight of material etc. In such a model it is possible to classify the material with different criterion. Inventory classification analysis is made for the materials which are used by electronic department. And the results are tabled and shown in APPENDIX-A. WTR910R is chosen to implement the model on. Also the model is investigated for the material RTD883D to see the other aspects of the results. The steps of the application:

1- To have a deterministic demand forecasting the study must be rephrased within 2 years past historical data to be used as demand data.
2- The forecasting accuracy of different methods are compared and the lowest MSD value is chosen
3- The input values of the problem are given to the new model and the optimal order quantity is found.
4- The total cost is found with respect to the optimal order quantity.
5- Other results are compared with the minimum total cost by considering transportation time, transportation holding cost.
6- Reorder point is found related to order quantity and lead time.
7- The results are our new model is compared with the old system which is used in company X.

5.2 Analyzing Historical Demand Data and Forecasting Study

The starting point of the inventory management is to have a good demand forecasting. The better predicted data for purchasing means better stock level especially in high lead times. In this study MINITAB software is used to forecast demand by using historical demand data. In my opinion, additional expert comments makes demand forecast more reliable. But in this study we used just historical demand data.WTR910R is a component which is used in manufacturing of LCD TV. This material is purchased from China and there are three transportation mode options for this material.

- Seaway
- Airway
- Highway

Table 5.11 Monthly consumption of material WTR910R in 2010 and 2011

WTR910R	2010 Monthly * Consumption	2011 Montly * Consumption
January MRP	13.261	14.854
February MRP	15.997	16.704
March MRP	12.293	12.656
April MRP	10.422	11.376
May MRP	10.085	9.981
June MRP	8.988	9.253
July MRP	8.483	9.011
Agust MRP	10.665	11.378
September MRP	12.315	11.447
Octonber MRP	13.111	13.874
November MRP	10.015	11.532
December MRP	10.221	9.447

* units / month

Figure 5.1 the consumption quantities of WTR910R in the last two years is given. It is possible to use the 2009 data too, but electronic components show changes from year to year and over than two years historical data can mislead the demand planner. For the historical demand data of WTR910R, different forecasting method will be applied. Then the forecast errors of the results will be compared.

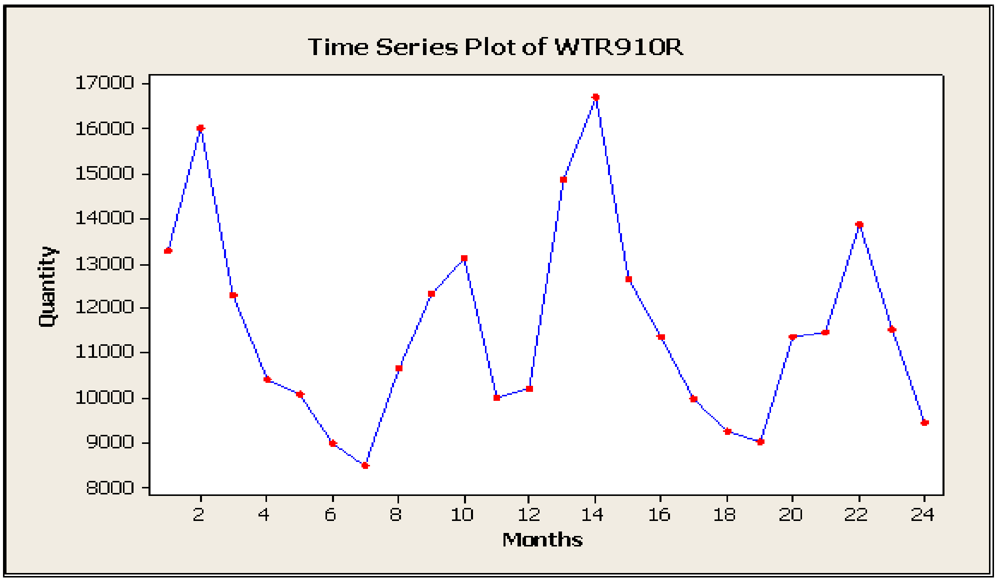

Figure 5.24 Time series plot of WTR910R

In Figure 5.1 the demand plot ofWTR910R in the past two years is observed. As the times series plot of WTR910R in first part of the year there is an increase in consumption of the material then a decreasing movement is realized. It is possible to say that there is seasonality in usage of WTR910R but there would be easier to have an idea about seasonality if the autocorrelation function is plotted.

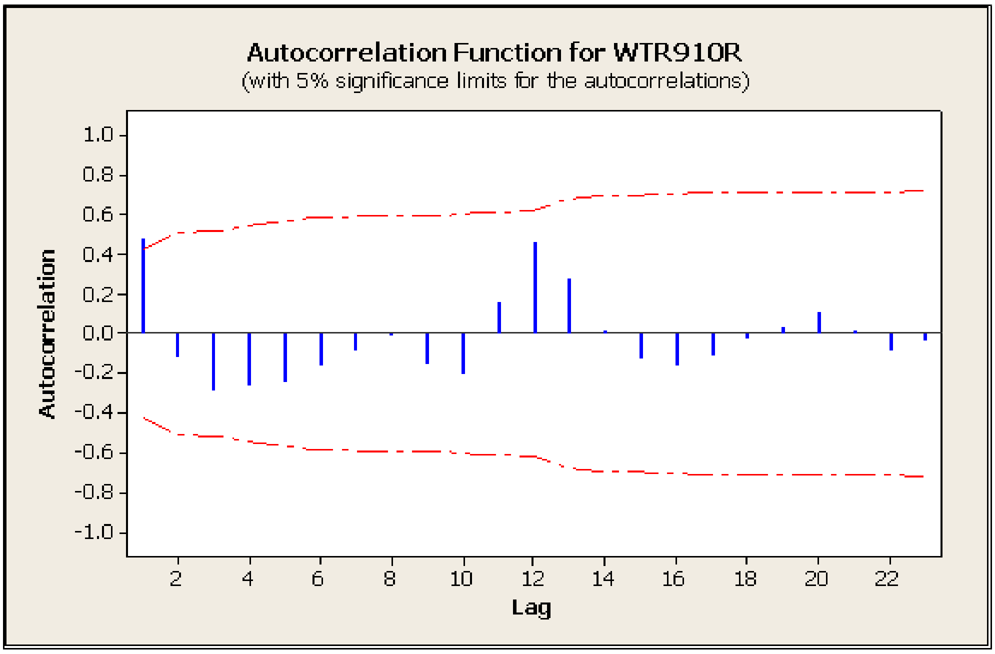

Figure 5.25 Autocorrelation Function of WTR910R

Table 5.12 Autocorrelation result of WTR910R data

Lag	CRF	Lag	CRF	Lag	CRF
r_1	0.4788	r_9	-0.1539	r_{17}	-0.1067
r_2	-0.1171	r_{10}	-0.1967	r_{18}	-0.0211
r_3	-0.2833	r_{11}	0.1568	r_{19}	0.0357
r_4	-0.2633	r_{12}	0.4595	r_{20}	0.1101
r_5	-0.2453	r_{13}	0.2722	r_{21}	0.0201
r_6	-0.1617	r_{14}	0.0150	r_{22}	-0.0830
r_7	-0.0836	r_{15}	-0.1259	r_{23}	-0.0317
r_8	-0.0125	r_{16}	-0.1625		

It is noticed that the autocorrelation at lag r_1 is higher than for the other lags. This is due to the seasonal pattern in the data. As the lag r_1 , r_{12} is the second highest value. After 12 periods, the lags show approximately same behavior. It can be commented that there is no 100 percent seasonality but it is clear that data pattern shows seasonality characteristics.

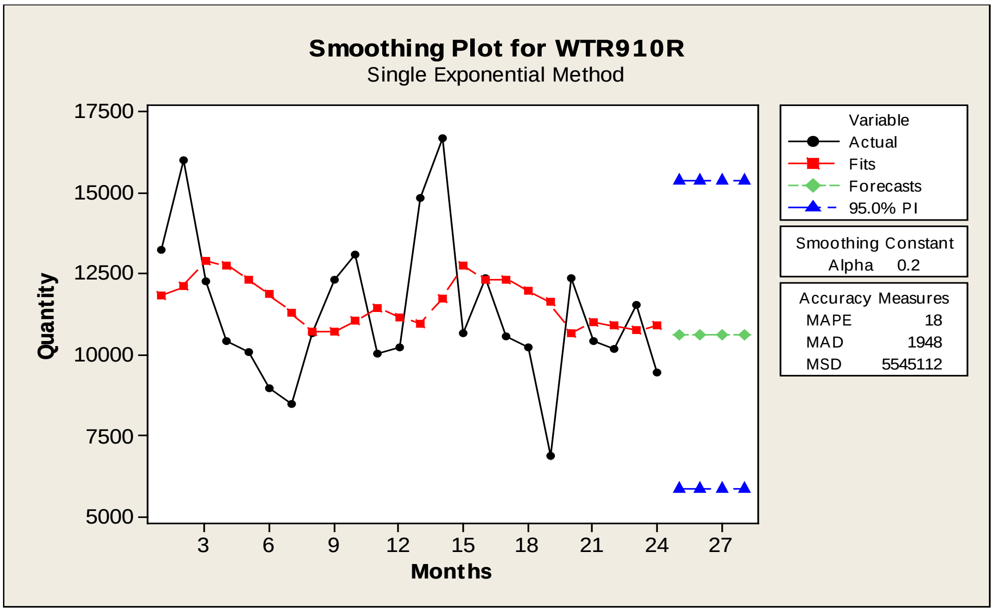

Figure 5.26 Smoothing plot of single exponential method for WTR910R

Length of period : 24

Smoothing constant alpha 1.23234

MAPE : 14

MAD : 1646

MSD : 4526139

Table 5.13 Forecast result of single exponential smoothing method

Period	FORECAST	UPPER	LOWER
25	9,120.95	13,152.40	5,089.51
26	9,120.95	13,152.40	5,089.51
27	9,120.95	13,152.40	5,089.51
28	9,120.95	13,152.40	5,089.51

