PARTNER SELECTION AND RESOURCE ALLOCATION IN SINGLE-CELL, MULTI-CELL AND COGNITIVE COOPERATIVE MULTIPLE ACCESS CHANNELS

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Abstract

Wireless communication has been widely used across the globe for several years. As the technology advances and gains popularity, more people start using it and more throughput is needed. For that reason, several techniques are being researched such as MIMO systems and cooperative networks. Wireless cooperative networks make use of the natural property of radiation of electromagnetic waves. Since the waves emitted by a user in the network can be heard by all users in the network, one cooperating partner uses this overheard information to increase throughput. However, to maximize the throughput of a system, cooperating partners must be selected intelligently. In this dissertation, first, we will summarize cooperative communication basics, techniques we used in convex optimization and graph theory. Then, we will show how these concepts can be used together to optimally maximize system throughput and propose lower complexity yet nearly-optimal partner selection algorithm will be proposed. Under the light of the results of this work, in the next chapters, we will introduce a novel fractional frequency reuse scheme which encourages users to cooperate and allow system to support more users. In the last chapter, a cognitive scenario will be used for one cell and we will present the optimal partner selection scheme for system throughput maximization.

TEK ALICILI, ÇOK ALICILI VE BİLİŞSEL İŞBİRLİKÇİ ÇOKLU ERİŞİM KANALLARINDA KAYNAK TAHSİSİ VE İŞBİRLİKÇİ PARTNER SEÇİMİ

Özet

Kablosuz haberleşme tüm dünyada gittikçe yaygınlaşan bir kullanıma sahiptir. Teknoloji ilerledikçe, artan kullanım doğrultusunda daha yüksek veri hızlarına çıkma ihtiyacı doğmaktadır. Bu sebeple, işbirlikçi haberleşme, çoklu alıcılı çoklu vericili sistemler geliştirilmektedir. İşbirlikçi haberleşme, elektromanyetik dalgaların yayılım prensibinden faydalanır. Yayılan dalgalar, sistemdeki tüm kullanıcılar tarafından duyulur ve bu durum, veri hızını artırmak amacı ile kullanılmaktadır. Buna rağmen, sistemin toplam veri hızını eniyilemek için, işbirlikçi partnerlerin seçimi de akıllıca yapılmalıdır. Bu tezde, ilk önce işbirlikçi telsiz haberleşme temellerini, kullanılan eniyileme ve çizge kuramı tekniklerini özetleyeceğiz. Sonrasında, bu tekniklerin sistemin toplam veri hızını eniyilemek için beraber kullanımını gösterip, düşük karmaşıklığa sahip eniyiye yakın sonuç veren algoritmalar önereceğiz. Bu çalışmanın ışığında, sonraki bölümlerde işbirliğini artırıp daha fazla kullanıcı desteklenmesini sağlayan özgün bir parçalı frekans tekrarı sistemi tanıtacağız. Son bölümde ise, işbirlikçi bilişsel tek hücreli telsiz sistemlede sistem veri hızını en iyileyen kaynak tahsisi ve partner seçimi algoritmaları tanıtılacaktır.

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To my family…

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List of Abbreviations

MAC	**M**ultiple **A**ccess **C**hannel
FFR	**F**ractional **F**requency **R**euse
GSM	**G**lobal **S**ystem for **M**obile Communications
SNR	**S**ignal to **N**oise **R**atio
OFDMA	**O**rthoganal **F**requency **D**ivision **M**ultiple **A**ccess
TDMA	**T**ime **D**ivision **M**ultiple **A**ccess
CDMA	**C**ode **D**ivision **M**ultiple **A**ccess
I.I.D.	**I**ndependent **I**dentically **D**istributed
MWM	**M**aximum **w**eighted **M**atching
KKT	**K**arush-**K**uhn-**T**ucker
MAC-GF	**M**ulti **A**ccess **C**hannel with **G**eneralized **F**eedback

Chapter 1

Introduction

The widespread use of wireless communication technologies in densely populated environments, brings along the need to revise the traditional frequency reuse and orthogonal multiple access techniques, and to build new models that accommodate more advanced opportunistic approaches. Therefore, the concept of cooperative communication arises naturally in wireless channels, due to their propagative properties. The users in a wireless network can overhear each other's signals, and with clever protocol design, they may aid each other's transmissions to combat the challenging channel conditions, in order to achieve better performance.

The term "user cooperation" is best suited for systems with mutually cooperating encoders, where all cooperating parties have their own messages to be transmitted. One of the pioneering works, which demonstrated the potential gains from user cooperation is [1], which deals with a two user fading Gaussian MAC with overheard information. It was shown in [1] that the users may increase their transmission rates considerably if they cooperate, and that the improvement in rates depends highly on the channel conditions in the system. In [2], the achievable rate region introduced in [1] was extended to include channel adaptive power allocation, and the optimum power control strategy was derived. In [2], authors have studied cooperation between two users yet, in practical wireless communication systems, more users are generally supported. Since in the network the channel conditions for different user groups are highly variable, in order to maximize the benefit from user cooperation, cooperating partners should be selected efficiently. This leads to the problem named "partner selection" in the literature. To this end, several strategies for partner selection in wireless networks have been developed in the literature. An SNR threshold based partner selection algorithm was proposed in [3] to reduce the error probability, or to increase the system throughput. A user location information based partner selection algorithm using maximum weighted matching for an amplify-and-forward relaying scheme was studied in [4] with the aim of minimizing total system transmission power.

The models used while dealing with the partnering problem usually involve some form of orthogonality across the user pairs, so that the pairs can cooperate without causing interference to each other. OFDMA, which has gained a lot of popularity in the recent years because of its several desirable properties, is a good candidate for realizing practical cooperation, due to its orthogonal structure. There is quite an extensive amount of work on power and subchannel allocation schemes for OFDMA, some examples of which are [5], [6], [7] and [8]. Yet, encoding techniques, and resource allocation for mutually cooperative OFDMA systems, have not been investigated much until rather recently. For cooperative OFDMA systems containing only two users, achievable rates based on mutual cooperation across subchannels were characterized in [9], and for such systems, optimal power allocation algorithms was developed in [10].

Partner selection in OFDMA has also been considered recently by several works in the literature. A related work [11] deals with a system which uses amplify-and-forward relaying scheme for OFDMA with only half-duplex user cooperation, where the benefit of partner selection is observed in the form of a significant reduction of total transmission power. The partner selection algorithm proposed in [12] applies a game theoretical approach on selecting partners for OFDMA systems.

In this dissertation, the work presented in Chapter 3 is focused on partner selection in cooperative wireless networks with power controlled OFDMA. The partner selection and resource allocation problems affecting each other and should be solved jointly. With the help the orthogonality provided by OFDMA, the power control policy can be applied to each cooperating pair independently of other pairs. Therefore, optimal partner selection algorithms can be applied after power control. These two steps of power control and partner selection algorithms are applied optimally. With the knowledge gained, some near-optimal practical heuristic algorithms are presented in order to decrease computational complexity. In this part of the study, one cell is considered and inter-cell interference is omitted.

As wireless communication usage increases across the globe, more users are needed to be supported, which leads engineers to focus on improving traditional frequency reuse methods. As a solution to this problem, fractional frequency reuse (FFR) is gaining popularity in the literature. FFR systems increases number of supported users with a cost of increased inter-cell interference, hence FFR systems becomes interference limited systems. The most common FFR scheme, "Strict FFR", divides users into two sets according to their distances to the receiver, and assigns different frequency bands to each sets. There are several studies such as [13] on finding the optimal distance rule to divide users according to inter-cell interference and throughput. However, "Strict FFR" is not designed with cooperation in mind, therefore cannot encourage the users to use cooperation for higher gains. In Chapter 4, we propose a novel FFR scheme called "Complementary FFR" which mainly focuses to encourage cell-edge users to cooperate with each other despite being in different cells. "Complementary FFR" with optimal partner selection, increases the total throughput of the system and provides improved fairness in comparison to the one-cell setup studied in Chapter 3. The work is presented in details in Chapter 4.

Through years, many frequency bands are licensed and actively in use. However, these limited usable frequencies start to fail to serve to high demand in wireless communications. Thus, several overcomings of this problem are studied such as cognitive radio. In cognitive radio, an additional user (secondary user) starts to send its own signals in the same frequency band of primary user, the owner of the communication channel. The main constraint that distinguishes cognitive radio from traditional systems is primary user must not be affected by the secondary users involvement to the communication channel. In Chapter 5, one-cell cognitive cooperative setup with optimal partner selection is studied thoroughly. Primary users communication is guarantied not to be affected negatively and possible transmission rate gain opportunities by cooperation are investigated. Selection of optimal partners consisting of primary and secondary users to maximize system total transmission rate is found.

Chapter 2

Background Theory

In this dissertation, we have solved various complex problems by using various techniques from multiple disciplines such as information theory, communication theory, convex optimization and graph theory. Each of the problems contain various concepts and it is useful to summarize the underlying theory briefly.

2.1 Gaussian Channel Capacity

Gaussian channel is the basis of modern communication theory introduced by Shannon. Gaussian channel is a time-discrete, continuous alphabet channel with independent identically distributed (i.i.d.) additive white noise at output.

$$Y_i = X_i + Z_i \quad \text{where} \quad Z \sim \mathcal{N}(0, N). \tag{2.1}$$

where X_i and Y_i are the input and output signals respectively and Z_i is the additive noise. The limit on mutual information between input and output is the

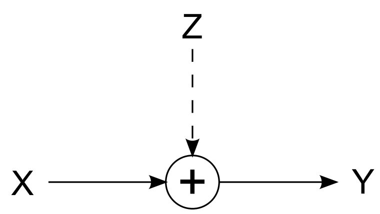


Figure 2.1: Gaussian channel model

information capacity of the Gaussian channel. The information capacity for a Gaussian channel with power constraint is defined as

$$C \triangleq \max_{E[X^2] \leq \bar{P}} I(X;Y) \tag{2.2}$$

where $\bar{P}$ is the power constraint and and $E[X^2]$ is expectation of the power of input signal X. We can calculate the information capacity by expanding mutual information between X and Y,

$$\begin{aligned} I(X;Y) &= h(Y) - h(Y|X) && (2.3) \\ &= h(Y) - h(X+Z|X) && (2.4) \\ &= h(Y) - h(Z|X) && (2.5) \\ &= h(Y) - h(Z) && (2.6) \end{aligned}$$

since Z is independent of X. Since $h(Z) = \frac{1}{2}\log 2\pi e N$ is known, we can calculate

$$E[Y^2] = E[(X+Z)^2] = E[X^2] + 2E[X]E[Z] + E[Z^2] = P + N. \tag{2.7}$$

By using this equality in equation (2.6), we get

$$\begin{aligned} I(X;Y) &= h(Y) - h(Z) && (2.8) \\ &\leq \frac{1}{2}\log\Big(2\pi e(P+N)\Big) - \frac{1}{2}\log(2\pi e N) && (2.9) \\ &= \frac{1}{2}\log\left(1 + \frac{P}{N}\right) && (2.10) \end{aligned}$$

Therefore, the capacity is found as

$$C \triangleq \max_{E[X^2] \leq P} I(X;Y) = \frac{1}{2}\log\left(1 + \frac{P}{N}\right) \tag{2.11}$$

since the maximum is achieved when $X \sim \mathcal{N}(0, P)$.

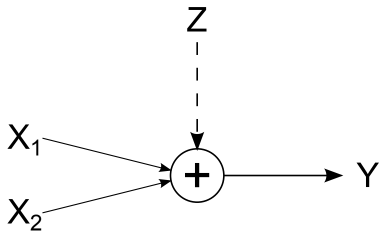


Figure 2.2: A simple two-user multiple access channel model.

2.2 Gaussian Multiple Access Channels

Multiple access channels (MAC) can be defined as a Gaussian channel, where more than one users are transmitting their own signals to be decoded by one receiver. Since the transmission channel is the same, the signals are received as sum of all signals at the receiver. The receiver can decode the signals one by one, treating other signals as noise. For a multiple access channel with two users transmitting to a common receiver, the received signal at time instant i is

$$Y_i = X_{1i} + X_{2i} + Z_i \tag{2.12}$$

where Z_i is i.i.d. random variable

$$Z \sim \mathcal{N}(0, N). \tag{2.13}$$

All the users in the channel has power constraints,

$$E[X_1^2] \leq \bar{P}_1 \tag{2.14}$$

$$E[X_2^2] \leq \bar{P}_2 \tag{2.15}$$

the capacity regions of users in Gaussian MAC is the convex hull of rates satisfying the constraints,

$$R_1 \le I(X_1;Y|X_2) \tag{2.16}$$

$$R_2 \le I(X_2;Y|X_1) \tag{2.17}$$

$$R_1 + R_2 \le I(X_1, X_2;Y) \tag{2.18}$$

for inputs $X_i \sim \mathcal{N}(0, P_i),\ i \in \{1, 2\}$.

The derivation of the rate regions is straightforward extension to equations (2.8)-(2.10).

$$\begin{aligned}
I(X_1;Y|X_2) &= h(Y) - h(Y|X_1, X_2) && (2.19)\\
&= h(X_1 + X_2 + Z|X_2) - h(X_1 + X_2 + Z|X_1, X_2) && (2.20)\\
&= h(X_1 + Z|X_2) - h(Z|X_1, X_2) && (2.21)\\
&= h(X_1 + Z|X_2) - h(Z) && (2.22)\\
&= h(X_1 + Z) - h(Z) && (2.23)\\
&= h(X_1 + Z) - \frac{1}{2}\log(2\pi eN) && (2.24)\\
&= \frac{1}{2}\log\Big(2\pi e(P_1 + N)\Big) - \frac{1}{2}\log(2\pi eN) && (2.25)\\
&\le \frac{1}{2}\log(1 + \frac{P_1}{N}) && (2.26)
\end{aligned}$$

similarly;

$$R_2 = \frac{1}{2}\log(1 + \frac{P_2}{N}). \tag{2.27}$$

For sum rate constraint,

$$
\begin{aligned}
R_1 + R_2 &\le I(X_1, X_2; Y) && (2.28)\\
&= h(Y) - h(Y|X_1, X_2) && (2.29)\\
&= h(X_1 + X_2 + Z) - h(X_1 + X_2 + Z|X_1, X_2) && (2.30)\\
&= h(X_1 + X_2 + Z) - h(Z|X_1, X_2) && (2.31)\\
&= h(X_1 + X_2 + Z) - h(Z) && (2.32)\\
&= \frac{1}{2}\log\Big(2\pi e(P_1 + P_2 + N)\Big) - \frac{1}{2}\log(2\pi e N) && (2.33)\\
&\le \frac{1}{2}\log(1 + \frac{P_1 + P_2}{N}) && (2.34)
\end{aligned}
$$

can be found.

User one and two generate codebooks of gaussian random codewords and send transmit them in the channel simultaneously. Since the transmitters send codewords from their private codebook arbitrarily, the receiver starts to decode one users signal treating other user's signal as noise.

$$R_1 \le \frac{1}{2}\log(1 + \frac{P_1}{P_2 + N}) \qquad (2.35)$$

After decoding the signal of user 1, the receiver subtracts first user's signal and decodes second users signal at the rate

$$R_2 \le \frac{1}{2}\log(1 + \frac{P_2}{N}) \qquad (2.36)$$

One of the most important concept about MAC is, if we generalize this channel to m users, the total rate will be

$$R_1 + R_2 + \ldots + R_m \le \frac{1}{2}\log(1 + \frac{mP}{N}), \qquad (2.37)$$

therefore, as $m \to \infty$, the sum rate also goes infinity, yet individual rates on the average will be

$$R_i \leq \frac{1}{2m} \log(1 + \frac{mP}{N}), \quad i \in \{1, 2, \ldots, m\}. \tag{2.38}$$

2.3 Parallel Gaussian Channels

In wireless communication systems, some techniques divide the communication channel into subchannels and transmitter is assigned to multiple subchannels for communication. Thus, transmitter needs to distribute powers according to a common power constraint. This approach is extremely important against nonwhite Gaussian noise since all subchannel components will represent different frequencies and be affected by noises at different levels. The capacity of the system can be found by optimal distribution of power to subchannels. The system can be represented as,

$$Y_j = X_j + Z_j, \quad j = 1, 2, \ldots, k, \tag{2.39}$$

where Z_j is i.i.d. as

$$Z_j \sim \mathcal{N}(o, N_j). \tag{2.40}$$

for channel j. The capacity of the system can be represented as,

$$C = \max_{\sum_1^k E[X_i^2] \leq \bar{P}} I(X_1, X_2, \ldots, X_k; Y_1, Y_2, \ldots, Y_k). \tag{2.41}$$

Similar to the capacity derivation of gaussian channel in equations (2.3)-(2.6), capacity of parallel Gaussian channels can be derived as,

$$I(X_1,X_2,\ldots,X_k;Y_1,Y_2,\ldots,Y_k) \tag{2.42}$$
$$= h(Y_1,Y_2,\ldots,Y_k) - h(Y_1,Y_2,\ldots,Y_k|X_1,X_2,\ldots,X_k) \tag{2.43}$$
$$= h(Y_1,Y_2,\ldots,Y_k) - h(Z_1,Z_2,\ldots,Z_k|X_1,X_2,\ldots,X_k) \tag{2.44}$$
$$= h(Y_1,Y_2,\ldots,Y_k) - h(Z_1,Z_2,\ldots,Z_k) \tag{2.45}$$
$$= \sum_i h(Y_i) - \sum_i h(Z_i) \tag{2.46}$$
$$\leq \sum_i h(Y_i) - h(Z_i) \tag{2.47}$$
$$\leq \sum_i \frac{1}{2}\log\left(1+\frac{P_i}{N_i}\right) \tag{2.48}$$

and maximum is achieved when X is i.i.d and $X_i \sim \mathcal{N}(0,P_i)$.

The optimal power allocation scheme that maximizes capacity subject to common power constraint in parallel Gaussian channels can be found by using Lagrange multipliers. Lagrangian can be expressed as

$$\mathcal{L}(P_1,\ldots,P_k) = \sum_i \frac{1}{2}\log\left(1+\frac{P_i}{N_i}\right) + \lambda\Big(\sum_i P_i\Big), \tag{2.49}$$

and differentiating with respect to P_i, we get

$$\frac{1}{2}\frac{1}{P_i+N_i} + \lambda = 0 \tag{2.50}$$

which can be turned into

$$P_i = (v - N_i) \tag{2.51}$$

where, $v = -\frac{1}{2\lambda}$. Since P_i cannot be negative, the equation becomes

$$P_i = (v - N_i)^+, \tag{2.52}$$

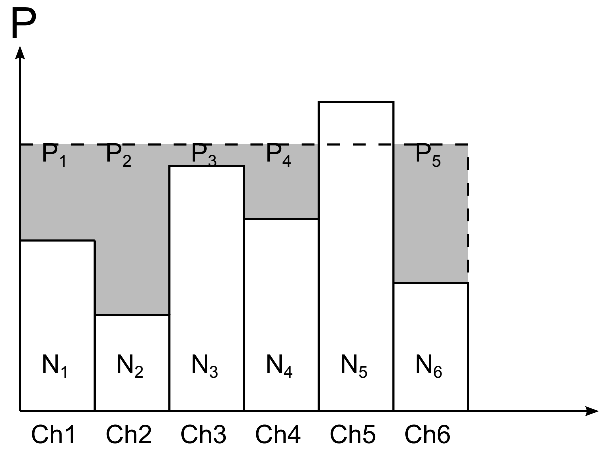


Figure 2.3: Waterfilling in parallel Gaussian channels. The dashed line represents the optimum power level and gray area under it is the powers used for each channel.

therefore we can choose v such that

$$\sum_{1}(v - N_i)^+ = \bar{P}. \tag{2.53}$$

The result of this derivation can be interpreted as, if there are multiple channels with different noise levels, the available power must be distributed such that, sum of noise and transmission powers in channels must be equal to each other, just like filling a pool with water. Therefore, this solution is referred as *waterfilling* in the literature.

2.4 Orthogonal Frequency Division Multiple Access

Wireless networks employ several techniques to assign channels to users in a multiple user scenario, for example, GSM systems use time division multiple access (TDMA), 3G networks use code division multiple access (CDMA) and 4G networks use Frequency division multiple access (FDMA). Each techniques have several advantages, yet FDMA is gaining popularity in recent years.

FDMA is mainly used to overcome problems caused by non-white noise in channels. In a system with frequency selective noise, optimizing the power distribution is a complicated task. To simplify this problem, we can divide the channel into subchannels with sufficiently small bandwidths so that the noise in each subchannel can be considered as white. Orthogonal frequency division multiple access (OFDMA) is a special case of OFDMA, where all the channels are orthogonal to each other and each user is assigned a subcarrier. This orthogonality isolates transmissions from each other and removes interference.

2.5 User Cooperation

Wireless communication is based on transmission of electromagnetic waves as information. If there are multiple transmitters are in the system, the transmitted signals overlap on each other and creates interference at the receiver. Although the interference seems to damage the performance of the system, by intelligent design using user diversity, the overlap of transmitted signals can be converted into an advantage. This groundbreaking cooperation system was proposed in [1]. The received signals at can be formulated as,

$$Y_0(t) = h_{10}X_1(t) + h_{20}X_2(t) + Z_0(t) \tag{2.54}$$

$$Y_1(t) = h_{21}X_1(t) + Z_1(t) \tag{2.55}$$

$$Y_2(t) = h_{12}X_1(t) + Z_2(t) \tag{2.56}$$

where $Y_0(t), Y_1(t), Y_2(t)$ are the baseband models of the received signals at receiver, user 1 and user 2 respectively. $X_i(t)$ is the signal transmitted by user i for $i = 1, 2$ and $Z_j(t)$ are the additive channel noise terms at receiver, user 1 and user 2 for $j = 0, 1, 2$ respectively. The noise terms are distributed as $Z_j \sim \mathcal{N}(0, \sigma_j^2)$, where in general we can assume $\sigma_0^2 = \sigma_1^2 = \sigma_2^2$. The fading coefficients h_{ij} are assumed to be constant over a symbol period, and perfectly known. The cooperation strategy is based on superposition block Markov [14] encoding and backward decoding [15, 16].

Each user in the system divides own message into three parts, for first user, X_{10} is transmitted to receiver directly, X_{12} is transmitted to the cooperating user to be sent to receiver and U_1 is the cooperation signal. The sent signal can be expressed as,

$$X_1 = X_{10} + X_{12} + U_1 \tag{2.57}$$

and respective powers,

$$P_1 = P_{10} + P_{12} + P_{U1}. \tag{2.58}$$

It should be noted that, the signal X_{12} must be perfectly decoded at second user since this transmission is basis for cooperation. Therefore, P_{12} must be selected according to that rule. As the number of transmitted blocks goes to infinity, under the light of findings in [17] and [18], the achievable rates can be calculated as

$$R_{12} \leq E\Big[\frac{1}{2}\log(1+\frac{h_{12}P_{12}}{h_{12}P_{10}+\sigma_2^2})\Big] \tag{2.59}$$

$$R_{21} \leq E\Big[\frac{1}{2}\log(1+\frac{h_{21}P_{21}}{h_{21}P_{20}+\sigma_1^2})\Big] \tag{2.60}$$

$$R_{10} \leq E\Big[\frac{1}{2}\log(1+\frac{h_{10}P_{10}}{\sigma_0^2})\Big] \tag{2.61}$$

$$R_{20} \leq E\Big[\frac{1}{2}\log(1+\frac{h_{20}P_{20}}{\sigma_0^2})\Big] \tag{2.62}$$

$$R_{10}+R_{20} \leq E\Big[\frac{1}{2}\log(1+\frac{h_{10}P_{10}+h_{20}P_{20}}{\sigma_0^2})\Big] \tag{2.63}$$

$$R_{10}+R_{20}+R_{12}+R_{21} \leq E\Big[\frac{1}{2}\log(1+\frac{h_{10}P_1+h_{20}P_2+2h_{10}h_{20}\sqrt{P_{U1}P_{U2}}}{\sigma_0^2})\Big] \tag{2.64}$$

where $P_1 = P_{10} + P_{12} + P_{U1}$ and $P_2 = P_{20} + P_{21} + P_{U2}$.

2.6 Maximum Weighted Matching

In graph theory, matching is defined as a set of independent edges without common vertices in a graph. On weighted graphs, each matching gets the weight of the vertice. Therefore, maximum weighted matching can be defined as the matching with maximum total weights of vertices. For maximum weighted matching, Jack Edmonds has found an algorithm presented in [19] and Harold Gabow has implemented the algorithm efficiently in [20]. There are several works in the literature, yet Gabow's implementation is the most popular one.

The powerful idea behind maximum weighted matching is, it searches for augmenting paths in the graph, then finds maximum weighted matching. Yet, to find augmenting paths, Edmond's algorithm searches for blossoms first, where a blossom is defined as a graph with n vertices in which every subgraph of $n1$ vertices has a perfect matching. Then, the algorithm morphs the blossoms into augmenting paths and continues the search. The worst case complexity of the algorithm is $O(n^3)$ in Gabow's implementation.

2.7 Karush-Kuhn-Tucker Conditions

In section 2.3, widely known optimization tool Lagrange multipliers were used. In this dissertation, we use a more generalized version of Lagrange multipliers, with Karush-Kuhn-Tucker (KKT) conditions. KKT conditions is an nonlinear optimization technique which ensures optimality provided that some regularity conditions are satisfied.

Let's consider a nonlinear optimization problem where we want to minimize function $L(x)$, with given constraint,

$$\min L(x) \tag{2.65}$$

$$\text{s.t.} \quad f(x) \leq 0 \tag{2.66}$$

and x_0 is the optimal value to be found. In that case, the constraint is active if

$$\left.\frac{\partial L}{\partial x}\right|_{x_0} = 0, \tag{2.67}$$

or the constraint is inactive for all admissible values of x when

$$\left.\frac{\partial L}{\partial x}\right|_{x \in S} = 0 \tag{2.68}$$

where S is the set of all admissible values of x. Therefore,

$$\frac{\partial L}{\partial x} + \lambda\frac{\partial f}{\partial x} = 0, \text{for} \lambda \geq 0. \tag{2.69}$$

We can write the Lagrangian as

$$\mathcal{L}(x, \lambda) = L(x) + \lambda f(x) \tag{2.70}$$

where the necessary conditions are

$$\frac{\partial \mathcal{L}}{\partial x} = 0 \tag{2.71}$$

$$f(x) \geq 0, \tag{2.72}$$

and

$$\lambda \geq 0, \text{if} f(x) = 0, \tag{2.73}$$

$$\lambda = 0, \text{if} f(x) < 0 \tag{2.74}$$

2.8 Fractional Frequency Reuse

The main consideration of wireless networks is using limited band of frequencies as efficient as possible. The system must be designed cleverly that it should support many users and all users should be able to use as much bandwidth as possible, yet still keep the interference at a acceptable level. It is a hard task and several approaches have been developed through the years.

In traditional cellular wireless networks, each receiver has a coverage area and a frequency band is used in that cell. The group of cells that use the whole bandwidth available are called and named cluster. The same frequency bands are reused in other cells in other clusters, however by intelligent design, these cells' distances are maximized to lower interference between these cells. The number of frequency bands that are repeatedly used is called frequency reuse factor. As this factor increases, the interference between cells employing the same frequency decreases at the cost of narrower bandwidth per cell. To optimize the frequency efficiency,the lowest frequency reuse factor that ensures acceptable interference between cells must be chosen. However, high demand to the wireless communication enforces engineers to find better ways to increase frequency efficiency. Fractional frequency reuse (FFR) technique is a possible solution that gained popularity over the years.

FFR moves the frequency efficiency to one step further by reusing some frequencies within fractions of cells. In FFR, in addition to assigning each cell in a cluster a different frequency band, fractions of cells uses another frequency which increases bandwidth per cell, resulting an increase in number of supported users within cell. These fractions are generally selected around the receiver, since the users closer to the receiver are affected less than other users by path loss, making them more tolerant to the increased interference due to reused frequency. There are various FFR schemes in the literature, however one of the widely known techniques in the literature is “Strict FFR” which is explained both verbally and visually in Chapter 4.

Chapter 3

Single-Cell Partner Selection in Cooperative OFDMA

Cooperative communication is a big step forward in wireless communication systems. With intelligently designed codewords, users can help each other and achieve better rates by increasing their overall immunity to channel quality fluctuations. From system-wide perspective, partner selection has a great impact on maximization the total achievable rates of the users in the system. Therefore, in this chapter, optimal partner selection in a cooperative wireless network with OFDMA is considered. The work focuses on one cell and ignores inter-cell interference. In this system setup, optimal partner selection algorithm has been found and several near-optimal yet simpler algorithms are investigated. This chapter forms a basis for Chapter 4 and Chapter 5.