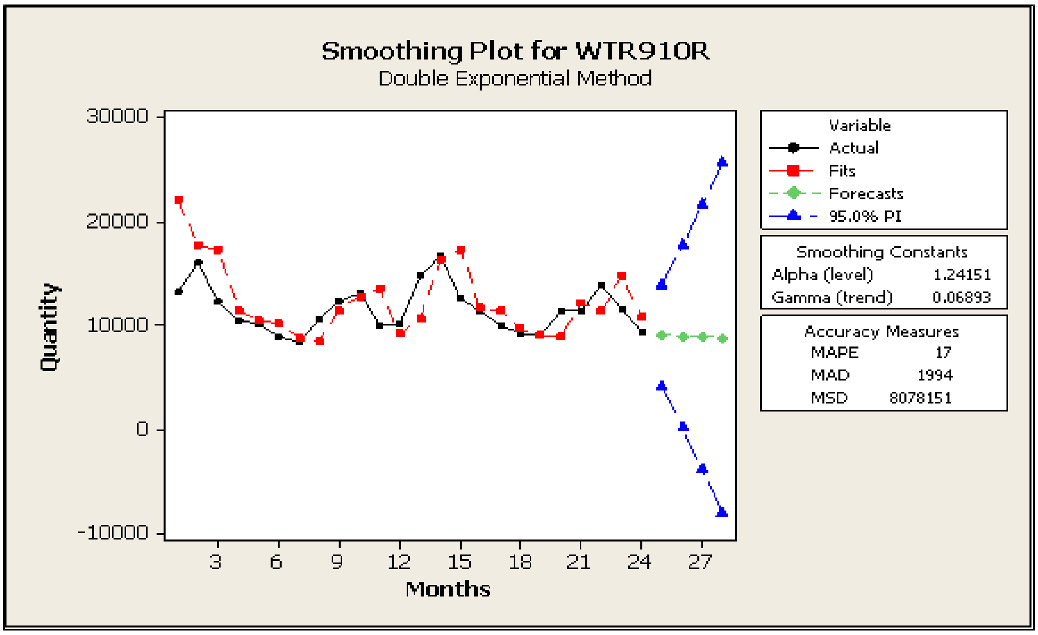

Figure 5.27 Smoothing plot of double exponential method for WTR910R

Length of period : 24

Smoothing constant alpha 1.24151

Smoothing constant beta 0.06893

MAPE : 17

MAD : 1994

MSD : 8078151

Table 5.14 Forecast results of double exponential smoothing method

Period	FORECAST	UPPER	LOWER
25	9,048.94	13,933.91	4,163.97
26	8,973.55	17,759.81	187.29
27	8,898.16	21,690.42	-3,894.10
28	8,822.77	25,651.13	-8,005.59

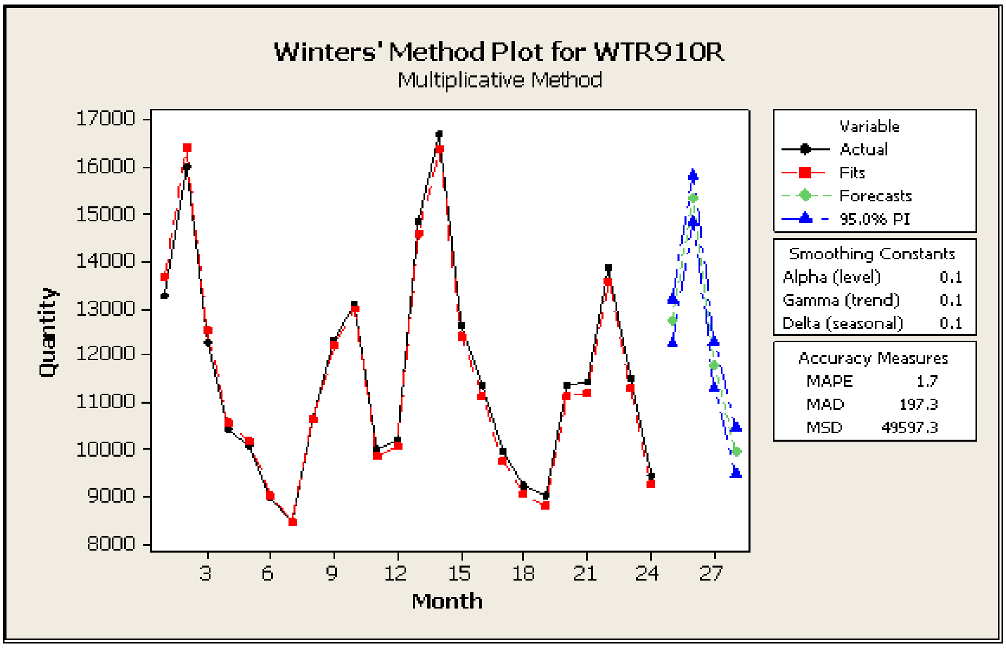

Figure 5.28 Winter's method plot for WTR910R

Length of period : 24

Smoothing constant alpha 0.1

Smoothing constant beta 0.1

Smoothing constant delta 0.1

MAPE : 1.7

MAD : 197.3

MSD : 49597.3

Table 5.15 Forecast results of Winter's method

Period	FORECAST	UPPER	LOWER
25	12,731.42	13,214.70	12,248.15
26	15,350.52	15,835.65	14,865.39
27	11,790.13	12,277.22	11,303.03
28	9,990.18	10,479.36	9,501.00

Table 5.16 Summary result of forecasting methods

	MAPE	MAD	MSD
Single Exponential Smoothing Method	14	1646	4526139
Double Exponential Smoothing Method	17	1994	8078151
Winter's Method	1,7	197,3	49597,3

In Table 5.5 the results of forecasting methods are presented. MSD value of Winter's method is very small in comparison to the with other forecasting methods. The Winter's method can be used to forecast demand of material WTR910R. But in real life expert demand planner should verify the forecast observations. Additionally if they change the value of forecast result with their professional experiences, it is possible to have a better forecast accuracy. In this study the forecast observations of Winter's method can be used for the next step of inventory planning. The forecast of next periods of Winter's methods are

Period 25 : 12731, Period 26 : 15350, Period 26 : 11790, Period 27 : 9990.

5.3 Implementation of EOQ Model with Transportation Cost on WTR910R

As it is mentioned in the previous part, WTR910R is purchased from China. From China to Turkey, it is possible to use three different transportation types. And there are two payment options. In such a case there are six different total cost possibilities.

There are lots of possibilities to supply the materials from outsource suppliers. Vendor may want to have the payment in cash. It could not be possible to choose a specific transportation type due to logistic problems or production plan. Therefore in first case six different total cost possibility will be calculated and the minimum cost value will be picked up. Then results will be analyzed.

$$TC(Q) = minimum(TC_1(Q),\ TC_2(Q),\ TC_3(Q), TC_4(Q), TC_5(Q), TC_6(Q))$$

Table 5.17 Total cost functions

Alternatives	$TC_1(Q)$	$TC_2(Q)$	$TC_3(Q)$	$TC_4(Q)$	$TC_5(Q)$	$TC_6(Q)$
Transportation Type	Seaway	Seaway	Airway	Airway	Highway	Highway
Payment	CIF	FOB	CIF	FOB	CIF	FOB

As a result of Winter's demand forecasting method in the next four months, demands are 12731, 15350, 11790and 9990.

Table 5.18 Input values of the problem

Demand per month		λ	unit / month	12.731
Order (setup cost)		K	$	50
Unit holding cost per month		h	$ / unit*month	0,035175
Unit purchasing cost		c	$ / unit	28,14
Unit weight of material		w	kg / unit	0,19
Transportation time of seaway	*	t	month	1
Transportation time of airway	**	t	month	0,1
Transportation time of highway	***	t	month	0,33
Seaway transportation cost		β	$ / kg	0,11
Airway transportation cost		β	$ / kg	0,99
Highway transportation cost		β	$ / kg	0,45
Fixed transportation cost		φ	$	100
Fixed importation cost		δ	$	450

* Seaway transportation duration is 30 days from China to Turkey
** Airway transportation duration is 3 days from China to Turkey
*** Highway transportation duration is 10 days from China to Turkey

Assumptions :
1- No shortages are allowed
2- No safety stcok will be held

Company X does not hold safety stock. Because safety stock means always extra cost and company X does not want to pay this cost although there is a risk of stock out. If there is a problem in transportation of the material or vendor cannot send the material in due date which is given to the buyer, manufacturer changes the production plan related to the material. For example if the material will arrive to the inventory with 3 days delay that means the usage of the material will be delayed 3 days and the production plan should be revised. In Table 5.8 the input values of the problem are presented. Now time interval of orders T, optimal order quantity Q*, transportation cost and total order cost can be calculated by the EOQ model with transportation cost model

- **$TC_1(Q)$ – Seaway and CIF payment**

In section 4.2 the total cost function is given as

$$TC(Q) = c * \lambda + K * \frac{\lambda}{Q} + h_1 \frac{Q}{2} + (ß * w * Q) * \frac{\lambda}{Q} + (\varphi + \delta) * \frac{\lambda}{Q} + h_2 * t * \lambda$$

The variables h_1 and h_2 are used for the ease of calculation. In reality they have the same values. Therefore we can use the variable h for both of h_1 and h_2.Then the formula takes the form

$$TC(Q) = c * \lambda + K * \frac{\lambda}{Q} + h \frac{Q}{2} + (ß * w * \lambda) + (\varphi + \delta) * \frac{\lambda}{Q} + h * t * \lambda$$

In CIF payment option buyers pays the money when they are received the goods to inventory. For this reason there wouldn't be any transportation holding cost and h_2 is 0.