3.1 System Model

We consider a fading Gaussian multiple access channel, with N users randomly distributed over a disk of radius R, where N is even. The receiver is assumed to be at the center of the circular cell. The users employ OFDMA in their transmissions, and also cooperate in pairs. Each cooperating pair, say $\{i, j\}$ where $i \in \{1, \dots, N\}$, $j \in \{1, \dots, N\}$ and $i \neq j$, is assigned M orthogonal subchannels $S_{ij} \subset \{1, \dots, NM/2\}$. This subchannel assignment is assumed to be made once, and is fixed throughout the transmission. We make no restrictive assumptions about the connectivity of the nodes, and consider all possible pairing combinations among all nodes; which also contains as special cases possible limited connectivity models. For each cooperating pair $\{i, j\}$, the signals received by the users i, j and the receiver (denoted by index 0), over each subchannel $s \in S_{ij}$, are respectively given by,

$$Y_i = \sqrt{h_{ji}^{(s)} d_{ij}^{-\alpha}} X_j^{(s)} + N_i^{(s)}, \tag{3.1}$$

$$Y_j = \sqrt{h_{ij}^{(s)} d_{ij}^{-\alpha}} X_i^{(s)} + N_j^{(s)}, \tag{3.2}$$

$$Y_0 = \sqrt{h_{i0}^{(s)} d_{i0}^{-\alpha}} X_i^{(s)} + \sqrt{h_{j0}^{(s)} d_{j0}^{-\alpha}} X_j^{(s)} + N_0^{(s)}. \tag{3.3}$$

In Equations (3.1)-(3.3), the noise components $N_i^{(s)}$, $N_j^{(s)}$ and $N_0^{(s)}$ over each subchannel are assumed to be independent, zero mean white Gaussian with variances ${\sigma_i^{(s)}}^2$, ${\sigma_j^{(s)}}^2$, ${\sigma_0^{(s)}}^2$. The symbols $X_i^{(s)}$ and $X_j^{(s)}$ denote the codewords trasmitted by users i and j. The fading over each subchannel is assumed to be independent and identically Rayleigh distributed. Hence, the instantaneous power fading coefficients $h_{ij}^{(s)}$, $h_{ji}^{(s)}$, $h_{i0}^{(s)}$ and $h_{j0}^{(s)}$ are i.i.d. exponential random variables. We assume that full channel state information, which we call $\mathbf{h}$, is available at each user pair and the receiver (instantaneous channel state information of users in other pairs will not be needed, once pairing is done based on the channel statistics.) The symbols d_{ij}, d_{i0} and d_{j0} denote the user i to user j, user i to receiver and user j to receiver distances respectively; and α denotes the path loss exponent. The self interference due to full duplex operation over each subchannel is removed by subtracting appropriately scaled versions of $X_i^{(s)}$ and $X_j^{(s)}$ from (3.1) and (3.2) respectively.

We employ mutual cooperation, i.e., both users involved in a cooperating pair decode and forward each other's messages, using the inter-subchannel cooperative encoding protocol introduced in [9]. Furthermore, each user is able to utilize the available channel state information to perform power control, in order to maximize the cooperating pair's sum rate, as in [10]. Accordingly, the transmitted codewords of users i and j over each subchannel s are formed using [10],

$$X_i^{(s)} = \sqrt{p_{i0}^{(s)}(\mathbf{h})}X_{i0}^{(s)} + \sqrt{p_{ij}^{(s)}(\mathbf{h})}X_{ij}^{(s)} + \sqrt{p_{U_i}^{(s)}(\mathbf{h})}U^{(s)}, \quad (3.4)$$

$$X_j^{(s)} = \sqrt{p_{j0}^{(s)}(\mathbf{h})}X_{j0}^{(s)} + \sqrt{p_{ji}^{(s)}(\mathbf{h})}X_{ji}^{(s)} + \sqrt{p_{U_j}^{(s)}(\mathbf{h})}U^{(s)}, \quad (3.5)$$

The component codewords $X_{i0}^{(s)}$, $X_{ij}^{(s)}$ and $U^{(s)}$ defined in (3.4), are used for direct transmission, common message generation, and cooperation purposes respectively. The variables $p_{i0}^{(s)}(\mathbf{h})$, $p_{ij}^{(s)}(\mathbf{h})$ and $p_{U_i}^{(s)}(\mathbf{h})$ simply denote the channel adaptive powers assigned to these codewords. The definitions for user j follow similarly. The powers of both users in the cooperating pair should satisfy the average power constraints,

$$\sum_{s \in S_{ij}} E\left[p_{i0}^{(s)}(\mathbf{h}) + p_{ij}^{(s)}(\mathbf{h}) + p_{U_i}^{(s)}(\mathbf{h})\right] \triangleq \sum_{s \in S_{ij}} E\left[p_i^{(s)}(\mathbf{h})\right] \leq \bar{p}_i,$$

$$\sum_{s \in S_{ij}} E\left[p_{j0}^{(s)}(\mathbf{h}) + p_{ji}^{(s)}(\mathbf{h}) + p_{U_j}^{(s)}(\mathbf{h})\right] \triangleq \sum_{s \in S_{ij}} E\left[p_j^{(s)}(\mathbf{h})\right] \leq \bar{p}_j.$$

The decoding at the receiver is performed using backwards decoding [1]. Extending the rate regions obtained in [10], to include the path loss based on inter-user and user-receiver distances, it is easy to show that the achievable sum rate for each cooperating pair, employing power adaptive inter-subchannel cooperative encoding, is given by the constraint (3.6).

3.2 Sum-rate-optimal partnering algorithm

In this section, we formulate and solve the jointly optimal power control and partner selection problem for the cooperative OFDMA system modeled in Section 3.1. The objective is to maximize the overall sum rate of the entire system, by optimally pairing the users. Let us denote by Γ the set of all possible 2-user partitions of the set $\{1, \ldots, N\}$ of users. To find the number of all possible 2-user partitions, consider the following approach. Fix an arbitrary user $n_1 \in \{1, \ldots, N\}$. There are $N-1$ possible partners $n_1' \in \{1, \ldots, N\} \setminus \{n_1\}$, for n_1. Once we select the

$$
\begin{aligned}
R_i + R_j \leq \min \Bigg\{ & \sum_{s \in S_{ij}} E \left[\log \left(1 + \frac{h_{i0}^{(s)} d_{i0}^{-\alpha} p_i^{(s)}(\mathbf{h}) + h_{j0}^{(s)} d_{j0}^{-\alpha} p_j^{(s)}(\mathbf{h})}{\sigma_0^{(s)2}} \right.\right. \\
& \left.\left. + \frac{2\sqrt{h_{i0}^{(s)} d_{i0}^{-\alpha} h_{j0}^{(s)} d_{j0}^{-\alpha} p_{u_i}^{(s)}(\mathbf{h}) p_{u_j}^{(s)}(\mathbf{h})}}{\sigma_0^{(s)2}} \right) \right], \\
& \sum_{s \in S_{ij}} E \left[\log \left(1 + \frac{h_{ij}^{(s)} d_{ij}^{-\alpha} p_{ij}^{(s)}(\mathbf{h})}{h_{ij}^{(s)} d_{ij}^{-\alpha} p_{i0}^{(s)}(\mathbf{h}) + \sigma_j^{(s)2}} \right) \right. \\
& \left. + \log \left(1 + \frac{h_{ji}^{(s)} d_{ji}^{-\alpha} p_{ji}^{(s)}(\mathbf{h})}{h_{ji}^{(s)} d_{ji}^{-\alpha} p_{j0}^{(s)}(\mathbf{h}) + \sigma_i^{(s)2}} \right) \right] \\
& + \sum_{s \in S_{ij}} E \left[\log \left(1 + \frac{h_{i0}^{(s)} d_{i0}^{-\alpha} p_{i0}^{(s)}(\mathbf{h}) + h_{j0}^{(s)} d_{j0}^{-\alpha} p_{j0}^{(s)}(\mathbf{h})}{\sigma_0^{(s)2}} \right) \right] \Bigg\}
\end{aligned}
\tag{3.6}
$$

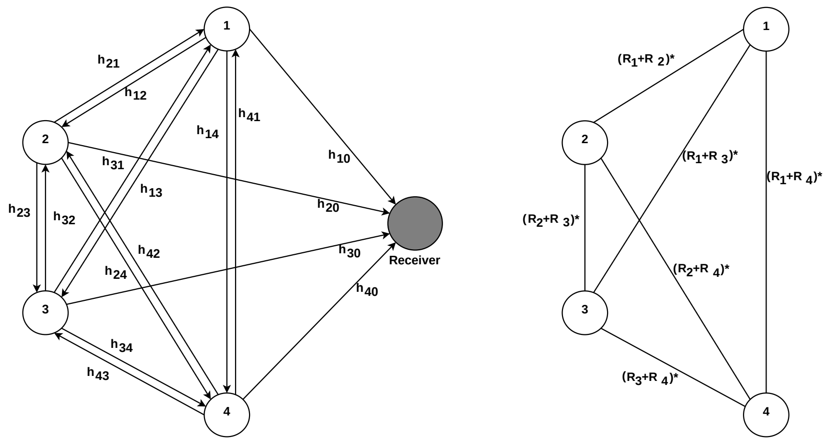
partner n_1', and remove n_1 and n_1' from the set of users, we have $N-2$ users remaining. Fix another user $n_2 \in \{1, \ldots, N\} \setminus \{n_1, n_1'\}$, for which there are $N-3$ possible partners. Repeating the same procedure until all partnerings are made, the number of all possible 2-user partitions can be found by,

$$
L = \prod_{n=1}^{N/2} (N - 2n + 1). \tag{3.8}
$$

Let Γ_l denote the lth 2-user partition of Γ, where $l \in \{1, \ldots, L\}$, and $\mathbf{p}(\mathbf{h})$ denote the vector of powers of all users, containing as its elements the non-negative powers $p_{i0}^{(s)}(\mathbf{h})$, $p_{ij}^{(s)}(\mathbf{h})$, $p_{U_i}^{(s)}(\mathbf{h})$, $\forall s$, $\forall i, j \in \{1, \ldots, N\}$ and $\forall \mathbf{h}$. Then, the sum rate maximization problem can be stated as,

$$
\begin{aligned}
\max_{\substack{\Gamma_l \in \Gamma, \\ \mathbf{p}(\mathbf{h})}} \quad & \sum_{\{i,j\} \in \Gamma_l} R_i + R_j \\
\text{s.t.} \quad & \sum_{s \in S_{ij}} E \left[p_{i0}^{(s)}(\mathbf{h}) + p_{ij}^{(s)}(\mathbf{h}) + p_{U_i}^{(s)}(\mathbf{h}) \right] \leq \bar{p}_i, \quad \forall \{i, j\} \in \Gamma_l \\
& R_i + R_j \text{ satisfy (3.6)}, \quad \forall \{i, j\} \in \Gamma_l.
\end{aligned}
\tag{3.9}
$$

In its present form, (3.9) seems rather difficult to solve, as the rates, which form



(a) 4-user OFDMA system model, with pairwise cooperation.

(b) 4-node undirected graph equivalent of the system in Figure 3.1(a)

Figure 3.1: Graph representations of the system

the objective function for power optimization, depend on the selected partnering strategy, while the partnering strategy that needs to be selected depends on the rates. Therefore, before we proceed, it is instructive to introduce a simple 4-user example, depicted in Figure 3.1(a), which will shed some light into the solution of the general problem. In Figure 3.1(a), all possible links which can be used for cooperation among all possible pairs are shown. Here, as suggested by (3.8) there are only three possible 2-user partitions of the set of users: $\{\{1,2\},\{3,4\}\}$, $\{\{1,3\},\{2,4\}\}$ and $\{\{1,4\},\{2,3\}\}$. The crucial observation is that, once one of these partitions is fixed, the sum rate of each pair in that partition depends solely on the channel gains on the subchannels used by that particular pair, and is not affected by the transmission policy of the remaining pair, thanks to the orthogonal nature of OFDMA. But then, since each pair's transmission rate is independent of the other, we can simply find the optimal power allocation, and the resulting sum rate separately for each pair, for each given partition. Afterwards, the optimal partition can be selected by performing a search over the L power optimized sum-rate values. This argument is obviously valid for an arbitrary number of pairs as well: going back to our original problem, our optimization problem (3.9) can be equivalently stated as a two step problem

$$
\begin{aligned}
\max_{\Gamma_l \in \Gamma,} \quad & \sum_{\{i,j\} \in \Gamma_l} \max_{\mathbf{p}_i(\mathbf{h}), \mathbf{p}_j(\mathbf{h})} (R_i + R_j), \\
\text{s.t.} \quad & \sum_{s \in S_{ij}} E\left[p_{i0}^{(s)}(\mathbf{h}) + p_{ij}^{(s)}(\mathbf{h}) + p_{U_i}^{(s)}(\mathbf{h})\right] \leq \bar{p}_i, \quad \forall \{i,j\} \in \Gamma_l \\
& R_i + R_j \text{ satisfy (3.6)}, \quad \forall \{i,j\} \in \Gamma_l.
\end{aligned} \tag{3.10}
$$

which can further be converted into

$$
\max_{\Gamma_l \in \Gamma,} \quad \sum_{\{i,j\} \in \Gamma_l} (R_i + R_j)^*, \tag{3.11}
$$

where $(R_i + R_j)^*$ is the power optimized sum rate of pair $\{i, j\}$, obtained by running the iterative algorithm proposed in [10]. While (3.11) is considerably simpler than (3.9), a brute force search over all possible partnering strategies would require factorial time, as evident from (3.8). However, given the sum rates achievable by each possible partnering, it is possible to model (3.11) as an equivalent matching problem in graph theory. Let us go back to our simple 4-user example, and create a complete undirected graph, where the users are the vertices, and the weights over the edges are the sum rate that is achievable by the pair of users connected by that particular edge, in case they are paired. The resulting graph is shown in Figure 3.1(b). In order to create all the weight information in this example, we need to compute six sum rates, each corresponding to one possible pair of users. However, note that since there are 4 users in this graph, we can simultaneously choose only 2 disjoint pairs, and the pairs for which the summation of the corresponding weights is maximized should be found. This problem is known as "maximum weighted matching" in graph theory, which can be solved by an efficient algorithm presented in [20].

The worst-case complexity of the maximum weighted matching algorithm is $O(N^3)$ [20]. Meanwhile, for a general system with N users, the complete graph consisting of all possible pairings of users contains only $N \times (N-1)/2$ edges. Since the cost of finding the weights $(R_i + R_j)^*$ on each edge based on power optimization is constant, the overall cost of generating the graph becomes negligible, compared to the cost of weighted matching as N grows. Note however that, for moderate number of users, which is typical in a wireless network, the fixed cost of computing these weights using iterative power optimization may become a time consuming computational burden. In practical networks, users are not necessarily stationary, and the topology of the network, and hence the channel conditions, may change frequently. Every time the topology changes, we may need a new matching. Therefore, in the next section, we propose alternative matching algorithms with the aim of obtaining even faster and more practical results.

3.3 Practical suboptimal pairing algorithms

In our model, the locations of the users, and their distances to each other are the major factors that effect their transmission rates. The impacts of Rayleigh fading and noise variances on the rates are negligible in comparison to path loss. This forces the power allocation and partner selection to be mostly dependent on the topology of the network, which means that a suboptimal but fast algorithm can be derived based only on user locations as an alternative to the maximum weighted matching algorithm. But then, the weights of the graph will not be needed to match the users, and this will decrease the time consumed by the matching algorithm drastically.

When we seek ways of utilizing user locations directly in partnering decisions, two contrasting approaches immediately come to mind: (i) the users close to each other being grouped together, and, (ii) the users at a disadvantage being grouped with users with stronger links. Also, it is clear that the partnering should depend on the user-receiver distances as well as the inter-user distances, hence it is of interest to see whether one should group the users starting with the nearest to or farthest from the receiver. Therefore, in what follows, we propose five algorithms that make partnering decisions based on differing criteria based on the relative locations of the users.

Algorithm A: Select Nearest to Receiver

The two users nearest to the receiver get matched. These users are removed from the pool, and the algorithm repeatedly matches the rest of users with the same method until every user is matched.

Algorithm B: Select Farthest from Receiver

The two user farthest from the receiver get matched. These users are removed from the pool, and the algorithm repeatedly matches the rest of users with the same method until every user is matched.

Algorithm C: Maximum Matching on Nearest Four to Receiver

The user nearest to the receiver is selected. Then, three users which are nearest to it are selected. Maximum weighted matching algorithm runs on those users and the users get matched. The algorithm repeatedly matches the rest of users with the same method until every user is matched.

Algorithm D: Maximum Matching on Farthest Four from Receiver

The user farthest from the receiver is selected. Then, three users which are nearest to it are selected. Maximum weighted matching algorithm runs on those users and the users get matched. The algorithm repeatedly matches the rest of users with the same method until every user is matched.

Algorithm E: Select Nearest and Farthest to Receiver

The user farthest to the receiver gets matched with the nearest to the receiver. These users are removed from the pool, and the algorithm repeatedly matches the rest of users with the same method until every user is matched.

The performance comparisons of the above algorithms are presented in the following section.

Table 3.1: Transmission rates of cooperating pairs obtained by a sample run of proposed algorithms

Pair	MWM	AlgoA	AlgoB	AlgoC	AlgoD	AlgoE
1	17.084	21.045	19.439	21.045	17.926	17.078
2	16.618	19.596	18.133	18.062	17.731	16.621
3	16.414	13.073	16.649	15.336	16.727	16.410
4	14.924	10.064	13.073	11.534	16.417	14.911
5	10.683	4.833	5.484	4.833	7.164	10.683
6	8.716	3.906	4.388	3.798	3.906	8.657
7	7.938	3.451	3.906	3.496	3.451	7.760
8	7.164	3.074	3.496	2.793	3.074	5.111
9	3.906	2.841	2.841	2.642	2.865	4.833
10	3.596	2.329	2.793	2.706	2.858	4.429
Total	107.043	84.211	90.202	86.245	92.117	106.494

3.4 Simulation Results

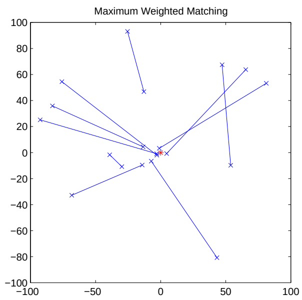
Fifty runs were taken from each of the algorithms proposed in Section 3.3, as well as from the weighted matching algorithm described in Section 3.2. In the simulations, $N = 20$ users were placed in a disk with radius $R = 100$m according to a uniform random distribution. The receiver was placed at the center of the disk. All of the users had the same transmission power and the same number $M = 3$ of Rayleigh fading subchannels. The path loss exponent in the simulations were set to $\alpha = 2$. The noise variances were normalized to unity. Users' transmission power before path loss and fading was set to $P = 10^4$. The simulations for lower signal to noise ratios (SNR) also yield similar relative performance results for the algorithms, although with decreasing SNR, the differences between the performances of the proposed algorithms become less pronounced.

In Table 3.1, a detailed comparison of the rates achieved by each cooperating pair is given for a sample run of all algorithms. We observe that, if the users close to the receiver are coupled first, these users' transmission rates are high, however the farther users' rates are so low that, the total is not as much as one can obtain by a more nearly equal distribution. This is the main problem encountered in Algorithm A. The same also applies to Algorithm B with a little bit of difference.

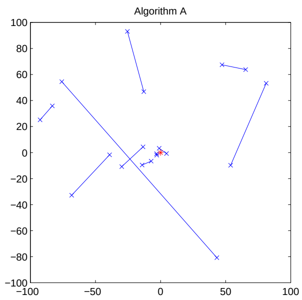
The users farther away from the receiver are selected as close as possible to each other, however, since the SNR goes down because of the path loss, the cooperation gain is still low for these users, and total rate becomes low. It is noteworthy that, algorithm B gives better results than algorithm A. Algorithm E, which is inspired by the optimal matching, performs surprisingly well.

In Figure 3.2, the matchings created by the algorithms are visually compared to maximum weighted matching. It is observed that, maximum weighted matching generally selects pairs such that, one of the users in the pair is close to the receiver, while the other user is far away from the receiver. This is rather surprising in that, the pairing that is optimal for the benefit of the entire system also happens to match users with best channel conditions with those with worst channel conditions. The achievable rates of the proposed algorithms are compared to the total transmission rate of maximum weighted matching, by defining the ratio of the sum rate achievable by each algorithm to the optimal sum rate of weighted matching in the form of a percentage, which we call the efficiency. We observe that, Algorithm E creates a matching which is closes to the maximum weighted matching, and hence achieves the best efficiency.

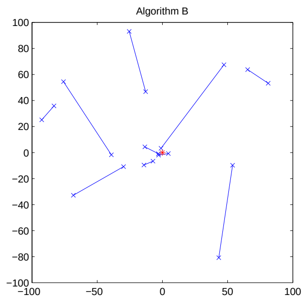
In Table 3.2, we provide the statistics of the efficiencies of our proposed algorithms. In our simulations, the efficiencies of the algorithms A and B are between 75% and 95%. Algorithms C and D include maximum weighted matching for subgroups of users as a subroutine, but they are still fast algorithms since subgroups include small numbers of users. Algorithm D gives better results than C, with efficiencies between 80% and 99%. Algorithm E is the best among the proposed heuristic algorithms in terms of efficiency, with efficiencies between 94% and 99%. Since one closer and one further user is paired with each other, for most user pairs, cooperative gain is average, but in total, this converges to the maximum transmission rate. Also, there is no maximum weighted matching routine in this algorithm, making it much faster.



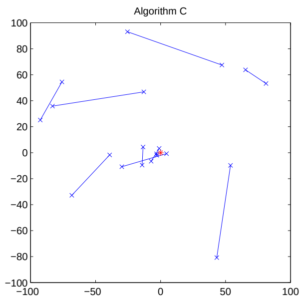
(a) Maximum weighted matching efficiency:100%



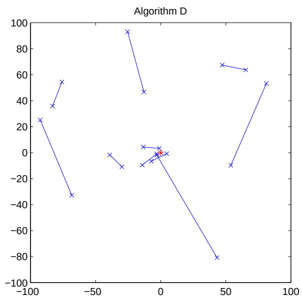
(b) Algorithm A efficiency:78.6705%



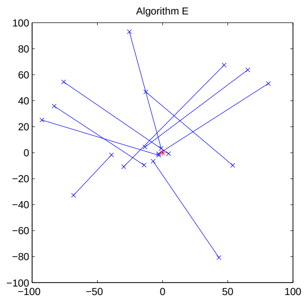
(c) Algorithm B efficiency: 84.2669%



(d) Algorithm C efficiency: 80.5707%



(e) Algorithm D efficiency: 86.0566%



(f) Algorithm E efficiency: 99.4869%

Figure 3.2: Maximum weighted matching and matchings created by different proposed algorithms.

Table 3.2: Statistics of proposed algorithms

Efficiencies	AlgoA	AlgoB	AlgoC	AlgoD	AlgoE
min	76.994	83.379	78.735	85.114	94.337
max	95.864	96.953	97.225	99.551	99.655
mean	87.109	90.483	88.874	94.236	97.527

3.5 Conclusion

Partner selection in wireless networks is a key consideration in rate maximization for cooperative networks. In this chapter, we formulated the joint power allocation and partner selection problem, with the goal of maximizing the sum-rate of a cooperative OFDMA network. It is shown that, the problem can be reduced into a maximum weighted matching problem which has a polynomial time solution. The result of the maximum weighted matching algorithm, inspired us to develop some heuristic algorithms with lower complexity. Hence, to further simplify the partnering problem, we proposed matching algorithms which only use the location information of the users. We demonstrated that, the algorithm which matches the users farthest away from the receiver to the ones closest to the receiver, gives a near-optimum solution, very fast.

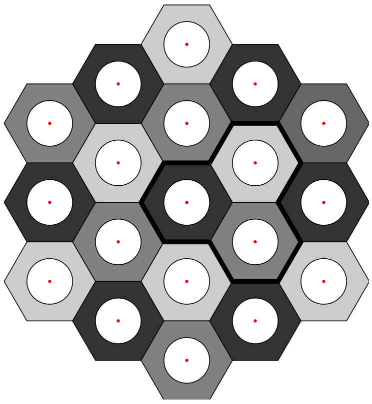
The main result of this part of the study is, the users nearer to the receiver uses further user's channel aggressively, reducing further users individual transmission rate in exchange for increasing total rate of the pair significantly. While this is meaningful from the sum rate maximization point of view, the system favors the nearer users reducing the fairness of the system in comparison to non-cooperative networks. In the following chapters, we are going to use the information gained by this part and try to increase fairness while maximizing the sum rate of the system.. In Chapter 4, we design a new frequency reuse scheme to force users to cooperate with closer users to increment fairness of the system and increase the cooperative gain. In Chapter 5, we are dealing with a cognitive scenario, where primary users transmission rate cannot be affected negatively by the existence of the secondary user, therefore the problem can be said to have a fairness constraint.

The study presented in this chapter is published at WCNC'12 conference in Paris, France [21].

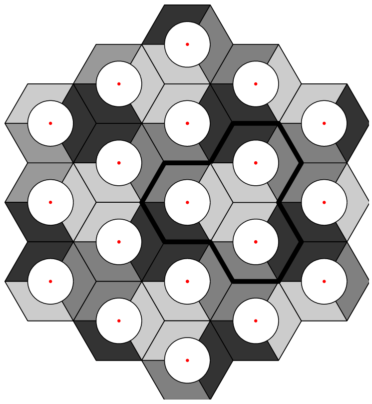
Chapter 4

Multi-Cell Partner Selection in Cooperative OFDMA with FFR

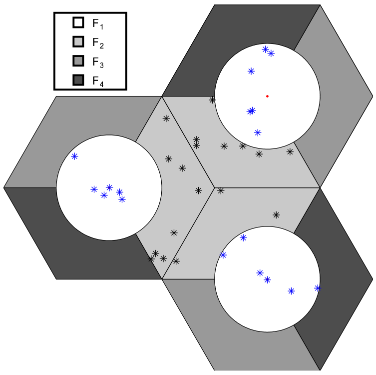
In order to maximize the capacity of a wireless system, usable frequency bands are limited and must be used effectively. Therefore, in traditional cellular systems, each cell employs a different frequency band and by intelligent design of frequency reuse pattern, limits the inter-cell interference under a desired level. Although this approach works quite well, it limits the frequencies used in each cell, therefore not all of the frequencies are supported at each cell. To overcome this problem, Fractional Frequency Reuse technique is developed and gaining popularity among researchers. In FFR, more frequency bands are used repeatedly, therefore more bandwidth is used effectively. The most popular FFR scheme in the literature is "Strict FFR" where each cell has its own frequency band as in traditional cellular systems, with an addition of distinct and repeatedly reused frequency band used in the areas near to the receivers in each cell. "Strict FFR" is an more effective in terms of frequency reuse, however not suitable enough for cooperation among users. In this chapter, we introduce a novel FFR scheme which is designed to increase fairness and cooperative gain among users, reducing inter-cell interference significantly by cell sectorization.



(a) Strict FFR Scheme.



(b) Complementary FFR Scheme.



(c) Complementary FFR: cluster of interest

Figure 4.1: Illustration of complementary FFR scheme, compared to strict FFR

4.1 Proposed Cooperation and Frequency Reuse Model

We consider a cellular multiple access setup, consisting of several fading Gaussian multiple access channels operating in parallel. Multiple access towards each base station is facilitated using OFDMA, and frequency reuse is employed to increase the user capacity of the system. Yet, in our model, the users are further assumed to cooperate in pairs based on overheard information. Note that if we allow receiver selection, two cell-edge users belonging to two distinct neighboring cells could be ideal candidates for a cooperating pair. Therefore, traditional multiple access and frequency reuse techniques, which target orthogonal transmissions and especially try to avoid interference from neighboring cells are not suitable in our cooperative setup. Hence, we first develop a novel frequency reuse and multiple access model, which is directly tailored for pairwise cooperation.

In Chapter 3, it was shown that in power controlled single cell cooperative OFDMA channels, optimal partner selection results in users close to base station being paired with cell edge users. As a result, cell center users abuse, rather than help, the cell edge users by taking over their subchannels while cooperating minimally, yielding a sum rate optimal but unfair resource allocation and partnering strategy. In a typical multi-cell environment, the cell edge users are more prone to interference and also suffer more from path loss; therefore fairer strategies compared to the partnering in Chapter 3 should be developed. Keeping this in mind, we propose to use a frequency reuse scheme which forces inner and outer users to cooperate in separate groups. This idea coincidentally leads to a fractional frequency reuse setup, an example of which is shown in Figure 4.1(a). In Figure 4.1(a) we assume three-cell clusters, which use four orthogonal frequency bands, F_1, F_2, F_3 and F_4, each denoted by different shades of grey. The main goal in FFR, is to increase the user capacity by allowing reuse of frequencies near the cell center, while still protecting cell edge users by assigning them orthogonal bands. Note however that orthogonalizing cell edge users in adjacent cells is completely against the spirit of user cooperation, as it rules out the possibility of cooperating across cells. Therefore, we propose the use of a rather unorthodox FFR scheme, called complementary fractional frequency reuse, which purposely assigns the same frequency sub-bands to neighboring cell sectors facing each other. This scheme is shown in Figure 4.1(b), where again distinct orthogonal frequency bands, F_1, F_2, F_3 and F_4 are used. Note that, the model in Figure 4.1(b) creates a translated frequency reuse pattern, with *pseudo-cells* that are composed of one sector from each cell being assigned a common frequency sub-band which is reused throughout. This not only enables cooperation across cells, but it also allows cooperating users to select an optimal receiver, as each pseudo-cell is now served by any one of the three base stations in the cluster. In our model, we divide the cells in the system into 3-cell clusters, and repeat the frequency reuse pattern over each cluster, as shown in Figure 4.1(b). We assume that there are $K = 12N$ users in a given cluster, where N is an integer, and that these users are uniformly distributed over the cluster surface, yielding $4N$ users per cell. Assuming hexagonal cells with radius r, each cell is divided into two concentric regions: the users inside a circle of radius $r_{in} = r/2$ surrounding the base station of each cell are called the inner users, and the remaining users are called outer users. Since the number of users is proportional to the area they are distributed on, there are on average N inner and $3N$ outer users in each cell. This also amounts to an average of $3N$ users per each pseudo-cell sharing the same frequency resource.