Table5.19 Input values of $TC_1(Q)$

Demand per month	λ	unit / month	12.731
Order (setup cost)	K	$	50
Unit holding cost per month	h_1	$ / unit * month	0,035175
Transportation holding cost	h_2	$ / unit * month	0
Unit purchasing cost	c	$ / unit	28,14
Unit weight of material	w	kg / unit	0,19
Seaway transportation cost	β	$ / kg	0,11
Fixed transportation cost	φ	$	100
Fixed importation cost	δ	$	450
Transportation time of seaway	t	month	1

To find out the stock holding cost we need some calculation. Interest rate of dollar for 1 month is approximately 0.00125 in Turkey. Unit purchasing cost of WTR910R is 28.14 $. Therefore

$$h_1 = 28.14 * 0.00125 = 0.035175 \ \$/month$$

$$Q^* = \sqrt{\frac{2 * (K + \delta + \varphi) * \lambda}{h}} = \sqrt{\frac{2 * (50 + 450 + 100) * 12731}{0.035175}}$$

$$Q^* = 20.840$$

Optimal order quantity is 20.840. And time interval is 1.63 months.

$$T = \frac{Q}{\lambda} = \frac{20.829}{12.718} = 1{,}63 \ months$$

In the first step optimal order quantity is found. Now the total cost can be calculated by using the formula:

$$TC(Q) = c * \lambda + K * \frac{\lambda}{Q} + h_1 \frac{Q}{2} + (ß * w * Q) * \frac{\lambda}{Q} + (\varphi + \delta) * \frac{\lambda}{Q} + h_2 * t * \lambda$$

$$TC(Q) = 28{,}14 * 12.731 + 50 * \frac{12.731}{20.840} + 0.035175 \frac{20.840}{2}$$

$$+ (0.11 * 0.19 * 20.840) * \frac{12.731}{20.840} + (100 + 450) * \frac{12.731}{20.840}$$

$$+ 0 * 1 * 12.731$$

$$TC(Q) = 359.249\ \$/month$$

Another point that has to be investigated is reorder point of the material. The time that is needed to be supplied of the material is 40 days. Transportation time of seaway shipment is 30 days. (1month)

Hence τ = 70 days = 2.33 months. The lead time of the material exceeds the cycle of order. In such a case, forming the ratio of τ / T, it is obtained that 1.42. This means that there are exactly 1.42 cycles in the lead time. Every order must be placed 1.42 cycles in advance.

For the purpose of computing the reorder point, this is exactly the same as placing the order 0.42 cycle in advance. This is true because the level of on-hand inventory is the same whether it is at a point 0.42 or 1.42 cycle before the arrival an order. In this case 0.42 cycle is 0.68 month. So reorder point

$r^* = \lambda * \tau_1$ = 12.731 pcs /month * 0.68 month = 8657 pcs.

Table 5.20 Solution result of $TC_1(Q)$

Demand per month	λ	unit / month	12.731
Order quantity	Q	unit	20.840
Order interval	T	month	1,64
Transportation cost		$ / month	602,06
Transportation holding cost		$ / month	0,00
Total material cost	c*λ	$ / month	358.250,34
Holding cost of inventory	h_1*Q/2	$ / month	366,53
Total ordering cost	K*λ/Q	$ / month	30,54
Grand total cost	$TC_1(Q)$	$ / month	359.249,48

In seaway transportation type 602$/month should be paid per order. If the payment option were FOB this value would be greater depending on transportation holding cost. In first part of the problem it is mentioned that there will be no safety stock in order not increase the total cost. Safety stock increase the total cost but conversely it reduces the risk of being stock out. The standard deviation of historical demand is 2.173 pieces.

The manufacture can supply this material in 52 days (1.73 months). Also seaway transportation needs 30 days (1 month). That means lead time of the material in this option is 2,73 months. If the company X wants to work with 95% customer service level :

$$SS = Z * \sigma * \sqrt{L} = 2.09 * 2173 * \sqrt{2.73} = 7503$$

σ is the standard deviation of the demand population, L is the constant lead time of the material and Z is the standard normal deviate and its value can be found on the basis of the service level requirement from standard normal distribution table. (Richard Tersine,1985)

Safety stock level should be 7.503 pieces. The material cost of 7.503 pieces is $211.159. Additional to this value, the loss of interest rate cost should be added. Company X does not want to pay this cost for safety stock. There are over than 2.000 different materials in the inventory of company X. If company X holds the money for the safety stock for each material in the inventory, there will be a huge amount of money for revised safety stock. This is strategically decision and no safety stock is held by company X.

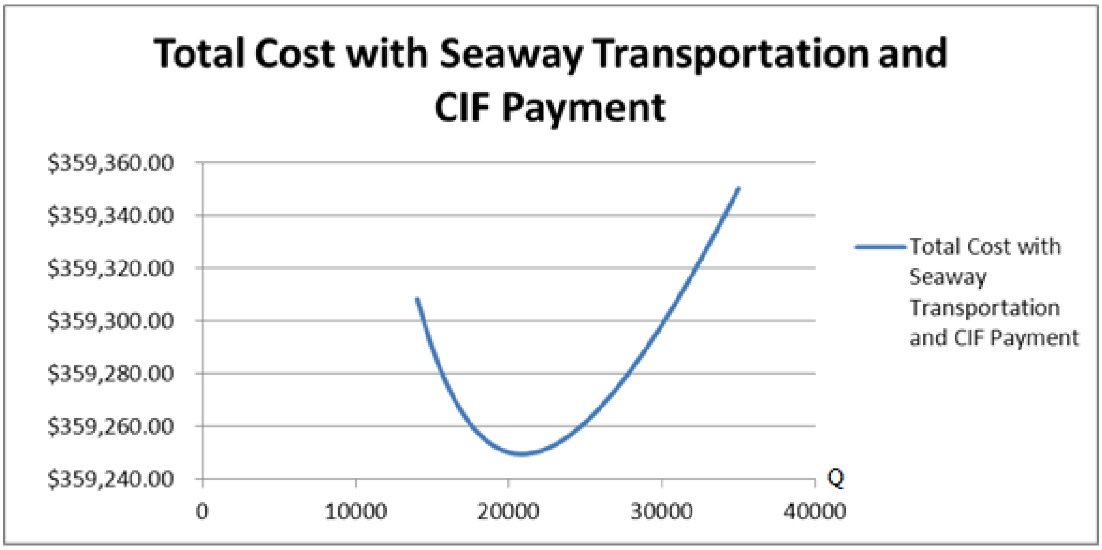

Figure 5.29 Plot of total cost with seaway transportation and CIF payment

❖ **$TC_2(Q)$ – Seaway and FOB payment**

Table 5.21 Input values of $TC_2(Q)$

Demand per month	λ	unit / month	12.731
Order (setup cost)	K	$	50
Unit holding cost per month	h_1	$ / unit * month	0,035175
Transportation holding cost	h_2	$ / unit * month	0,035175
Unit purchasing cost	c	$ / unit	28,14
Unit weight of material	w	kg / unit	0,19
Seaway transportation cost	β	$ / kg	0,11
Fixed transportation cost	φ	$	100
Fixed importation cost	δ	$	450
Transportation time of seaway	t	month	1

In this problem there is a transportation holding cost different from $TC_1(Q)$. Because in FOB payment option the purchasing cost of the material is paid before receiving them. The payment is done when the materials are loaded to ship. Transportation duration is 30 days and that means there will be loss of interest rate cost. It is denoted as h_2. Optimal order quantity is always same for each total cost function. For this option optimal order quantity is 20.840.

$$TC(Q) = c * \lambda + K * \frac{\lambda}{Q} + h_1 \frac{Q}{2} + (ß * w * Q) * \frac{\lambda}{Q} + (\varphi + \delta) * \frac{\lambda}{Q} + h_2 * t * \lambda$$

$$TC(Q) = 28{,}14 * 12.731 + 50 * \frac{12.731}{20.840} + 0.035175 \frac{20.829}{2}$$

$$+(0.11 * 0.19 * 20.829) * \frac{12.731}{20.840} + (100 + 450) * \frac{12.731}{20.840}$$

$$+0.035175 * 1 * 12.731$$

$$TC(Q) = 359.697\ \$/month$$

The reorder point is the same in comparison to previous part. Because the difference between TC_1 and TC_2 is only the payment option. Therefore it is not necessary to calculate all steps again.

r* = λ*τ_1 = 12.731 pcs /month * 0.68 month = 8657 pcs.