Due to symmetry, it is sufficient to focus on a single cluster, which is highlighted by the bold boundary in Figure 4.1(b), and shown separately in Figure 4.1(c). The light gray region at the center of the cluster, consisting of one sector from each cell, will be our pseudo-cell of interest. A sample user distribution is also given in Figure 4.1(c), showing only the set of outer users, U_{out}, belonging to the pseudo cell of interest, and the inner users, $U_{in,b}$ in each cell, where $b = \{1, 2, 3\}$ is the receiver, or equivalently, cell index. Other outer users may be communicating with receivers from a different cluster, and hence are not shown on Figure 4.1(c).

The receiver of each cell in the cluster is located at the center of the cell. The inner users in cell b, labeled $U_{in,b}$, are to be grouped in cooperating pairs, exclusively

$$
\begin{aligned}
(R_i + R_j)_b \leq \min \Bigg\{ & \sum_{s \in S_{ij}} E\left[\log\left(1 + \frac{h_{ib}^{(s)} d_{ib}^{-\alpha} p_i^{(s)}(\mathbf{h}) + h_{jb}^{(s)} d_{jb}^{-\alpha} p_j^{(s)}(\mathbf{h})}{{\sigma_b^{(s)}}^2 + I_b}\right.\right. \\
& + \left.\left.\frac{2\sqrt{h_{ib}^{(s)} d_{ib}^{-\alpha} h_{jb}^{(s)} d_{jb}^{-\alpha} p_{u_i}^{(s)}(\mathbf{h}) p_{u_j}^{(s)}(\mathbf{h})}}{{\sigma_b^{(s)}}^2 + I_b}\right)\right], \\
& \sum_{s \in S_{ij}} E\left[\log\left(1 + \frac{h_{ij}^{(s)} d_{ij}^{-\alpha} p_{ij}^{(s)}(\mathbf{h})}{h_{ij}^{(s)} d_{ij}^{-\alpha} p_{ib}^{(s)}(\mathbf{h}) + {\sigma_j^{(s)}}^2 + I_j}\right)\right. \\
& + \left.\log\left(1 + \frac{h_{ji}^{(s)} d_{ji}^{-\alpha} p_{ji}^{(s)}(\mathbf{h})}{h_{ji}^{(s)} d_{ji}^{-\alpha} p_{jb}^{(s)}(\mathbf{h}) + {\sigma_i^{(s)}}^2 + I_i}\right)\right] \\
& + \sum_{s \in S_{ij}} E\left[\log\left(1 + \frac{h_{ib}^{(s)} d_{ib}^{-\alpha} p_{ib}^{(s)}(\mathbf{h}) + h_{jb}^{(s)} d_{jb}^{-\alpha} p_{jb}^{(s)}(\mathbf{h})}{{\sigma_b^{(s)}}^2 + I_b}\right)\right] \Bigg\}
\end{aligned} \tag{4.1}
$$

within that cell; i.e., there is no inter-cell cooperation for inner users. Each pair of users $\{i, j\} \in U_{in,b} \times U_{in,b}$ is assigned a distinct set of sub-channels $S_{ij} \subset F_1$, and both users of the pair simultaneously utilize these sub-channels. The outer users U_{out} in the pseudo-cell shared by receivers $b = \{1, 2, 3\}$ are also to be grouped in cooperating pairs. If a cooperating pair has users from two different cells, an intended receiver is also to be selected optimally. Each pair $\{i, j\} \in U_{out} \times U_{out}$ is assigned a distinct set of sub-channels $S_{ij} \in F_2$, and both users of the pair simultaneously utilize these subchannels. It is easy to check that, assuming n subchannels are assigned to each pair, there needs to be a total of $nN/2$ sub-channels in F_1, and $3nN/2$ subchannels in F_2. This subchannel assignment is assumed to be made once, and is fixed throughout the transmission.

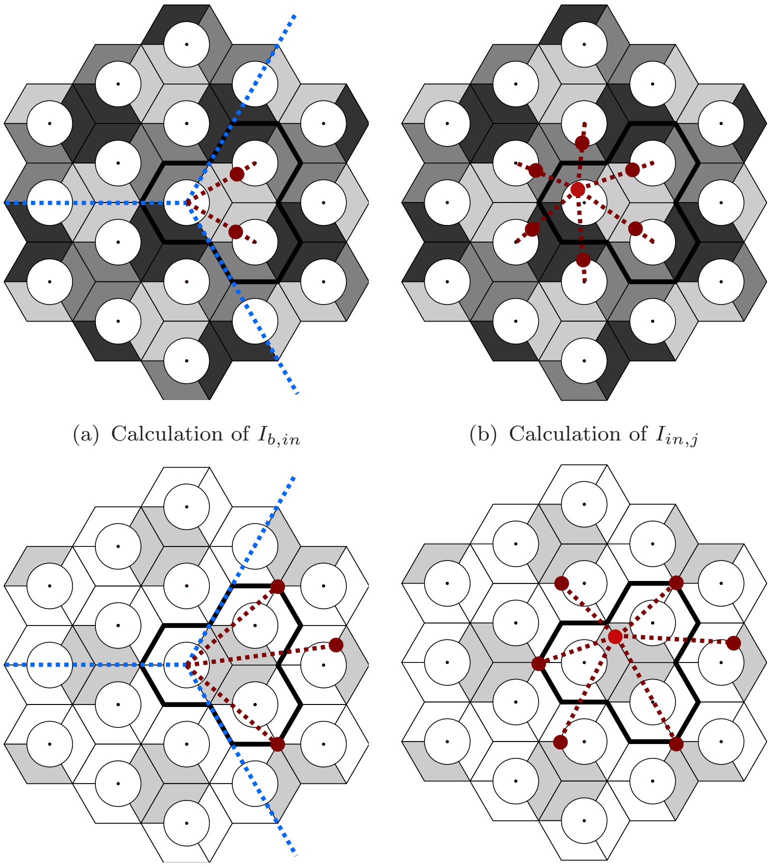
Regardless of the cooperating pair being an inner or outer pair, the signals received by the users i, j and the receiver b, over each subchannel $s \in S_{ij}$, are respectively given by,

$$Y_i = \sqrt{h_{ji}^{(s)} d_{ij}^{-\alpha}} X_j^{(s)} + I_i^{(s)} + N_i^{(s)}, \tag{4.2}$$

$$Y_j = \sqrt{h_{ij}^{(s)} d_{ij}^{-\alpha}} X_i^{(s)} + I_j^{(s)} + N_j^{(s)}, \tag{4.3}$$

$$Y_b = \sqrt{h_{ib}^{(s)} d_{ib}^{-\alpha}} X_i^{(s)} + \sqrt{h_{jb}^{(s)} d_{jb}^{-\alpha}} X_j^{(s)} + I_b^{(s)} + N_b^{(s)}, \tag{4.4}$$

where, for each subchannel s, $N_i^{(s)}$, $N_j^{(s)}$ and $N_b^{(s)}$ denote independent, zero mean white Gaussian noise components, having variances ${\sigma_i^{(s)}}^2$, ${\sigma_j^{(s)}}^2$, ${\sigma_b^{(s)}}^2$; $I_i^{(s)}$, $I_j^{(s)}$ and $I_b^{(s)}$ denote intercell interference at users i, j and receiver b; $X_i^{(s)}$ and $X_j^{(s)}$ denote the codewords transmitted by users i and j; $h_{ij}^{(s)}$, $h_{ji}^{(s)}$, $h_{ib}^{(s)}$ and $h_{jb}^{(s)}$ are i.i.d. exponential power fading coefficients. The variables d_{ij}, d_{ib} and d_{jb} denote the user i to user j, user i to receiver and user j to receiver distances respectively; and α denotes the path loss exponent. We assume that pairwise channel state information $\mathbf{h} = \left\{h_{ij}^{(s)}, h_{ji}^{(s)}, h_{ib}^{(s)}, h_{jb}^{(s)}, \ \forall s \in S_{ij}\right\}$, is only available at the corresponding cooperating pair and the receiver, and pairing is done at the receiver, based only on the channel statistics. The calculation of intercell interference terms, I_i, I_j and I_b require special attention, and will be discussed in the following section.



(a) Calculation of $I_{b,in}$ (b) Calculation of $I_{in,j}$

(c) Calculation of $I_{b,out}$ (d) Calculation of $I_{out,j}$

Figure 4.2: Interferer locations for inner and outer users. Only the interfering pseudo-cells are shown for outer users (Figures 4.2(c) and 4.2(d)). Cell sectors, shown by dashed blue lines, help reduce the interference at the receivers, but the interference at the users is affected by all first tier interferers.

4.2 Encoding, Decoding and Achievable Rates

Let us assume that users i and j are paired, and assigned a set of subchannels S_{ij} and a base station b. The cooperation then proceeds according to the power controlled inter-subchannel cooperative OFDMA model of [10]. Namely, the users employ block Markov superposition encoding, and decode each other's message at the end of each block, and the receiver decodes the user messages using backwards decoding after receiving all blocks of information. The transmitted codeword, consisting of direct transmission, common message generation and common message transmission components, $X_{ib}^{(s)}$, $X_{ij}^{(s)}$ and $U^{(s)}$ respectively is

$$X_i^{(s)} = \sqrt{p_{ib}^{(s)}(\mathbf{h})} X_{ib}^{(s)} + \sqrt{p_{ij}^{(s)}(\mathbf{h})} X_{ij}^{(s)} + \sqrt{p_{U_i}^{(s)}(\mathbf{h})} U^{(s)} \tag{4.5}$$

where the powers, assigned to the codewords selected from zero mean Gaussian distributions, should satisfy the long term average constraint

$$\sum_{s \in S_{ij}} E\left[p_{ib}^{(s)}(\mathbf{h}) + p_{ij}^{(s)}(\mathbf{h}) + p_{U_i}^{(s)}(\mathbf{h})\right] \triangleq \sum_{s \in S_{ij}} E\left[p_i^{(s)}(\mathbf{h})\right] \leq \bar{P}_i,$$

For each pair $\{i, j\}$, and receiver b, the resulting achievable sum rate can be obtained by extending the rate regions in [10] and Chapter 3, to include the inter-cell interference parameters I_i, I_j and I_b, which are modelled as Gaussian, resulting in (4.1) at the bottom of this page. Note however that, while the general form of the sum rate expression does not depend on whether we are dealing with an inner or outer pair, the interference terms do. Due to the cooperative nature of our model, and the geometry of complementary FFR, four different interference terms arise: inner user to base station, $I_{b,in}$; inner user to inner user j, $I_{in,j}$; outer user to base station, $I_{b,out}$; and outer user to outer user j, $I_{out,j}$.

The geometries used in the calculation of each interference term are shown in Figures 4.2(a)-4.2(d). While computing interference, we only consider first tier interferers, and assume worst case scenarios for the positions of the interferers. An important observation is, since we already use cell sectoring as a part of our complementary FFR setup, we can further exploit the sectorized structure to limit the interference at the base stations by adjusting the receive antenna beams. As a result, while computing $I_{b,in}$, we have only two first tier interferers, and for $I_{b,out}$, we have only three first tier interferers, see Figures 4.2(a) and 4.2(c). This way, the increased interference for inner users, which is typical for FFR, is significantly reduced, as a byproduct of our cooperative model. Since the users cannot do receive beamforming, we need to consider six interferers, while computing $I_{in,j}$ and $I_{out,j}$.

Since our main goal is to optimize the powers, partnering strategies and receiver selection; and channel state information at the transmitters is limited, we take the powers of the interferers outside the cluster of interest to be equal to their average, say $\bar{P}$, while computing total interference at each channel state, which is a common assumption. This way, the convexity of the optimization problem is preserved. Also, we assume that the fading from the interferers is averaged out, and we only consider a simplified path loss model from the interferers. The resulting average interference powers are given by,

$$I_{b,in} = 2 \times \bar{P}/(r\sqrt{3} - r_{in})^{\alpha} \tag{4.6}$$

$$I_{b,out} = 2 \times \bar{P}/(r\sqrt{7})^{\alpha} + \bar{P}/(r\sqrt{10})^{\alpha} \tag{4.7}$$

$$I_{in,j} = \sum_{m=1}^{6} \bar{P}/d_{jm,in}^{\alpha} \tag{4.8}$$

$$I_{out,j} = \sum_{m=1}^{6} \bar{P}/d_{jm,out}^{\alpha} \tag{4.9}$$

where $d_{jm,in}$ (respectively, $d_{jm,out}$) is the distance of the mth first tear inner (respectively, outer) interferer to inner (respectively, outer) user j, and depends on user coordinates. Finally, if $\{i, j\} \in U_{in,b} \times U_{in,b}$, we set $\{I_b, I_i, I_j\} = \{I_{b,in}, I_{in,i}, I_{in,j}\}$; if $\{i, j\} \in U_{out} \times U_{out}$ we set $\{I_b, I_i, I_j\} = \{I_{b,out}, I_{out,i}, I_{out,j}\}$ in (4.1).

4.3 Jointly Optimum Power, Cooperating Partner and Receiver Selection

The sum rate of the system can be written as a sum of inner and outer user pair rates, and due to the orthogonality of the subchannels, the sum rate of inner users and outer users can be optimized separately. As far as inner user sum rate maximization is concerned, there is no issue of base station selection, and for each inner cell, the problem can be reduced to the joint partnering and power control problem in Chapter 3, by adding the intercell interference powers computed in the previous section to noise variances. Hence, we will focus here on the outer user rate maximization, which is considerably more involved. Note that the sum rate maximization for each pseudo-cell can be solved separately, thanks to orthogonality supplied by OFDMA. The goal is then to solve,

$$\begin{aligned}\max_{\substack{\Gamma_l\in\Gamma,\\ b_{ij}\in\{1,2,3\},\\ \mathbf{p}(\mathbf{h})}} \quad & \sum_{\{i,j\}\in\Gamma_l} (R_i+R_j)_{b_{ij}} \\ \text{s.t.} \quad & \sum_{s\in S_{ij}} E\left[p^{(s)}_{ib_{ij}}(\mathbf{h})+p^{(s)}_{ij}(\mathbf{h})+p^{(s)}_{U_i}(\mathbf{h})\right]\leq \bar{P}_i, \\ & (R_i+R_j)_{b_{ij}} \text{ satisfies (4.1)}, \quad \forall\{i,j\}\in\Gamma_l,\end{aligned} \tag{4.10}$$

where Γ_l is a two user partition of the set U_{out} of users in the pseudo-cell of interest, Γ is the set of all such distinct partitions Γ_l, b_{ij} is the receiver selected by $\{i,j\}$ and $\mathbf{p}(\mathbf{h})$ denotes the vector of all power variables at all channel states.

The joint maximization problem is rather difficult to solve, as the channel gains, distances, and hence the sum rates themselves depend on which users are paired, and which base station is selected. A brute force search clearly results in a combinatoric problem, and is not a viable option. The key to solving (4.10) is to realize that like its single-cell counterpart of the system presented in Chapter 3, it can be reduced to a maximum weighted matching problem on a graph, if the sum rate obtainable by each pair of users and the selected receiver, after power control, is viewed as the weights assigned to the edges of the graph. A simple four user, three receiver example is shown in Figure 4.3(a). The resulting weighted graph is shown in Figure 4.3(b). Each of the three parallel edges connecting each user pair corresponds to selecting a distinct receiver. Clearly, in the final solution, each pair should be assigned only one edge, so that it is served by only one base station. The trick is to realize that the edge selection for each potential pair may in fact be done before solving the matching problem: one can simply keep only the edge corresponding to the most powerful receiver for each pair, and delete the other two, without considering which partners or receivers are selected by the other users. This can be shown easily by contradiction. Let us assume we know that users i and j are paired in the optimal strategy, and let them be served by base station b. Now, if there exists $b' \neq b$, for which $(R_i + R_j)_b < (R_i + R_j)_{b'}$ in the original graph, the edge between i and j corresponding to receiver b can be removed, and selecting b' as the new receiver will result in a strictly better system sum rate, as the sum rate of the other users remain unchanged. This contradicts the optimality of b, and shows that any edge corresponding to such inferior b can be removed initially, without compromising optimality. As a result, the model in Figure 4.3(c) is obtained, and the structure of the problem once again reduces to that of single cell partnering. The jointly optimal partnering, receiver selection and power allocation problem can therefore be stated as an equivalent three stage problem,

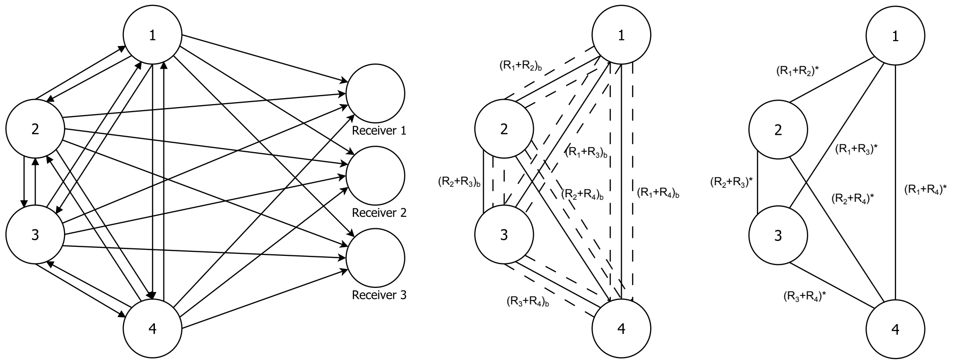
$$\begin{aligned} \max_{\Gamma_l \in \Gamma,} \quad & \sum_{\{i,j\} \in \Gamma_l} \max_{b} \max_{\mathbf{p}_i(\mathbf{h}), \mathbf{p}_j(\mathbf{h})} (R_i + R_j)_b, \\ \text{s.t.} \quad & \sum_{s \in S_{ij}} E\left[p_{ib}^{(s)}(\mathbf{h}) + p_{ij}^{(s)}(\mathbf{h}) + p_{U_i}^{(s)}(\mathbf{h}) \right] \leq \bar{P}_i, \\ & (R_i + R_j)_b \text{ satisfies (4.1)}, \quad \forall \{i,j\} \in \Gamma_l. \end{aligned} \tag{4.11}$$

which can further be converted to

$$\max_{\Gamma_l \in \Gamma,} \quad \sum_{\{i,j\} \in \Gamma_l} (R_i + R_j)^*, \tag{4.12}$$

and being a maximum weighted matching problem on a complete graph, (4.12) can be solved in polynomial time using methods such as Edmonds algorithm [20]. Algorithm 1 below summarizes the stages of our three step optimization.

Instead of calculating the optimum powers to obtain the graph weights for each pair of users, it is also possible to resort to some heuristic distance based algorithms to perform the matching step. We now propose such an algorithm: the distances among each pair of outer users in each pseudo-cell are computed and sorted. The users closest to each other are matched, removed from the list of users, then the same procedure is applied to the remaining users. Once the matching is found, power allocation and receiver selection steps are performed.



(a) 3 receiver system model (b) Weighted graph, $b = 1, 2, 3$ for each pair (c) Reduced graph, after selecting b^* for each pair

Figure 4.3: Determination of weights for edge cell users

The distance based matching for the inner users is identical to single cell matching, and is performed using Algorithm E in Chapter 3. The performance of the heuristic algorithm will be evaluated in the following section.

Algorithm 1 Algorithm for outer cell users

for all $(i, j) \in U_{out}$ **do**
  for all receivers $b \in \{1, 2, 3\}$ **do**
    Compute optimal powers using the algorithm from [10]
    Calculate $(R_i + R_j)_b$ by equation (4.1)
  end for
  Select $b^* = \arg\max (R_i + R_j)_b$,
  Use $(R_i + R_j)^* \triangleq (R_i + R_j)_{b^*}$ as graph weights
end for
Run MWM algorithm on weighted graph for optimal pairing.

4.4 Simulation Results

We simulate our proposed frequency reuse, partner selection, base station selection and power allocation strategy for a system with $4N = 24$ users per cell, $r = 2r_{in} = 100$m. We assume that the average power of each user is unity, and the fading is exponential with mean 1. Each user is assigned an average of one subchannel, that is, in the cooperative scenario, the user pair is assigned two subchannels and share both of these subchannels. This amounts to a total of 60 subchannels reused in the system. Note that, if FFR, whether complementary or

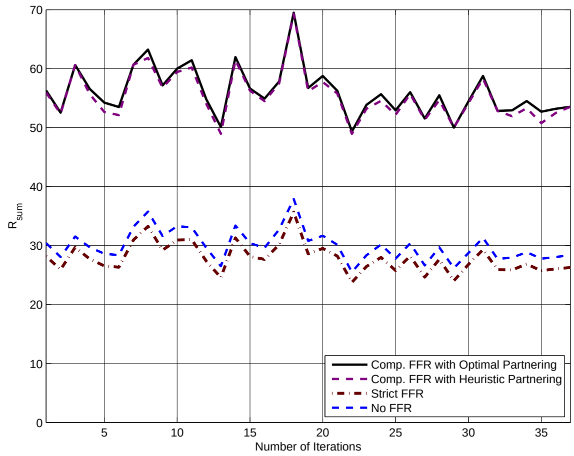


Figure 4.4: Sum rate comparison of proposed model with non-cooperative models.

strict, is not used, each cell can support only 20 users, in which case the worst 4 users should be blocked. The rate maximization is carried out for only the outer users in the central pseudo-cell of the cluster, and the inner users; and per cell sum rate is found by averaging.

In Figure 4.4, we compare the sum rates of four strategies: our proposed jointly optimized strategy, our heuristic strategy, strict FFR with single user power control but no cooperation, and power control only (no FFR). Each index on the horizontal axis refer to a different user geometry. While the use of non-cooperative strict FFR increases the user capacity, it yields less sum rate compared to no FFR, due to the added interference at the inner users. In fact, it was noted in [13] that when $r_{in} = r/2$, FFR and no FFR give nearly the same rate, as validated here. However, our proposed strategy, as well as the heuristic partnering approach nearly double the rates of both non-cooperative techniques, thanks to the gain from cooperation, reduction of interference due to the sectorized complementary FFR model, and flexibility in choosing partners and receivers.

In Figure 4.5, we give the optimal partnering strategy for a sample geometry. Dividing the cell into two has the effect of increasing the connectivity of the users, and encourages cooperation, compared to a single cell setup presented in Chapter 3. As a result, especially the cell edge users with comparable direct link gains tend to pair with close-by helpers, as opposed to the observations in the

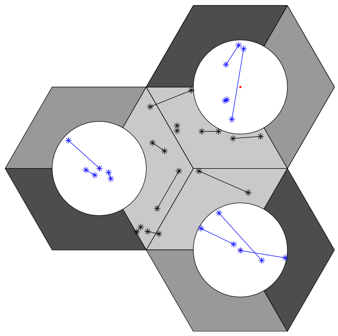


Figure 4.5: Sample optimal partnering strategy obtained by MWM.

User Pair	No FFR User Rates	Strict FFR User Rates	Comp. FFR Sum rate
12-16	1.20 - 1.14	1.20 - 1.14	4.92
6-17	0.99 - 1.21	0.99 - 1.21	4.29
2-5	0.98 - 0.84	0.98 - 0.84	4.24
3-10	0.75 - 0.78	0.75 - 0.78	3.88
4-14	0.71 - 0.75	0.71 - 0.75	3.76
8-11	1.01 - 1.10	1.01 - 1.10	3.44
1-9	0.69 - x	0.69 - 0.65	3.37
15-18	x - x	0.68 - 0.59	3.13
7-13	0.80 - x	0.80 - 0.68	3.07

Table 4.1: Comparison of user rates for cooperative vs. noncooperative protocols.

single cell scenario Chapter 3. This leads to a fairer solution and higher rates for cell edge users. This is further illustrated in Table I, where we tabulate the rates of the outer users, falling into the pseudo-cell in Figure 4.5. In non-cooperative strict FFR and no FFR scenarios, each user is assigned a single subchannel and performs single user optimal power control, leading to the individual rates given in Table I. Note that, without FFR, only $7N/3 = 14$ of the $3N = 18$ outer users can be supported, hence the worst four users are denied access to the channel. In the cooperative FFR scenario, each pair is assigned two subchannels, and their sum rate is shown. The pairs shown on Table I correspond to optimal partnering obtained by MWM. It can be observed from Table I that the worst case users benefit more from cooperation, as the sum rates of user pairs are more nearly equal compared to the non-cooperative setup.

4.5 Conclusion

We proposed a new fractional frequency reuse technique, to be used in conjunction with pairwise cooperation in cooperative multicell multiple access channels. This technique allows cell edge users, potentially from adjacent cells, to share the same subchannels, and select their receiver, which is also convenient for soft hand-off scenarios. We obtained the jointly optimal partner selection, power allocation and receiver selection policy, and demonstrated that this policy not only doubles the system sum rate compared to non-cooperative techniques, but also it provides a fairer rate distribution for cell edge users.

Chapter 5

Cognitive Cooperative Networks and Partner Selection

With the increasing demand for wireless networks, physically limited usable frequency bands' values go higher by time. Therefore, new techniques are constantly under research to increase efficiency of wireless communication channels. However, most of the already-in-use licensed frequency bands are standardized long ago and can't take advantage of these new technologies. This creates an efficiency problem that can be solved with cognitive radio approach. With cognitive radio, primary user who has the channel use rights transmits its message, and a secondary user joins into the channel, transmits its own message with one rule; do not affect primary users communication negatively. Therefore, although the channel is occupied by one user, secondary user can still communicate with intelligent design of communication scheme.

In addition to cognitive networks, cooperation between users may increase throughput of the system significantly. The cooperation idea is rooted from relay model which first introduced in [22]. However, in relay channel, the relay only transmits other user's signals. In cooperative networks, relays have their own messages to be transmitted, therefore the cooperative system may be considered as a special case of relay networks, yet can also be modeled as a multiple access channel with generalized feedback (MAC-GF). This model was first introduced in [18], however in [1] extended to the wireless networks and power optimization was done in [2].

Although in this model users are treated as equals in contrast to the cognitive setup, in this chapter, the model will be altered to fit in cognitive radio concept.

Cognitive radio was first designed to be used in occupied frequency bands as in [23], however now the idea has expanded to all wireless networks. With clever design, the users can transmit their messages by tracing the other user's channel conditions and sends messages according to the other user's power allocation strategy. The systems where the second user transmits signals according to first user's power in the channel without negatively affecting first user's rate is called underlay. Underlay systems are studied fairly wall in the literature, and now the newer and more popular cognitive radio approach is overlay. In overlay cognitive radio, second user decodes first user's messages, may use them to create codewords, and even may relay this message to the receiver. With this approach, second user may have more flexibility on transmission strategy, which may result into increased throughput.

In a cellular system with more than one primary and one secondary users, matching of users also have affect on throughput of the system and sum rate of secondary users. Therefore, partner selection must be done cleverly. In this chapter, a cooperative cognitive radio setup with joint optimal power control is considered and optimum partner selections to maximize system sum rate and secondary users' total rate are found. The proposed system is compared to underlay non-cooperative cognitive radio network.

5.1 System Description

We consider a fading cognitive cooperative multiple access channel with K primary and K secondary users. The users are to be divided into K disjoint pairs, each consisting of one primary and one secondary user. Each pair is then assigned one of K orthogonal subchannels using OFDMA, and users in a given pair communicate with each other, as well as with the receiver, over their assigned subchannel. This creates K cognitive cooperative multiple access channels [24] in parallel, having one primary and one secondary user each. The primary and secondary users are randomly placed on a disk of radius d. The locations of the users, and the statistics of the channel gains among all users and the receiver are assumed to be known at the receiver. The receiver uses this information, and the fact that users in each group will use channel adaptive power control, to determine the optimal cooperating pair assignment, which is then fixed throughout the transmission. We assume that the subchannel assignment is fixed at the beginning of transmissions, and is not optimized instantaneously. Once the cooperating pairs are determined, it is sufficient to assume that each cooperating pair of users have only their own channel state information (inter-user and user-receiver). Likewise, the receiver only needs the instantaneous CSI of the cooperating pairs, the CSI among non-cooperating primary-secondary users is not needed.

We denote by P_i the ith primary user, and by S_j the jth secondary user, where $i, j \in \{1, \ldots, K\}$. A sample system with $K = 5$ is shown in Figure 5.1, along with one possible pairing strategy. Once the pairing is fixed, the received signals $Y_{r_{ij}}$ at the receiver, and Y_{s_j} at the the secondary user for each cooperating pair $\{P_i, S_j\}$ can be written respectively as

$$Y_{r_{ij}} = \sqrt{h_{p_i r} d_{ir}^{-\alpha}} X_{p_i} + \sqrt{h_{s_j r} d_{jr}^{-\alpha}} X_{s_j} + N_r, \tag{5.1}$$

$$Y_{s_j} = \sqrt{h_{p_i s_j} d_{ij}^{-\alpha}} X_{p_i} + N_{s_j}, \tag{5.2}$$

In (5.1)-(5.2), X_{p_i} and X_{s_j} denote the codewords sent by the primary and secondary users; $h_{p_i r}$, $h_{s_j r}$ and $h_{p_i s_j}$ denote the PU_i to receiver, SU_j to receiver and PU_i to SU_j channel power gains due to frequency flat fading; N_r and N_{s_j} denote the zero mean additive white Gaussian noise components at the receiver and SU_j respectively. The noise variances are σ_r^2 and $\sigma_{s_j}^2$.