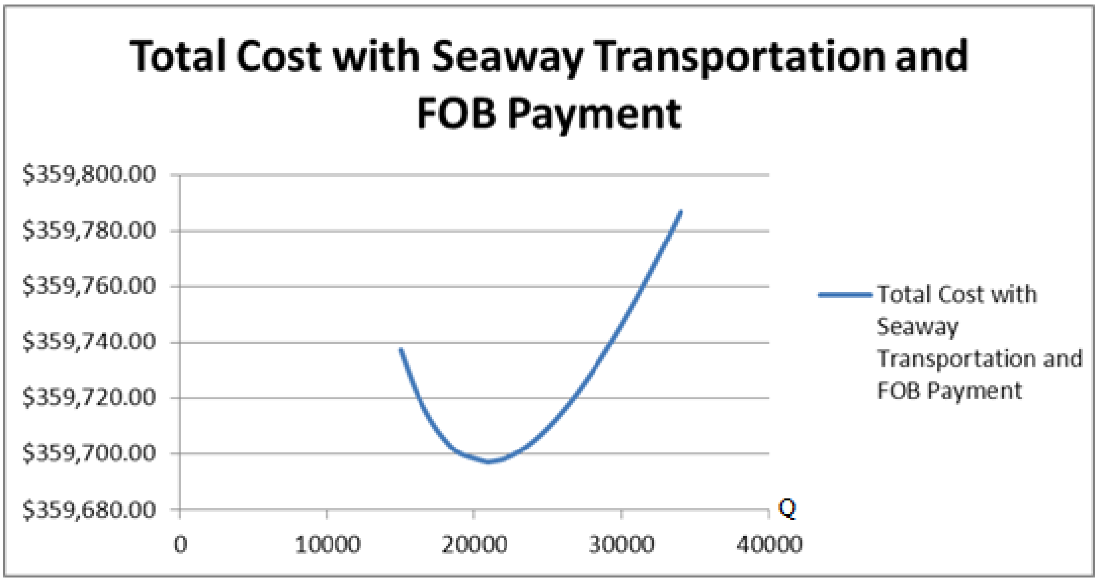

Figure 5.30 Plot of total cost with seaway transportation and FOB payment

$TC_1(Q) = 359.249 \ \$ \ / \ month$

$TC_2(Q) = 359.697 \ \$ \ / \ month$

There is a 448$ deviation because of CIF and FOB payment option. In FOB payment we should pay extra 448 $ if the vendor insists for FOB payment.

Table5.22Solution result of$TC_2(Q)$

Demand per month	λ	unit / month	12.731
Order quantity	Q	unit	20.840
Order interval	T	month	1,64
Transportation cost		$ / month	602,06
Transportation holding cost		$ / month	447,81
Total material cost	c*λ	$ / month	358.250,34
Holding cost of inventory	h_1*Q/2	$ / month	366,53
Total ordering cost	K*λ/Q	$ / month	30,54
Grand total cost	$TC_2(Q)$	$ / month	359.697,29

- **$TC_3(Q)$ – Airway and CIF payment**

Table 5.23 Input values of $TC_3(Q)$

Demand per month	λ	unit / month	12.731
Order (setup cost)	K	$	50
Unit holding cost per month	h_1	$ / unit * month	0,035175
Transportation holding cost	h_2	$ / unit * month	0
Unit purchasing cost	c	$ / unit	28,14
Unit weight of material	w	kg / unit	0,19
Airway transportation cost	β	$ / kg	0,99
Fixed transportation cost	φ	$	100
Fixed importation cost	δ	$	450
Transportation time of airway	t	month	0,33

To calculate $TC_3(Q)$, optimal order quantity should be determined. There are no change changes for setup cost, fixed transportation cost, fixed importation cost and demand rate. Because of that optimal order quantity would not be changed. EOQ is 20.840 units. The main impact on total cost is CIF payment and airway transportation time. Transportation duration of airway option is approximately 3 days. We can assume this time as constant. In CIF payment there won't be a loss of interest rate cost because payment will be done after receiving the materials. For this reason h_2 is assumed to be 0. Thus transportation duration is no sense. As it is denoted while determining $TC_1(Q)$ and $TC_2(Q)$ company X does not want to hold safety stock. So safety stock won't be calculated for $TC_3(Q)$.

$$TC(Q) = c * \lambda + K * \frac{\lambda}{Q} + h_1 \frac{Q}{2} + (ß * w * Q) * \frac{\lambda}{Q} + (\varphi + \delta) * \frac{\lambda}{Q} + h_2 * t * \lambda$$

$$TC(Q) = 28{,}14 * 12.731 + 50 * \frac{12.731}{20.840} + 0.035175 * \frac{20.829}{2}$$

$$+(0.99 * 0.19 * 20.840) * \frac{12.731}{20.840} + (100 + 450) * \frac{12.731}{20.840}$$

$$+0 * 0.1 * 12.731$$

$$TC(Q) = 361.378\ \$/month$$

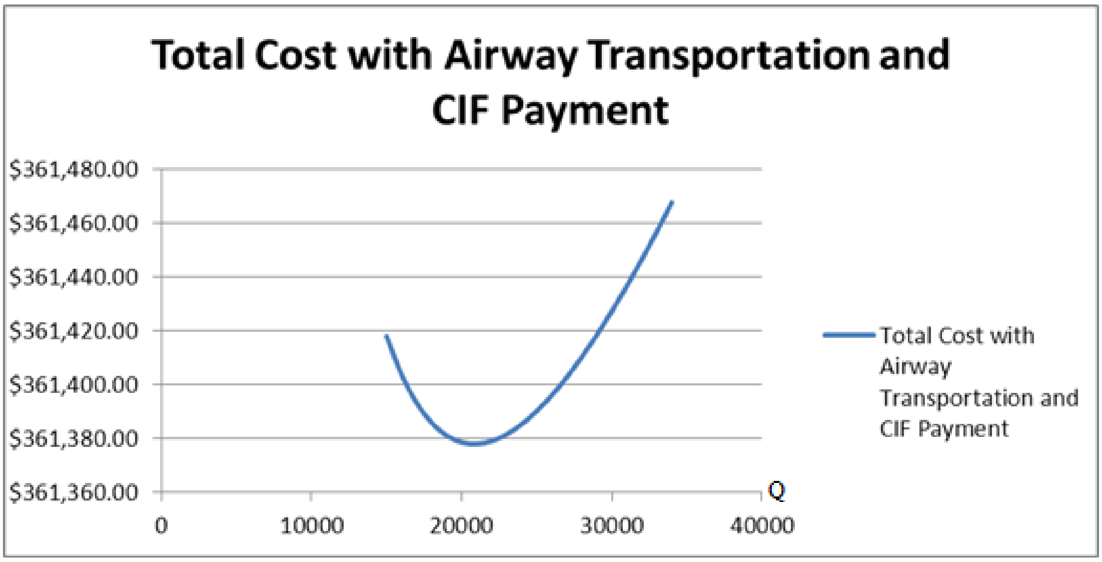

Figure 5.31 Plot of total cost with airway transportation and CIF payment

Total cost airway transportation and CIF payment is 361738 $. As it is denoted in previous chapter, CIF payment is always advantages for buyers. Because you may pay after the goods are delivered. In the solution part of the problem we will determine the total cost airway transportation and FOB payment.

The manufacturing time of the material is 40 days. Transportation time of airway shipment is 3 days. (0.1month) And so τ = 43 days = 1.43 months. In such case to calculate reorder point is much easier.

$r^* = \lambda^*\tau_1$ = 12.731 pcs /month * 1.43 month = 18.205 pcs.

Table5.24Solution result of $TC_3(Q)$

Demand per month	λ	unit / month	12.731
Order quantity	Q	unit	20.840
Order interval	T	month	1,64
Transportation cost		$ / month	366,53
Transportation holding cost		$ / month	0,00
Total material cost	$c^*\lambda$	$ / month	358.250,34
Holding cost of inventory	$h_1{}^*Q/2$	$ / month	366,53
Total ordering cost	$K^*\lambda/Q$	$ / month	30,54
Grand total cost	$TC_3(Q)$	$ / month	361.358,10

❖ **$TC_4(Q)$ – Airway and FOB payment**

Setup cost, fixed importation cost, fixed transportation cost are constant and EOQ is 20.840 units. The difference for this part is payment option compared to $TC_3(Q)$ is as fallows. Transportation duration is 3 days and payment will be realized before 3 days. That means that there will be loss of interest rate cost of 3 days (0.1 month). 3 days loss of interest rate cost is 44.78 $.

$Loss\ of\ interest\ rate\ cost = 0.035175 * 0.1 * 12.731 = 44.78$ $

$TC(Q) = 361.423\ \$/month$

Table 5.25 Input values ofTC4(Q)

Demand per month	λ	unit / month	12.731
Order quantity	Q	unit	20.840
Order interval	T	month	1,64
Transportation cost		$ / month	366,53
Transportation holding cost		$ / month	44,78
Total material cost	c*λ	$ / month	358.250,34
Holding cost of inventory	h1*Q/2	$ / month	366,53
Total ordering cost	K*λ/Q	$ / month	30,54
Grand total cost	TC4(Q)	$ / month	361.422,88

The reorder point of the material is exactly like it is found in TC3. Because payment option does not have role in exploring of the reoder point.