This channel model is a generalization of the model introduced in [24] which can be considered as a relay channel, where the relay also has its own messages to transmit, or a special case of a MAC with generalized feedback [1, 18], where the

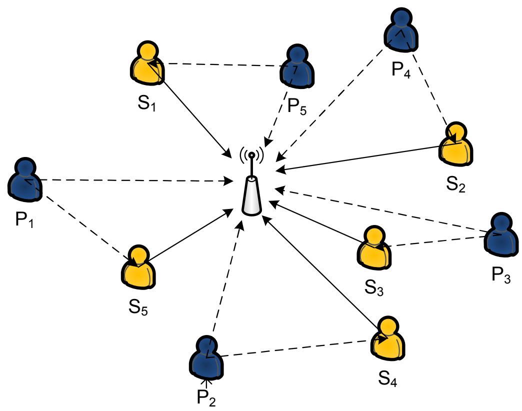


Figure 5.1: Multi-User Cooperative Cognitive Gaussian MAC

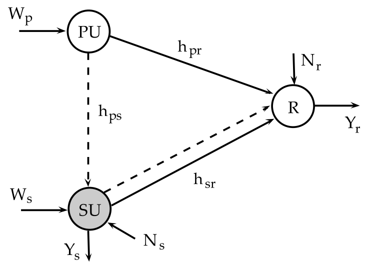


Figure 5.2: Two-User Cooperative Cognitive Gaussian MAC

cooperation signals from one of the users is disabled. We take the latter approach, and modify the superposition block Markov encoding strategy in [1]: we divide the PU's message into two submessages, i.e., $W_{p_i} = (W_{p_i r}, W_{p_i s_j})$. The submessage $W_{p_i r}$ is the information sent directly to the receiver, and the submessage $W_{p_i s_j}$ is the part that can be decoded by both the SU and the receiver. The SU message is not partitioned, as the PU should not aid the SU, due to the cognitive setup. Then, these messages are mapped to randomly generated codewords, whose entries are selected from unit Gaussian distributions, i.e.,

$$X_{s_j r}(W_{s_j}(b), W_{p_i s_j}(b-1)) \tag{5.3}$$

$$X_{p_i r}(W_{p_i r}(b), W_{p_i s_j}(b-1)) \tag{5.4}$$

$$X_{p_i s_j}(W_{p_i s_j}(b), W_{p_i s_j}(b-1)) \tag{5.5}$$

$$C(W_{p_i s_j}(b-1)) \tag{5.6}$$

where $X_{s_j r}$ and $X_{p_i r}$ are used to transmit fresh information $W_{s_j r}(b)$ and $W_{p_i r}(b)$

directly intended for the receiver in block b, $X_{p_i s}$ is signal transmitted by the PU to allow potential cooperation from the SU in the next block, and C is the common signal which is used by both users to cooperatively transmit the PU's information $W_{p_i s_j}(b-1)$ from the previous block. The resulting codewords of the users are formed by superposition, where we also take into account the possibility of power control as in [2], as a function of the available channel state information, denoted by the the channel state vector $\mathbf{h} = [h_{p_i r}, h_{p_i s_j}, h_{s_j r}]$:

$$X_{p_i} = \sqrt{P_{p_i r}(\mathbf{h})}X_{p_i r} + \sqrt{P_{p_i s_j}(\mathbf{h})}X_{p_i s_j} + \sqrt{P_{p_i c}(\mathbf{h})}C, \tag{5.7}$$

$$X_{s_j} = \sqrt{P_{s_j r}(\mathbf{h})}X_{s_j r} + \sqrt{P_{s_j c}(\mathbf{h})}C. \tag{5.8}$$

The powers are required to satisfy the average power constraints,

$$P_{p_i}(\mathbf{h}) = P_{p_i r}(\mathbf{h}) + P_{p_i s_j}(\mathbf{h}) + P_{p_i c}(\mathbf{h}) \tag{5.9}$$

$$P_{s_j}(\mathbf{h}) = P_{s_j r}(\mathbf{h}) + P_{s_j c}(\mathbf{h}) \tag{5.10}$$

$$E\Big[P_n(\mathbf{h})\Big] \leq \bar{P}_n \text{ where } n \in \{p, s\} \tag{5.11}$$

The achievable rate constraints are as follows:

$$\begin{aligned} R_{p_i} \leq\ & E\Big\{\log(1+s_{p_i r}P_{p_i r}(\mathbf{h}))|s_{p_i s_j} < s_{p_i r}\Big\}Pr[s_{p_i s_j} < s_{p_i r}] \\ & + E\Big\{\log(1+s_{p_i s_j}P_{p_i s_j}(\mathbf{h}))|s_{p_i s_j} > s_{p_i r}\Big\}Pr[s_{p_i s_j} > s_{p_i r}] \end{aligned} \tag{5.12}$$

$$R_{s_j} < E\Big\{\log\Big[1+s_{s_j r}P_{s_j r}(\mathbf{h})\Big]\Big\} \tag{5.13}$$

$$\begin{aligned} R_{p_i}+R_{s_j} \leq \min \Bigg\{ & E\Big\{\log\Big[1+s_{p_i r}P_{p_i r}(\mathbf{h})+s_{s_j r}P_{s_j r}(\mathbf{h})\Big]|s_{p_i s_j} < s_{p_i r}\Big\}Pr[s_{p_i s_j} < s_{p_i r}] \\ & + E\Big\{\log\Big[1+s_{p_i s_j}P_{p_i s_j}(\mathbf{h})\Big]|s_{p_i s_j} > s_{p_i r}\Big\}Pr[s_{p_i s_j} > s_{p_i r}] \\ & + E\Big\{\log\Big[1+s_{s_j r}P_{s_j r}(\mathbf{h})\Big]|s_{p_i s_j} > s_{p_i r}\Big\}Pr[s_{p_i s_j} > s_{p_i r}], \\ & E\Big\{\log(A)\Big\}\Bigg\} \end{aligned} \tag{5.14}$$

where $A = 1 + s_{p_i r}P_{p_i}(\mathbf{h}) + s_{s_j r}P_{s_j}(\mathbf{h}) + 2\sqrt{s_{p_i r}s_{s_j r}P_{p_i c}P_{s_j c}}$. In (5.12)-(5.14), R_{p_i} and R_{s_j} denote the rates of primary and secondary users; and the channel fading coefficients, normalized by the noise powers are denoted as $s_{ij} = h_{ij}^2/\sigma_j^2$, where $i \in \{p, s\}$ and $j \in \{s, r\}$, $i \neq j$.

Now, we describe the crucial twist from the cooperative communication framework, due to the cognitive setup: not all rates satisfying the above constraints are necessarily achievable, as we should also guarantee that the PU's achievable rate is no worse than what it would be, had the PU been transmitting alone. Moreover, we have to assume that the PU would be able to use optimal power allocation [25], which is single user waterfilling, while computing the worst case rate requirement of the PU. Therefore, we need the constraint:

$$R_{p_i} \geq E\Big\{ \log \Big[1 + P_{p_i}^{\star}(h_{p_i r}) s_{p_i r}\Big]\Big\} \triangleq B^* \qquad (5.15)$$

where $P_{p_i}^{\star}(h_{pr})$ is the optimal power level for single user transmission, with power constraint $E[P_{p_i}^{\star}(h_{p_i r})] = \bar{P}_{p_i}$; and B^* is the resulting maximum data rate achievable by the PU, without cooperation.

In the next section, we solve the optimal power allocation problem for the cognitive cooperative scenario, with two separate objectives: sum rate maximization, which creates an extra incentive for the PU to allow cooperation, and SU rate maximization, which aims to accommodate as much rate for the cognitive user as possible, while still providing a maximum single-user rate guarantee for the PU.

5.2 Maximization of the Achievable Rates

We start by noting, also in light of the findings in [2] for the non-cognitive cooperative MAC, that for channel states which satisfy $s_{p_i s_j} > s_{p_i r}$, the optimal strategy is to set $P_{p_i r}(\mathbf{h}) = 0$, meaning no additional power should be allocated by the PU for direct transmission. Due to symmetry, for channel states $s_{p_i s_j} < s_{p_i r}$, the optimal strategy becomes $P_{p_i s_j}(\mathbf{h}) = 0$, where primary user should transmit its own message to the receiver directly. Therefore, the maximization of achievable rates can be separated in two cases depending on the channel states with joint power constraints. These power settings ensures convexity of the problem and the optimization problem can be stated in convex form as follows:

$$\max_{\mathbf{P(h)}} \alpha R_{p_i} + R_{s_j} \tag{5.16}$$

$$\text{s.t.} \quad R_{p_i} \leq E\Big\{\log(1+s_{p_ir}P_{p_ir}(\mathbf{h}))|s_{p_is_j}<s_{p_ir}\Big\}Pr[s_{p_is_j}<s_{p_ir}]$$
$$+ E\Big\{\log(1+s_{p_is_j}P_{p_is_j}(\mathbf{h}))|s_{p_is_j}>s_{p_ir}\Big\}Pr[s_{p_is_j}>s_{p_ir}] \tag{5.17}$$

$$R_s < E\Big\{\log\Big[1+s_{s_jr}P_{s_jr}(\mathbf{h})\Big]\Big\} \tag{5.18}$$

$$R_{p_i}+R_{s_j} \leq \min\Bigg\{E\Big\{\log\Big[1+s_{p_ir}P_{p_ir}(\mathbf{h})+s_{s_jr}P_{s_jr}(\mathbf{h})\Big]|s_{p_is_j}<s_{p_ir}\Big\}Pr[s_{p_is_j}<s_{p_ir}]$$
$$+ E\Big\{\log\Big[1+s_{p_is_j}P_{p_is_j}(\mathbf{h})\Big]|s_{p_is_j}>s_{p_ir}\Big\}Pr[s_{p_is_j}>s_{p_ir}]$$
$$+ E\Big\{\log\Big[1+s_{s_jr}P_{s_jr}(\mathbf{h})\Big]|s_{p_is_j}>s_{p_ir}\Big\}Pr[s_{p_is_j}>s_{p_ir}],$$
$$E\Big\{\log(A)\Big\}\Bigg\} \tag{5.19}$$

$$R_{p_i} \geq B^\star \tag{5.20}$$

$$E\Big[P_{p_ir}(\mathbf{h}) + P_{p_is_j}(\mathbf{h}) + P_{pc}(\mathbf{h})\Big] \leq \bar{P}_{p_i} \tag{5.21}$$

$$E\Big[P_{s_jr}(\mathbf{h}) + P_{sc}(\mathbf{h})\Big] \leq \bar{P}_{s_j} \tag{5.22}$$

$$P_{p_is_j}(\mathbf{h}), P_{p_ic}(\mathbf{h}), P_{s_jr}(\mathbf{h}), P_{s_jc}(\mathbf{h}) \geq 0 \tag{5.23}$$

where, K_l denotes lth cooperating pair of K cooperating pairs in the system, $l \in \{1, \ldots, K\}$. Note that, by setting $\alpha = 1$ in (5.16), we obtain the sum rate maximization for cognitive MAC, and by setting $\alpha = 0$, we obtain the SU rate maximization. We will treat both problems in parallel, and discuss their differences as they become apparent. First, by associating several Lagrange multipliers with the constraints in (5.17)-(5.23), we write the Lagrangian,

$$
\begin{aligned}
\mathcal{L} =& \alpha R_{p_i} + R_{s_j} \\
& + \gamma_1 \Bigg\{ E\Big\{ \log(1+s_{p_ir}P_{p_ir}(\mathbf{h}))|s_{p_is_j}<s_{p_ir} \Big\} Pr[s_{p_is_j}<s_{p_ir}] \\
& + E\Big\{ \log(1+s_{p_is_j}P_{p_is_j}(\mathbf{h}))|s_{p_is_j}>s_{p_ir} \Big\} Pr[s_{p_is_j}>s_{p_ir}] - R_{p_i} \Bigg\} \\
& + \gamma_2 \Bigg\{ E\Big\{ \log\Big(1 + s_{s_jr}P_{s_jr}(\mathbf{h})\Big)\Big\} - R_{s_j} \Bigg\} \\
& + \gamma_3 \Bigg\{ E\Big\{ \log(A) \Big\} - R_{p_i} - R_{s_j} \Bigg\} \\
& + \gamma_4 \Bigg\{ E\Big\{ \log\Big[1 + s_{p_ir}P_{p_ir}(\mathbf{h}) + s_{s_jr}P_{s_jr}(\mathbf{h})\Big] |s_{p_is_j}<s_{p_ir} \Big\} Pr[s_{p_is_j}<s_{p_ir}] \\
& + E\Big\{ \log\Big[1+s_{p_is_j}P_{p_is_j}(\mathbf{h})\Big] |s_{p_is_j}>s_{p_ir} \Big\} Pr[s_{p_is_j}>s_{p_ir}] \\
& + E\Big\{ \log\Big[1+s_{s_jr}P_{s_jr}(\mathbf{h})\Big] |s_{p_is_j}>s_{p_ir} \Big\} Pr[s_{p_is_j}>s_{p_ir}] - R_{p_i} - R_{s_j} \Bigg\} \\
& + \gamma_5 \Bigg\{ R_{p_i} - B^{\star} \Bigg\} \\
& + \lambda_1 \Bigg\{ P_{p_i} - E\Big[P_{p_ir}(\mathbf{h}) + P_{p_is_j}(\mathbf{h}) + P_{p_ic}(\mathbf{h}) \Big] \Bigg\} \\
& + \lambda_2 \Bigg\{ P_{s_j} - E\Big[P_{s_jr}(\mathbf{h}) + P_{s_jc}(\mathbf{h}) \Big] \Bigg\} \\
& + \mu_1 P_{p_ir}(\mathbf{h}) + \mu_2 P_{p_is_j}(\mathbf{h}) + \mu_3 P_{p_ic}(\mathbf{h}) + \mu_4 P_{s_jr}(\mathbf{h}) + \mu_5 P_{s_jc}(\mathbf{h})
\end{aligned}
\tag{5.24}
$$

For simplicity, we can divide the Lagrange function into two cases, according to channel states of $s_{p_is_j}$ and s_{p_ir}. Case 1 can be defined strong primary to secondary user link in comparison to primary to receiver link, i.e., $s_{p_is_j} > s_{p_ir}$ and case 2 is the exact opposite, $s_{p_is_j} < s_{p_ir}$. Taking the partial derivatives with respect to the power components of primary and secondary users, as well as the rate variables, and employing complementary slackness constraint in both cases, it is easy to show that the following KKT conditions are necessary and sufficient for optimality:

Case 1:

$$\lambda_1 \geq (\gamma_1 + \gamma_4)\frac{s_{p_i s_j}}{1 + s_{p_i s_j} P_{p_i s_j}(\mathbf{h})} + \gamma_3 \frac{s_{p_i r}}{A} \tag{5.25}$$

$$\lambda_2 \geq (\gamma_2 + \gamma_4)\frac{s_{s_j r}}{1 + s_{s_j r} P_{s_j r}(\mathbf{h})} + \gamma_3 \frac{s_{s_j r}}{A} \tag{5.26}$$

Case 2:

$$\lambda_1 \geq (\gamma_1 + \gamma_4)\frac{s_{p_i r}}{1 + s_{p_i r} P_{p_i r}(\mathbf{h})} + \gamma_3 \frac{s_{p_i r}}{1 + s_{p_i r} P_{p_i r} + s_{s_j r} P_{s_j r}} \tag{5.27}$$

$$\lambda_2 \geq (\gamma_2 + \gamma_4)\frac{s_{s_j r}}{1 + s_{s_j r} P_{s_j r}(\mathbf{h})} + \gamma_3 \frac{s_{s_j r}}{1 + s_{p_i r} P_{p_i r} + s_{s_j r} P_{s_j r}} \tag{5.28}$$

Case 1 and 2:

$$\lambda_1 \geq \gamma_3 \frac{s_{p_i r}\sqrt{P_{p_i c}(\mathbf{h})} + \sqrt{s_{p_i r} s_{s_j r} P_{s_j c}(\mathbf{h})}}{A\sqrt{P_{p_i c}(\mathbf{h})}} \tag{5.29}$$

$$\lambda_2 \geq \gamma_3 \frac{s_{s_j r}\sqrt{P_{s_j c}(\mathbf{h})} + \sqrt{s_{p_i r} s_{s_j r} P_{p_i c}(\mathbf{h})}}{A\sqrt{P_{s_j c}(\mathbf{h})}} \tag{5.30}$$

$$1 = \gamma_2 + \gamma_3 + \gamma_4 \tag{5.31}$$

$$\alpha + \gamma_5 = \gamma_1 + \gamma_3 + \gamma_4 \tag{5.32}$$

The constraints (5.25) - (5.28) are satisfied with equality, if the powers $P_{p_i r}(\mathbf{h})$, $P_{p_i s_j}(\mathbf{h})$,$P_{s_j r}(\mathbf{h})$,$P_{p_i c}(\mathbf{h})$,$P_{s_j c}(\mathbf{h})$ are positive.