$r^* = \lambda^*\tau_1$ = 12.731 pcs /month * 1.43 month = 18.205 pcs.

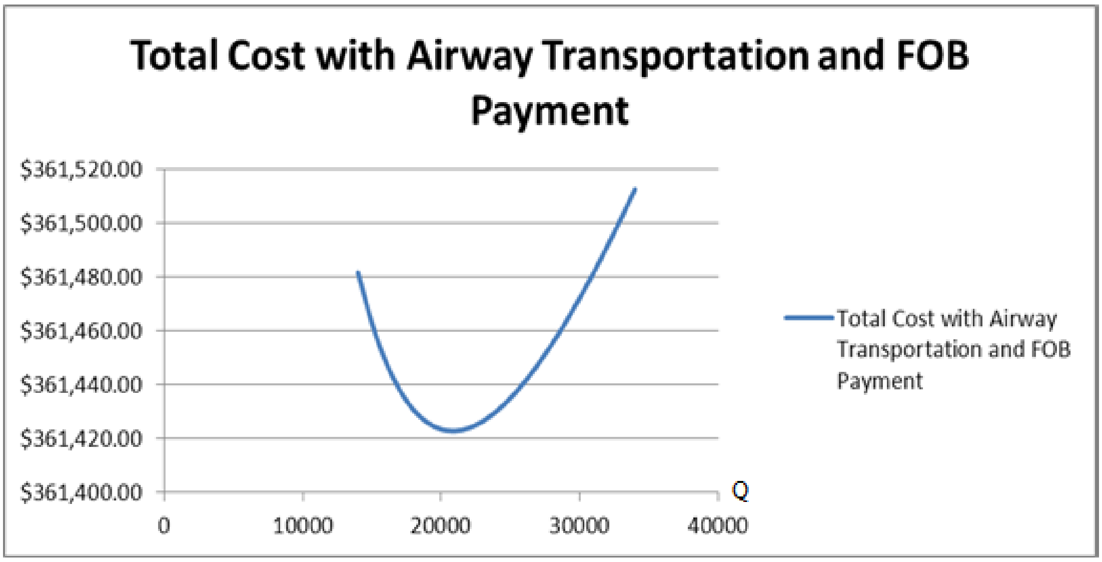

Figure 5.32Plot of total cost with airway transportation and FOB payment

❖ **TC5(Q) – Highway and CIF payment**

Optimal order quantity is valid for TC5(Q) and TC6(Q) and it is 20840 units. Highway transportation cost per kg is 0.45 $. Although highway transportation is cheaper than airway option, it is more expensive than seaway transportation.

Conversely transportation duration is longer than airway transportation but shorter than airway transportation.

Table 5.26 Input values of$TC_5(Q)$

Demand per month	λ	unit / month	12.731
Order (setup cost)	K	$	50
Unit holding cost per month	h_1	$ / unit * month	0,035175
Transportation holding cost	h_2	$ / unit * month	0
Unit purchasing cost	c	$ / unit	28,14
Unit weight of material	w	kg / unit	0,19
Highway transportation cost	β	$ / kg	0,45
Fixed transportation cost	φ	$	100
Fixed importation cost	δ	$	450
Transportation time of highway	t	month	0,33

$TC_5(Q)$ does not contain transportation holding cost because CIF payment will be do not in this option. Because of that h_2 is 0 transportation duration does not matter.

$$TC(Q) = c * \lambda + K * \frac{\lambda}{Q} + h_1 \frac{Q}{2} + (ß * w * Q) * \frac{\lambda}{Q} + (\varphi + \delta) * \frac{\lambda}{Q} + h_2 * t * \lambda$$

$$TC(Q) = 28{,}14 * 12.731 + 50 * \frac{12.731}{20.840} + 0.035175 * \frac{20.829}{2}$$

$$+(0.45 * 0.19 * 20.840) * \frac{12.731}{20.840} + (100 + 450) * \frac{12.731}{20.840}$$

$$+0 * 0.33 * 12.731$$

$$TC(Q) = 360.072\ \$/month$$

The manufacturing time of the material is 40 days. Transportation time of highway shipment is 10 days. (0.1 month) And so

τ = 50 days = 1.66 months.

That means lead time exceeds the cycle length of the material. In such a case, forming the ratio of τ / T, it is obtained that 1.01. This means that there are exactly 1.01 cycles in the lead time. Every order must be placed 1.01 cycles in advance.

For the purpose of computing the reorder point, this is exactly the same as placing the order 1.01 cycles in advance. This is true because the level of on-hand inventory is the same whether it is at a point 1.01 or 0.01 cycle before the arrival an order. In this case 0.01 cycles is 0.0163 month. So reorder point

$r^* = \lambda^* \tau_1$ = 12.731 pcs /month * 0.0163 month = 207 pcs.

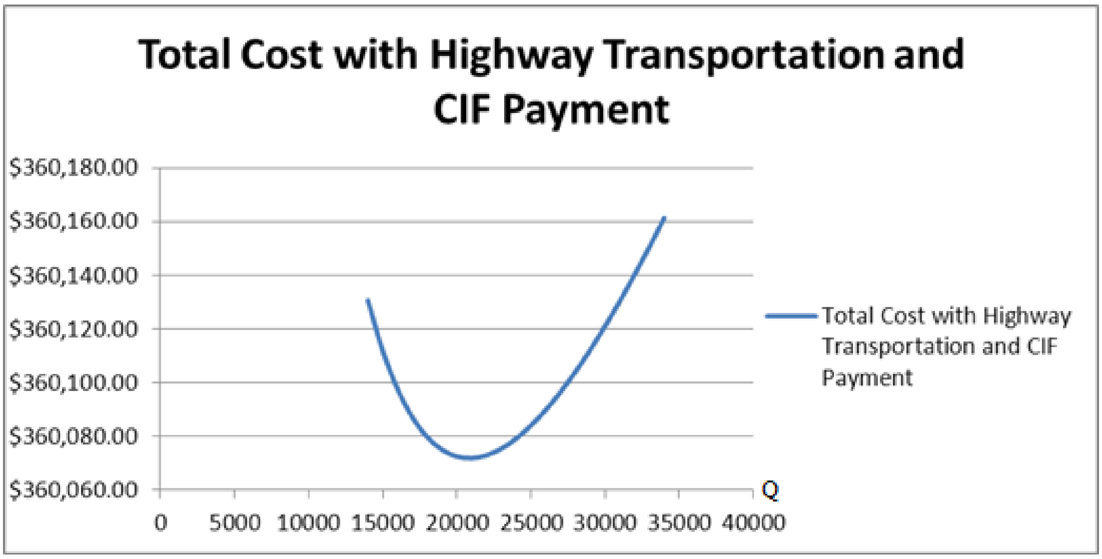

Figure 5.33Plot of total cost with highway transportation and CIF payment

- **$TC_6(Q)$ – Highway and FOB payment**

Highway transportation duration is 10 days (0.33 month) form China to Turkey. $TC_6(Q)$contains additional extra loss of interest rate cost of highway transportation duration. Optimal order quantity is 20840 units and same with other alternatives.

Table 5.27 Input values of$TC_6(Q)$

Demand per month	λ	unit / month	12.731
Order (setup cost)	K	$	50
Unit holding cost per month	h_1	$ / unit * month	0,035175
Transportation holding cost	h_2	$ / unit * month	0,035175
Unit purchasing cost	c	$ / unit	28,14
Unit weight of material	w	kg / unit	0,19
Highway transportation cost	β	$ / kg	0,45
Fixed transportation cost	φ	$	100
Fixed importation cost	δ	$	450
Transportation time of highway	t	month	0,33

$$Loss\ of\ interest\ rate\ cost = \ 0.035175 * 0.33 * 12.731 = 147{,}78\ \$$$

$TC(Q) = 360.220\ \$/month$

The reorder point is the same value which is found in $TC_5(Q)$. r* = 207 pcs.

In FOB payment, company X should pay extra 147,78 dollar. This is cause from 10 days highway transportation duration.

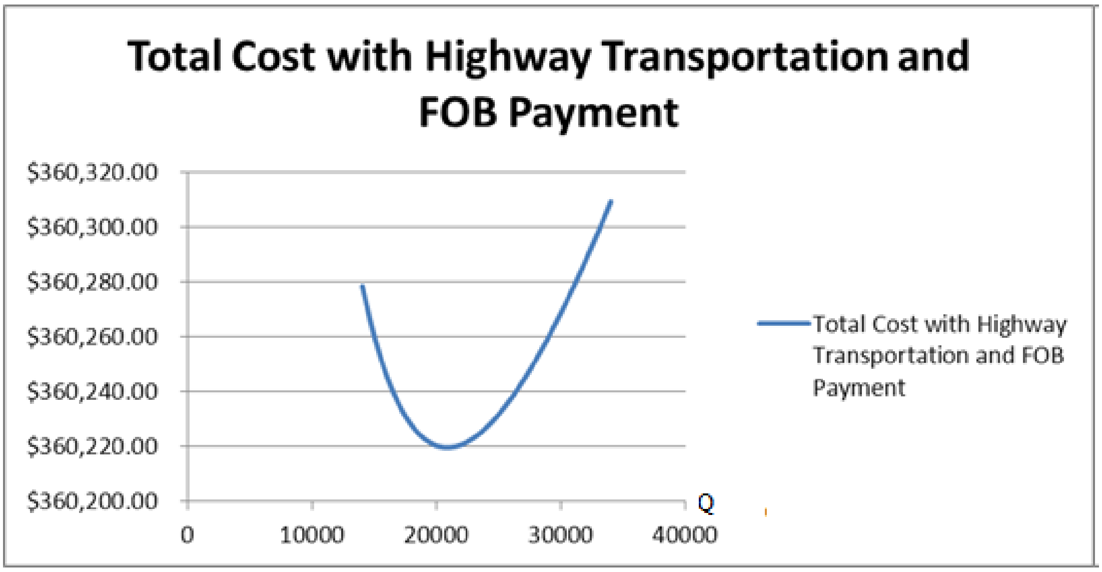

Figure 5.34Plot of total cost with highway transportation and FOB payment

5.4 Comparison Total Cost of Different Transportation Type

In previous part, in the first step the optimal order quantity is calculated. After that for each transportation mode and payment option total cost is determined.

Table 5.28 All possible total cost results

Alternatives	$TC_1(Q)$	$TC_2(Q)$	$TC_3(Q)$	$TC_4(Q)$	$TC_5(Q)$	$TC_6(Q)$
Transportation Type	Seaway	Seaway	Airway	Airway	Highway	Highway
Payment	CIF	FOB	CIF	FOB	CIF	FOB
Result	$359.249,48	$359.697,29	$361.378,10	$361.422,88	$360.071,90	$360.219,68

The above table shows the all possible total cost results. It is denoted as possible because in real life sometimes buyers have no choice to choose the payment option or transportation mode.

In six different purchasing option Seaway and CIF payment is in first row because of low total cost result in compare to other options. If the results are sorted in descending sequence, the sorted list is shown in Table 6.19.

It is clearly seen that CIF payment is always advantage in compare to FOB payment. Because the payment is done after getting the materials and there is no loss of interest rate cost. In the second analyze seaway transportation is always cheaper than other alternatives. Because transportation cost of seaway is 0.11 $ per kg and airway is 0.99 $/kg and highway is 0.45$/kg.

Table 6.29 Total cost results in descending sequence

Transportation Option	Trasnportation Duration	Payment Option	Extra Costs
Seaway	30 days	CIF	$359.249,48
Seaway	30 days	FOB	$359.697,29
Highway	10 days	CIF	$360.071,90
Highway	10 days	FOB	$360.219,68
Airway	3 days	CIF	$361.378,10
Airway	3 days	FOB	$361.422,88

If buyer has chance to select the payment option normally CIF payment is selected and seaway transportation is used as transportation mode. As total cost this option is most advantages case but seaway transportation is much longer than other alternatives. In other words demand forecasting for each month should be made earlier in order to compare to airway or highway transportation. That can affect demand forecasting accuracy negatively. Another effective point is interest rate. In Turkey interest rate is very fluctuated and high in comparison to European countries and USA. In the short term if the interest rate changes too much, there would be an important risk depending on payment option.

Table 5.30 The other costs out of purchasing cost

Transportation Option	Trasnportation Duration	Payment Option	Extra Costs
Seaway	30 days	CIF	$999,14
Seaway	30 days	FOB	$1.446,95
Highway	10 days	CIF	$1.821,56
Highway	10 days	FOB	$1.969,34
Airway	3 days	CIF	$3.127,76
Airway	3 days	FOB	$3.172,54

The main question is "May the other transportation options can be selected in for different materials which is not heavy and expensive?"

$$TC(Q) = c * \lambda + K * \frac{\lambda}{Q} + h_1 \frac{Q}{2} + (ß * w * Q) * \frac{\lambda}{Q} + (\varphi + \delta) * \frac{\lambda}{Q} + h_2 * t * \lambda$$

$$Q^* = \sqrt{\frac{2 * (K + \delta + \varphi) * \lambda}{h}}$$

In this new approach we can see that the weight of the material w, transportation duration t and unit purchase price c has not got role while determining optimal order quantity. These values affect the total cost after determining optimal order quantity. In this master thesis the implementation of the new approach is done on WTR910R. The unit purchasing price of the material is 28.14 $ and weight of the material is 0.19

kg. For this material all options are considered and seaway and CIF payment has the lowest total cost. If any other material is more expensive and lighter than WTR910R any other transportation option can be used.

Table 5.31 The characteristic of RTD883D

RTD883D	
Demand Rate	7.280 units / month
Purchasing Cost	88,36 $ / unit
Weight of Material	0,145 kg / unit
Transportation Time of Seaway	2,00 month
Transportation Time of Airway	0,10 month
Transportation Time of Highway	0,50 month

RTD883D is material which is used on the main board of LED television to control electrical voltage for other electronic components. The weight of the material is 0.145 kg per one RTD883D. This material is very expensive in comparison to other materials and unit purchase price is 88.36 $. Demand forecasting study is made like for the material WTR910R and it is 7280 units/month.

There are two alternative ways to purchase this item. The first possible vendor is form China. Second option is from Norwegian. The vendor from China stops the production of the material in 2011 because optimizing the assembly line of factory. Because of that company X should purchase the material from Norwegian. Seaway transportation duration is 2 months. This time is longer according to shipment from China because, the frequency of seaway transportation between Norwegian and Turkey is not often.

Generally companies choose the partial freight option and this extends the transportation time. Airway transportation changes between 2 and 3 days and 3 days can be accepted as constant. And lastly highway transportation duration is 15 days and it is 0.5 month.

If all solution process of the WTR910R is made exactly for the RTD883D.

Table 5.32 Total cost results of RTD883D

RTD883D			
Seq.	Transportation Mode	Payment	Total Cost
1	Seaway	CIF	$644.359,20
2	Highway	CIF	$644.718,11
3	Airway	CIF	$645.288,13
1	Highway	FOB	$645.120,15
2	Airway	FOB	$645.368,54
3	Seaway	FOB	$645.967,36

In Table 5.22 it is seen that total cost results for all possible transportation and payment options. For the CIF payment seaway transportation option has the lowest total cost like WTR910R. This result can be generalized for all materials if payment is CIF because transportation cost per kg of seaway option is the lowest.

In FOB payment the situation is different because of transportation duration. RTD883D is purchased from Norwegian and seaway transportation time is so long. In 2 months buyer has extra loss of interest rate cost. This time increases total cost much more in comparison to other transportation options. Highway transportation option has lowest cost when company X should have to pay with FOB. Then airway transportation is profitable 599$ according to the seaway option.

Although it is seen that seaway transportation is always cheaper than others and this example shows that sometimes other alternatives can be profitable. In FOB payment we can prove that other highway or airway transportation modes may be more profitable in comparison to seaway transportation numerically. In Appendix B details result are tabled for each option.

5.5 Comparison old System and New System of Company X

The current system which is used in company X is given in Table 5.22. As it is seen in Table 5.22 that the company X needs 12.709 pieces of WTR910R in May. In our forecasting study we found that 12.731 pieces are needed in May. This proves that our forecasting study is not bad. But it does not guarantee that always there is a high forecasting accuracy. This implies forecasting study is the one of the most important part of the problem.

In section 3.3 the forecasting study, purchasing and transportation process of company X are mentioned. company X does not buy 12.654 pieces commonly. Every day MRP is run and company X see that how many material they need in the coming period.

Table 5.33 Material requirement plan of WTR910R after periodic MRP run

MATERIAL REQUIREMENT PLAN AFTER MRP RUN					
WTR910R	January	February	March	April	May
January (01.01.2011) MRP	-	14.963	8.811	5.339	0
January (15.01.2011) MRP	-	15.062	11.927	9.498	0
February (01.02.2011) MRP			12.999	10.897	6.892
February (15.02.2011) MRP			13.098	11.177	8.270
March (01.03.2011) MRP				13.661	9.983
March (15.03.2011) MRP				13.696	11.074
April (01.04.2011) MRP					12.003
April (15.04.2011) MRP					12.709

The benefits of forecasting the sales of May begin to be obtained in the beginning of February. Then company X begins to place order to China. In February company X sees that 8.500 pieces can be ordered to the vendor and this amount of quantity will be delivered by seaway shipment. The material planner of company X observes that the usage of material of the material WTR910R will increase in the coming days and he orders 8.500 pieces although ERP system shows 6892 pieces.

In the first day of March material planner sees that the benefits of the WTR910R 9.983 pieces. The first order 8.500 pieces are not enough to supply the production and he has to order again. For the seaway shipment, there is no time because seaway transportation duration needs 30 days. And vendor needs at least 1 month to produce the second order. Material planner ordered 2.600 pieces of WTR910R and the second order will be delivered by highway transportation. In the second order there is no time for seaway transportation shipment.

In the beginning of April the amount of requirement of the material is still increasing and ERP shows that company X needs 12.003 pieces in May. In the previous two orders 11.100 pieces was ordered. Company X still needs 903 pieces. The problem is there is not enough time to produce the material by vendor and for seaway shipment and highway transportation. In such a situation company X propose the vendor to pay extra cost for additional work time and tooling cost. These additional costs are present as extra cost in Table 5.23. The vendor accepts 903 pieces extra order with extra cost. And this order should be delivered by airway freight.

As it is mentioned before MRP is launched every day and in the middle of April company X sees that extra 706 pieces are needed in May. Company X proposes again to vendor the production of material with extra cost. This proposal is accepted by vendor.

Table 5.34 The cost results of existing system

Order Date	Trasportation Mode	Order Quantity	Total Cost	Extra Cost
February	Seaway	8.500	$240.416	$0
March	Highway	2.600	$74.062	$0
April	Airway	903	$26.199	$1.150
April	Airway	706	$20.615	$7.300
			Grand Total Cost	$369.742

The vendor which produces WTR910R accepts only FOB payment. In Table 5.24 all orders are presented with costs. The grand total cost for May is 369.742 $. If the payment is made as CIF this value will decrease 335 dollar and company X could pay 369.407 $. In all cases the new system is more profitable.

Table 5.35 Comparison of total cost between new system and existing system

Alternatives	$TC_1(Q)$	$TC_2(Q)$	$TC_3(Q)$	$TC_4(Q)$	$TC_5(Q)$	$TC_6(Q)$
Transportation Type	Seaway	Airway	Highway	Seaway	Airway	Highway
Payment	FOB	FOB	FOB	CIF	CIF	CIF
New System	359.697,29	361.422,88	360.219,68	359.249,48	361.378,10	360.071,90
Old System	369.742,21	369.742,21	369.742,21	369.407,38	369.407,38	369.407,38
Difference	10.044,92	8.319,33	9.522,53	10.157,90	8.029,28	9.335,48

Chapter 6

6. Conclusion and Future Research

The company X needs a general inventory planning procedure to reduce the inventory costs and freight cost. Additional to these targets company X wants to work with no safety stock. Because company X works with over than 3000 different material and the amount of safety stock value of material holds huge amount of money.

In real life there are different possibilities and numerical analysis are not valid in some cases. CIF payment can be more profitable as a calculation but if vendor accepts only FOB payment, the existing study is not meaningful. In such a case the all step of the calculation must be done again according to the constraints of the problem.

In this master thesis we determine all cost possibilities related by using the new approach of the EOQ model and we have seen that CIF payment is always more profitable. When observed in detail seaway shipment is advantageous in comparison to airway and highway transportation as total cost. But on the other hand company X should forecast the demand of the period earlier than other transportation options. The reason of that transportation duration of seaway is longer than other alternatives.

If the payment is FOB highway or airway transportation options may have lower total cost in compare to seaway transportation option. Because loss of interest rate cost affects negative total cost of seaway transportation. For this reason it is necessary to calculate and investigate all possible condition to make a decision when supplying the components for production. So according to the different situation there will be different strategy to implement.

Another important point is to have good demand forecast. Because qualitative and high quality demand forecast affects positively stock level of inventory, the frequency of freight and low transportation cost. In order to make long term inventory planning, demand forecasting studies are the key part of the optimization.

In all cases the new system is more profitable than existing system. Demand forecasting accuracy of company X is very bad and it causes extra orders to be made. For each extra order company X should face to pay extra cost like additional work cost, tooling cost and expensive airfreight cost. This costs increases total cost. This analyzes show that demand forecasting studies are very important to reduce total cost.

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Curriculum Vitae

I was in born in Sivas in 23.07.1986. I received my high school degree in İstanbul Erkek Lisesi. I got German education five years long during gymnasium. In 2005 I passed OSS exam and got %100 scholarship from Isık University in Computer Engineering department. After completion of Computer Engineering, I began to educate my MSc degree of Industrial Engineering in Işık University.

I worked as intern in Information Process Center of Işık University during my Bs degree. During this trainee period, I worked on writing flash banners, assisting university network problems, solving network and hardware problems of computer laboratories.

Between 2009 and 2010 I worked as planning engineer in Cihan Mimarlık ve Fuar Hizmetleri. I was preparing daily and weekly production plans and assisting capacity analyze studies.

Between 01.2011 and 07.2011 I have been accepted to company X as a graduate thesis student intern. I was member of project group which work on optimizing the material flows from supplier to inventory. In this period we worked on analyzing quality rate of materials and due-date success of suppliers, determination of due-date of new orders related the previous du-date success of suppliers, minimizing transportation cost with binding close orders, minimizing lot size of orders by using equivalent materials in considering price, weight etc, and classification inventory with AHP II method.

From 05.2012 I work as production planning engineer in Bosch and Siemens Home Appliances Company. I prepare daily and weekly production plan of six different assembly lines.

APPENDIXES A

INVENTORY CLASSIFICATION ANALYSIS

Table A.1 Reuslt of ABC Analysis for Electronic Department – 1

Material Code	Price ($/1000 unit)	Price ($/1 unit)	Annual Consumption (Unit)	Annual Cost ($)	Rank	Percentage %	Cum.%	Class
RTD883D	88.360,00	88,3600	87.360	7.719.129,60	1	0,41	41,22%	A
WTA910R	28.140,00	28,1400	138.684	3.902.567,76	2	0,21	62,06%	A
XGG107R	1.533,65	1,5337	923.004	1.415.565,08	3	0,08	69,62%	A
VRR107R	5.087,50	5,0875	220.796	1.123.299,65	4	0,06	75,61%	A
45TT813R	3.237,50	3,2375	306.544	992.436,20	5	0,05	80,91%	A
454FF1R	12.025,00	12,0250	45.565	547.919,13	6	0,03	83,84%	B
FLMT107R	1.303,70	1,3037	399.484	520.805,29	7	0,03	86,62%	B
453MMSD0R-1	257,15	0,2572	1.056.081	271.571,23	8	0,01	88,07%	B
457MBN6R-1	2.442,00	2,4420	75.071	183.323,38	9	0,01	89,05%	B
453RT077R-5	162,80	0,1628	1.105.295	179.942,03	10	0,01	90,01%	B
7ZY107R	900,58	0,9006	191.988	172.900,55	11	0,01	90,93%	B
VAL107R-A	2.035,00	2,0350	73.880	150.345,80	12	0,01	91,74%	B
282473R	136,90	0,1369	1.072.643	146.844,83	13	0,01	92,52%	B
453MY745R	2.201,50	2,2015	53.324	117.392,79	14	0,01	93,15%	B
452975R-1	775,15	0,7752	149.474	115.864,77	15	0,01	93,77%	B
453MR412R	296,00	0,2960	370.219	109.584,82	16	0,01	94,35%	B
XGS107R-LC	1.572,50	1,5725	68.846	108.260,34	17	0,01	94,93%	B
281FT487R	46,25	0,0463	2.082.135	96.298,74	18	0,01	95,44%	B
281BW488R	66,60	0,0666	1.382.527	92.076,30	19	0,00	95,93%	B
252GH154R	89,36	0,0894	876.569	78.325,82	20	0,00	96,35%	B
282TR103R	42,18	0,0422	1.800.202	75.932,52	21	0,00	96,76%	C
303TR420R	53,65	0,0537	1.065.401	57.158,76	22	0,00	97,06%	C
303TR306R	75,85	0,0759	729.187	55.308,83	23	0,00	97,36%	C
453TR095R-3	101,75	0,1018	511.434	52.038,41	24	0,00	97,64%	C
453TR263R-4	181,30	0,1813	282.390	51.197,31	25	0,00	97,91%	C
170404R	2,92	0,0029	15.390.318	45.014,37	26	0,00	98,15%	C
XKU107R	1.498,50	1,4985	21.169	31.721,75	27	0,00	98,32%	C
303278R	129,50	0,1295	220.789	28.592,18	28	0,00	98,47%	C
454408R	527,25	0,5273	53.324	28.115,08	29	0,00	98,62%	C
281490R	118,40	0,1184	186.515	22.083,38	30	0,00	98,74%	C
170404G	2,92	0,0029	7.328.685	21.435,30	31	0,00	98,85%	C
304424R	70,30	0,0703	299.554	21.058,65	32	0,00	98,97%	C
5FZ107R	627,34	0,6273	29.396	18.441,14	33	0,00	99,07%	C

Table A.2 Result of ABC Analysis for Electronic Department - 2

Material Code	Price ($/1000 unit)	Price ($/1 unit)	Annual Consumption (Unit)	Annual Cost ($)	Rank	Percentage %	Cum.%	Class
173116R	0,48	0,0005	33.299.744	16.140,39	34	0,00	99,15%	C
281491R	38,85	0,0389	385.180	14.964,24	35	0,00	99,23%	C
179008R	0,55	0,0005	23.703.325	13.023,79	36	0,00	99,30%	C
303894R	12,77	0,0128	868.225	11.082,89	37	0,00	99,36%	C
251478R	8,88	0,0089	1.156.184	10.266,91	38	0,00	99,42%	C
304431R	12,03	0,0120	821.647	9.880,31	39	0,00	99,47%	C
173100R	0,55	0,0005	14.114.646	7.755,29	40	0,00	99,51%	C
172117R	0,66	0,0007	11.457.882	7.588,56	41	0,00	99,55%	C
251116R	8,88	0,0089	824.410	7.320,76	42	0,00	99,59%	C
303189R	18,50	0,0185	379.622	7.023,01	43	0,00	99,63%	C
303843G	13,69	0,0137	499.878	6.843,33	44	0,00	99,66%	C
303532R	22,57	0,0226	255.710	5.771,37	45	0,00	99,69%	C
172104R	0,55	0,0005	10.037.051	5.514,86	46	0,00	99,72%	C
304250R	2,00	0,0020	2.682.480	5.359,60	47	0,00	99,75%	C
170132R	2,92	0,0029	1.783.479	5.216,41	48	0,00	99,78%	C
250115R	8,88	0,0089	469.353	4.167,85	49	0,00	99,80%	C
179001G	0,98	0,0010	3.834.903	3.760,12	50	0,00	99,82%	C
172114R	0,48	0,0005	7.097.099	3.439,96	51	0,00	99,84%	C
170220R	0,54	0,0005	6.023.795	3.231,77	52	0,00	99,86%	C
173223R	0,48	0,0005	6.645.130	3.220,89	53	0,00	99,87%	C
173223G	0,46	0,0005	6.925.333	3.202,97	54	0,00	99,89%	C
250220R	9,99	0,0100	312.946	3.126,33	55	0,00	99,91%	C
170476G	0,46	0,0005	5.630.050	2.603,90	56	0,00	99,92%	C
252467R	46,62	0,0466	50.271	2.343,63	57	0,00	99,94%	C
252244R	26,64	0,0266	65.672	1.749,50	58	0,00	99,94%	C
172336R	0,55	0,0005	2.878.123	1.581,38	59	0,00	99,95%	C
VKK920R	8.510,00	8,5100	184	1.565,84	60	0,00	99,96%	C
171478G	0,46	0,0005	3.016.701	1.395,22	61	0,00	99,97%	C
173123R	1,02	0,0010	1.285.305	1.305,42	62	0,00	99,98%	C
453962R	777,00	0,7770	1.648	1.280,50	63	0,00	99,98%	C
171472R	0,55	0,0005	2.293.868	1.260,37	64	0,00	99,99%	C
170406R	3,44	0,0034	364.556	1.252,41	65	0,00	100,00%	C
173153R	0,55	0,0005	1.367.163	751,19	66	0,00	100,00%	C
			Total Cost ($)	**18.727.542,76**				

APPENDIXES B

APPLICATION OF THE MODEL FOR RTD883D

Table B.1 General Information about RTD883D

Demand per month		λ	unit / month	7.280
Order (setup cost)		K	$	50
Unit holding cost per month		h	$ / unit*month	0,110450
Unit purchasing cost		c	$ / unit	88,36
Unit weight of material		w	kg / unit	0,145
Transportation time of seaway	*	t	month	2
Transportation time of airway	**	t	month	0,1
Transportation time of highway	***	t	month	0,5
Seaway transportation cost		β	$ / kg	0,11
Airway transportation cost		β	$ / kg	0,99
Highway transportation cost		β	$ / kg	0,45
Fixed transportation cost		φ	$	100
Fixed importation cost		δ	$	450

* Seaway transportation duration is 30 days from Norwegian to Turkey

** Airway transportation duration is 3 days from Norwegian to Turkey

*** Highway transportation duration is 10 days from Norwegian to Turkey

Assumptions :

1- No shortages are allowed

2- No safety stcok will be held

Table B.2 Input values for $TC_1(Q)$ – Seaway and CIF Payment

Demand per month	λ	unit / month	7.280
Order (setup cost)	K	$	50
Unit holding cost per month	h_1	$ / unit * month	0,11045
Transportation holding cost	h_2	$ / unit * month	0
Unit purchasing cost	c	$ / unit	88,36
Unit weight of material	w	kg / unit	0,145
Seaway transportation cost	β	$ / kg	0,11
Fixed transportation cost	φ	$	100
Fixed importation cost	δ	$	450
Transportation time of seaway	t	month	2

Table B.3 Results of $TC_1(Q)$ – Seaway and CIF Payment

Demand per month	λ	unit / month	7.280
Order quantity	Q	unit	8.894
Order interval	T	month	1,22
Transportation cost		$ / month	566,33
Transportation holding cost		$ / month	0,00
Total material cost	c*λ	$ / month	643.260,80
Holding cost of inventory	h_1*Q/2	$ / month	491,14
Total ordering cost	K*λ/Q	$ / month	40,93
Grand total cost	$TC_1(Q)$	$ / month	644.359,20

Table B.4 Input values for $TC_2(Q)$ – Seaway and FOB Payment

Demand per month	λ	unit / month	7.280
Order (setup cost)	K	$	50
Unit holding cost per month	h_1	$ / unit * month	0,11045
Transportation holding cost	h_2	$ / unit * month	0,11045
Unit purchasing cost	c	$ / unit	88,36
Unit weight of material	w	kg / unit	0,145
Seaway transportation cost	β	$ / kg	0,11
Fixed transportation cost	φ	$	100
Fixed importation cost	δ	$	450
Transportation time of seaway	t	month	2

Table B.5 Results of $TC_2(Q)$ – Seaway and FOB Payment

Demand per month	λ	unit / month	7.280
Order quantity	Q	unit	8.894
Order interval	T	month	1,22
Transportation cost		$ / month	566,33
Transportation holding cost		$ / month	1.608,15
Total material cost	c*λ	$ / month	643.260,80
Holding cost of inventory	h_1*Q/2	$ / month	491,14
Total ordering cost	K*λ/Q	$ / month	40,93
Grand total cost	$TC_2(Q)$	$ / month	645.967,36

Table B.6 Input values for $TC_3(Q)$ – Airway and CIF Payment

Demand per month	λ	unit / month	7.280
Order (setup cost)	K	$	50
Unit holding cost per month	h_1	$ / unit * month	0,11045
Transportation holding cost	h_2	$ / unit * month	0
Unit purchasing cost	c	$ / unit	88,36
Unit weight of material	w	kg / unit	0,145
Airway transportation cost	β	$ / kg	0,99
Fixed transportation cost	φ	$	100
Fixed importation cost	δ	$	450
Transportation time of airway	t	month	0,1

Table B.7 Results of $TC_3(Q)$ – Airway and CIF Payment

Demand per month	λ	unit / month	7.280
Order quantity	Q	unit	8.894
Order interval	T	month	1,22
Transportation cost		$ / month	1.495,26
Transportation holding cost		$ / month	0,00
Total material cost	c*λ	$ / month	643.260,80
Holding cost of inventory	h_1*Q/2	$ / month	491,14
Total ordering cost	K*λ/Q	$ / month	40,93
Grand total cost	$TC_3(Q)$	$ / month	645.288,13

Table B.8 Input values for $TC_4(Q)$ – Airway and FOB Payment

Demand per month	λ	unit / month	7.280
Order (setup cost)	K	$	50
Unit holding cost per month	h_1	$ / unit * month	0,11045
Transportation holding cost	h_2	$ / unit * month	0,11045
Unit purchasing cost	c	$ / unit	88,36
Unit weight of material	w	kg / unit	0,145
Airway transportation cost	β	$ / kg	0,99
Fixed transportation cost	φ	$	100
Fixed importation cost	δ	$	450
Transportation time of airway	t	month	0,1

Table B.9 Results of $TC_4(Q)$ – Airway and FOB Payment

Demand per month	λ	unit / month	7.280
Order quantity	Q	unit	8.894
Order interval	T	month	1,22
Transportation cost		$ / month	1.495,26
Transportation holding cost		$ / month	80,41
Total material cost	c*λ	$ / month	643.260,80
Holding cost of inventory	h_1*Q/2	$ / month	491,14
Total ordering cost	K*λ/Q	$ / month	40,93
Grand total cost	$TC_4(Q)$	$ / month	645.368,54

Table B.10 Input values for $TC_5(Q)$ – Highway and CIF Payment

Demand per month	λ	unit / month	7.280
Order (setup cost)	K	$	50
Unit holding cost per month	h_1	$ / unit * month	0,11045
Transportation holding cost	h_2	$ / unit * month	0
Unit purchasing cost	c	$ / unit	88,36
Unit weight of material	w	kg / unit	0,145
Highway transportation cost	β	$ / kg	0,45
Fixed transportation cost	φ	$	100
Fixed importation cost	δ	$	450
Transportation time of highway	t	month	0,5

Table B.11 Results of $TC_5(Q)$ – Highway and CIF Payment

Demand per month	λ	unit / month	7.280
Order quantity	Q	unit	8.894
Order interval	T	month	1,22
Transportation cost		$ / month	925,24
Transportation holding cost		$ / month	0,00
Total material cost	c*λ	$ / month	643.260,80
Holding cost of inventory	h_1*Q/2	$ / month	491,14
Total ordering cost	K*λ/Q	$ / month	40,93
Grand total cost	$TC_5(Q)$	$ / month	644.718,11

Table B.12 Input values for $TC_6(Q)$ – Highway and FOB Payment

Demand per month	λ	unit / month	7.280
Order (setup cost)	K	$	50
Unit holding cost per month	h_1	$ / unit * month	0,11045
Transportation holding cost	h_2	$ / unit * month	0,11045
Unit purchasing cost	c	$ / unit	88,36
Unit weight of material	w	kg / unit	0,145
Highway transportation cost	β	$ / kg	0,45
Fixed transportation cost	φ	$	100
Fixed importation cost	δ	$	450
Transportation time of highway	t	month	0,5

Table B.13 Results of $TC_6(Q)$ – Highway and FOB Payment

Demand per month	λ	unit / month	7.280
Order quantity	Q	unit	8.894
Order interval	T	month	1,22
Transportation cost		$ / month	925,24
Transportation holding cost		$ / month	402,04
Total material cost	c*λ	$ / month	643.260,80
Holding cost of inventory	h_1*Q/2	$ / month	491,14
Total ordering cost	K*λ/Q	$ / month	40,93
Grand total cost	$TC_6(Q)$	$ / month	645.120,15