Let us first consider the sum rate maximization, i.e., $\alpha = 1$. From (5.31) and (5.32), we have $1 + \gamma_5 = \gamma_2 + \gamma_3 + \gamma_4$ and $1 = \gamma_1 + \gamma_3 + \gamma_4$. The crucial trick is to consider two cases separately: when $\gamma_5 = 0$, (5.20) is inactive, meaning the PU rate already satisfies the cognitive transmission constraint. Then, we are back to the non-cognitive scenario as in [2], and after some lengthy manipulations of

(5.25)-(5.30), with $\gamma_3 = 1 - (\gamma_2 + \gamma_4) = 1 - (\gamma_1 + \gamma_4)$ and $\gamma_1 = \gamma_2$, we get

Case 1:

$$P_{p_i s_j}(\mathbf{h}) = \left((\gamma_1 + \gamma_4) \frac{(\lambda_2 s_{p_i r} + \lambda_1 s_{s_j r})}{\lambda_1^2 s_{s_j r}} - \frac{1}{s_{p_i s_j}} \right)^+, \tag{5.33}$$

$$P_{s_j r}(\mathbf{h}) = \left((\gamma_2 + \gamma_4) \frac{(\lambda_2 s_{p_i r} + \lambda_1 s_{s_j r})}{\lambda_2^2 s_{s_j r}} - \frac{1}{s_{s_j r}} \right)^+, \tag{5.34}$$

$$P_{p_i c}(\mathbf{h}) = \frac{\gamma_3 \frac{\left(s_{p_i r} + \lambda_1 s_{s_j r}/\lambda_2\right)}{\lambda_1} - D_1}{(s_{p_i r} + \lambda_1 s_{s_j r}/\lambda_2)^2} s_{p_i r}, \tag{5.35}$$

$$P_{s_j c}(\mathbf{h}) = \frac{\gamma_3 \frac{\left(s_{s_j r} + \lambda_2 s_{p_i r}/\lambda_1\right)}{\lambda_2} - D_1}{(s_{s_j r} + \lambda_2 s_{p_i r}/\lambda_1)^2} s_{s_j r}, \tag{5.36}$$

Case 2:

$$P_{p_i r}(\mathbf{h}) = \left((\gamma_1 + \gamma_4) \frac{(\lambda_2 s_{p_i r} + \lambda_1 s_{s_j r})}{\lambda_1^2 s_{s_j r}} - \frac{1}{s_{p_i s_j}} \right)^+, \tag{5.37}$$

$$P_{s_j r}(\mathbf{h}) = \left((\gamma_2 + \gamma_4) \frac{(\lambda_2 s_{p_i r} + \lambda_1 s_{s_j r})}{\lambda_2^2 s_{s_j r}} - \frac{1}{s_{s_j r}} \right)^+, \tag{5.38}$$

$$P_{p_i c}(\mathbf{h}) = \frac{\gamma_3 \frac{\left(s_{p_i r} + \lambda_1 s_{s_j r}/\lambda_2\right)}{\lambda_1} - D_2}{(s_{p_i r} + \lambda_1 s_{s_j r}/\lambda_2)^2} s_{p_i r}, \tag{5.39}$$

$$P_{s_j c}(\mathbf{h}) = \frac{\gamma_3 \frac{\left(s_{s_j r} + \lambda_2 s_{p_i r}/\lambda_1\right)}{\lambda_2} - D_2}{(s_{s_j r} + \lambda_2 s_{p_i r}/\lambda_1)^2} s_{s_j r}, \tag{5.40}$$

where $D_1 = 1 + s_{p_i r} P_{p_i s_j}(\mathbf{h}) + s_{s_j r} P_{sr}(\mathbf{h})$ and $D_2 = 1 + s_{p_i r} P_{p_i r}(\mathbf{h}) + s_{s_j r} P_{sr}(\mathbf{h})$, provided $P_{p_i c}(\mathbf{h})$ and $P_{s_j c}(\mathbf{h})$ obtained from equations (5.35), (5.39) and (5.36), (5.40) are positive. Otherwise, $P_{p_i c}(\mathbf{h})$ and $P_{s_j c}(\mathbf{h})$ both have to be set to zero in respective case as there is no other alternative, as having only one cooperative power non-zero would be strictly suboptimal. For case 1, one should re-solve (5.25) and (5.26) for $P_{p_i s_j}(\mathbf{h})$ and $P_{s_j r}(\mathbf{h})$, which are the positive roots of the following quadratic equation:

$$a_i P_{ij}(\mathbf{h})^2 + b_i P_{ij}(\mathbf{h}) + c_i = 0, \quad \{i, j\} \in \{\{p, s\}, \{s, r\}\} \tag{5.41}$$

where the coefficients are given by

$$
\begin{aligned}
(a_{p_i}; b_{p_i}; c_{p_i}) = (&\lambda_1 s_{p_i r} s_{p_i s_j}; \lambda_1 (s_{p_i r} + s_{p_i s_j} + s_{p_i s_j} s_{s_j r} P_{s_j r}(\mathbf{h}) \\
&- s_{p_i r} s_{p_i s_j}); \lambda_1 (1 + s_{s_j r} P_{s_j r}(\mathbf{h})) \\
&- (\gamma_2 + \gamma_4)(s_{p_i s_j} + s_{p_i s_j} s_{s_j r} P_{s_j r}(\mathbf{h}) - s_{p_i r}) - s_{p_i r}) \\
(a_{s_j}; b_{s_j}; c_{s_j}) = (&\lambda_2 s_{s_j r}^2; \lambda_2 (2 s_{s_j r} + s_{p_i r} s_{s_j r} P_{p_i s_j}(\mathbf{h})) - s_{s_j r}^2; \\
&\lambda_2 (1 + s_{p_i r} P_{p_i s_j}(\mathbf{h})) - (\gamma_2 + \gamma_4)(s_{p_i r} s_{s_j r} P_{p_i s_j}(\mathbf{h})) - s_{s_j r})
\end{aligned}
$$

For case 2, the equations which should be re-solved are (5.27) and (5.28) for $P_{p_i r}(\mathbf{h})$ and $P_{s_j r}(\mathbf{h})$, which are the positive roots of the following quadratic equation:

$$
\begin{aligned}
(a_{p_i}; b_{p_i}; c_{p_i}) = (&\lambda_1 s_{p_i r}^2; \lambda_1 (2 s_{p_i r} + s_{p_i r} s_{s_j r} P_{s_j r}(\mathbf{h}) \\
&- s_{p_i r}^2); \lambda_1 (1 + s_{s_j r} P_{s_j r}(\mathbf{h})) \\
&- (\gamma_2 + \gamma_4) s_{p_i r} s_{s_j r} P_{s_j r}(\mathbf{h}) - s_{p_i r}) \\
(a_{s_j}; b_{s_j}; c_{s_j}) = (&\lambda_2 s_{s_j r}^2; \lambda_2 (2 s_{s_j r} + s_{p_i r} s_{s_j r} P_{p_i r}(\mathbf{h})) - s_{s_j r}^2; \\
&\lambda_2 (1 + s_{p_i r} P_{p_i r}(\mathbf{h})) - (\gamma_2 + \gamma_4)(s_{p_i r} s_{s_j r} P_{p_i r}(\mathbf{h})) - s_{s_j r})
\end{aligned}
$$

Note that, because of the equalities $\gamma_3 = 1 - (\gamma_2 + \gamma_4) = 1 - (\gamma_1 + \gamma_4)$ and $\gamma_1 = \gamma_2$, only one Lagrange multiplier search will be sufficient to reach to the optimal solution.

Now, we go back to the second possible case, $\gamma_5 > 0$, meaning that (5.20) is satisfied with equality, i.e., the PU rate is fixed to its minimum possible value. The key observation here is that the sum rate maximization problem then becomes equivalent to SU rate maximization, and solving the SU maximization problem will also complete the solution of the sum rate maximization. To do so, we may as well set $\alpha = 0$, and force equality in (5.20), by varying γ_5. Luckily, the KKT conditions, and the resulting optimal power expressions are almost identical to the previous sum rate maximization case, except now the equality $\gamma_1 = \gamma_2$ does not hold. Therefore, by using equations (5.31) and (5.32), we now need to search for two Lagrange multipliers, $\gamma_1 + \gamma_4$ and $\gamma_2 + \gamma_4$, rather than one. Since we are searching for the sum of Lagrange multipliers instead of individual values, we can treat $\gamma_1 + \gamma_4$ and $\gamma_2 + \gamma_4$ as two individual Lagrange multipliers. Once again, when the cooperative powers turn out to be negative, we set them to zero, and we instead solve $P_{p_i s_j}(\mathbf{h})$, $P_{s_j r}(\mathbf{h})$ in case 1, $P_{p_i r}(\mathbf{h})$, $P_{s_j r}(\mathbf{h})$ in case 2, that maximize the SU rate, with again by roots of the following quadratic equation:

$$a_i P_{ir}(\mathbf{h})^2 + b_i P_{ir}(\mathbf{h}) + c_i = 0, \quad i \in \{p, s\} \tag{5.42}$$

Case 1:

$$\begin{aligned}
(a_{p_i}; b_{p_i}; c_{p_i}) = (&\lambda_1 s_{p_i r} s_{p_i s_j}; \lambda_1(s_{p_i r} + s_{p_i s_j} + s_{p_i s_j} s_{s_j r} P_{s_j r}(\mathbf{h})) \\
&- (\gamma_1 - \gamma_2 + 1) s_{p_i r} s_{p_i s_j}; \lambda_1(1 + s_{s_j r} P_{s_j r}(\mathbf{h})) \\
&- \gamma_1(s_{p_i s_j} + s_{p_i s_j} s_{s_j r} P_{s_j r}(\mathbf{h})) - (1 - \gamma_2 - \gamma_4) s_{p_i r}) \\
(a_{s_j}; b_{s_j}; c_{s_j}) = (&\lambda_2 s_{s_j r}^2; \lambda_2(2 s_{s_j r} + s_{p_i r} s_{s_j r} P_{p_i s_j}(\mathbf{h})) - s_{s_j r}^2; \\
&\lambda_2(1 + s_{p_i r} P_{p_i s_j}(\mathbf{h})) - (\gamma_2 + \gamma_4)(s_{p_i r} s_{s_j r} P_{p_i s_j}(\mathbf{h})) - s_{s_j r})
\end{aligned}$$

Case 2:

$$\begin{aligned}
(a_{p_i}; b_{p_i}; c_{p_i}) = (&\lambda_1 s_{p_i r}^2; \lambda_1(2 s_{p_i r} + s_{p_i r} s_{s_j r} P_{s_j r}(\mathbf{h})) \\
&- (\gamma_1 - \gamma_2 + 1) s_{p_i r}^2; \lambda_1(1 + s_{s_j r} P_{s_j r}(\mathbf{h})) \\
&- (\gamma_1 + \gamma_4)(s_{p_i r} + s_{p_i r} s_{s_j r} P_{s_j r}(\mathbf{h})) - (1 - \gamma_2 - \gamma_4) s_{p_i r}) \\
(a_{s_j}; b_{s_j}; c_{s_j}) = (&\lambda_2 s_{s_j r}^2; \lambda_2(2 s_{s_j r} + s_{p_i r} s_{s_j r} P_{p_i r}(\mathbf{h})) - s_{s_j r}^2; \\
&\lambda_2(1 + s_{p_i r} P_{p_i r}(\mathbf{h})) - (\gamma_2 + \gamma_4)(s_{p_i r} s_{s_j r} P_{p_i r}(\mathbf{h})) - s_{s_j r})
\end{aligned}$$

Convex optimization represented in this chapter can be done numerically with iterative implementation of search for Lagrange multipliers. In the next section, numerical implementation will be used for each primary and secondary user pairs and optimal partner selection algorithm will be applied on the resulting multiuser network afterwards.

5.3 Optimal Partner Selection

In this chapter, we consider a multiuser cognitive cooperative network employing OFDMA as multiple access technique. Due to cognitive setup, users are divided into two disjoint sets; set of primary users $\S_p$ and set of secondary users $\S_s$ and each primary user should be matched with a secondary user as partners and employ joint power control employing cooperative communication scheme. In this setup with power allocation scheme proposed in section 5.2, we will deal with two system wide maximization problems; sum rate maximization and secondary user rates maximization. Since in both maximization problems primary users have a lower bound on their transmission rates, secondary users' rates play great role and the partner selection must be done wisely. Due to the orthogonality provided by OFDMA, rate maximization of partners can be done independently from other cooperating partner, which leads us to divide the problem into two parts; partnerwise rate maximization and partner selection. The maximization problems can be expressed as follows:

$$\max_{\Gamma_l \in \Gamma} \quad \sum_{\{i,j\} \in \Gamma_l} \max_{\mathbf{P(h)}} \; \alpha R_{p_i} + R_{s_j} \tag{5.43}$$

$$\text{s.t.} \quad \sum_{s \in S_{ij}} E\left[P^{(s)}_{p_i r}(\mathbf{h}) + P^{(s)}_{p_i s_j}(\mathbf{h}) + P^{(s)}_{p_i c}(\mathbf{h})\right] \le \bar{P}_{p_i}, \quad \forall \{i,j\} \in K_l$$

$$R_i + R_j \text{ satisfy (5.14)}, \quad \forall \{i,j\} \in K_l. \tag{5.44}$$

where Γ_l is the matchings of primary and secondary users, Γ is set of all distinct partitions of Γ_l. Note that, by setting $\alpha = 0$, the maximization problem becomes secondary user rate maximization problem. The sum rate or the secondary user rate maximization problems are explained in detail in the previous section, hence in this section we will only focus on partner selection algorithm.

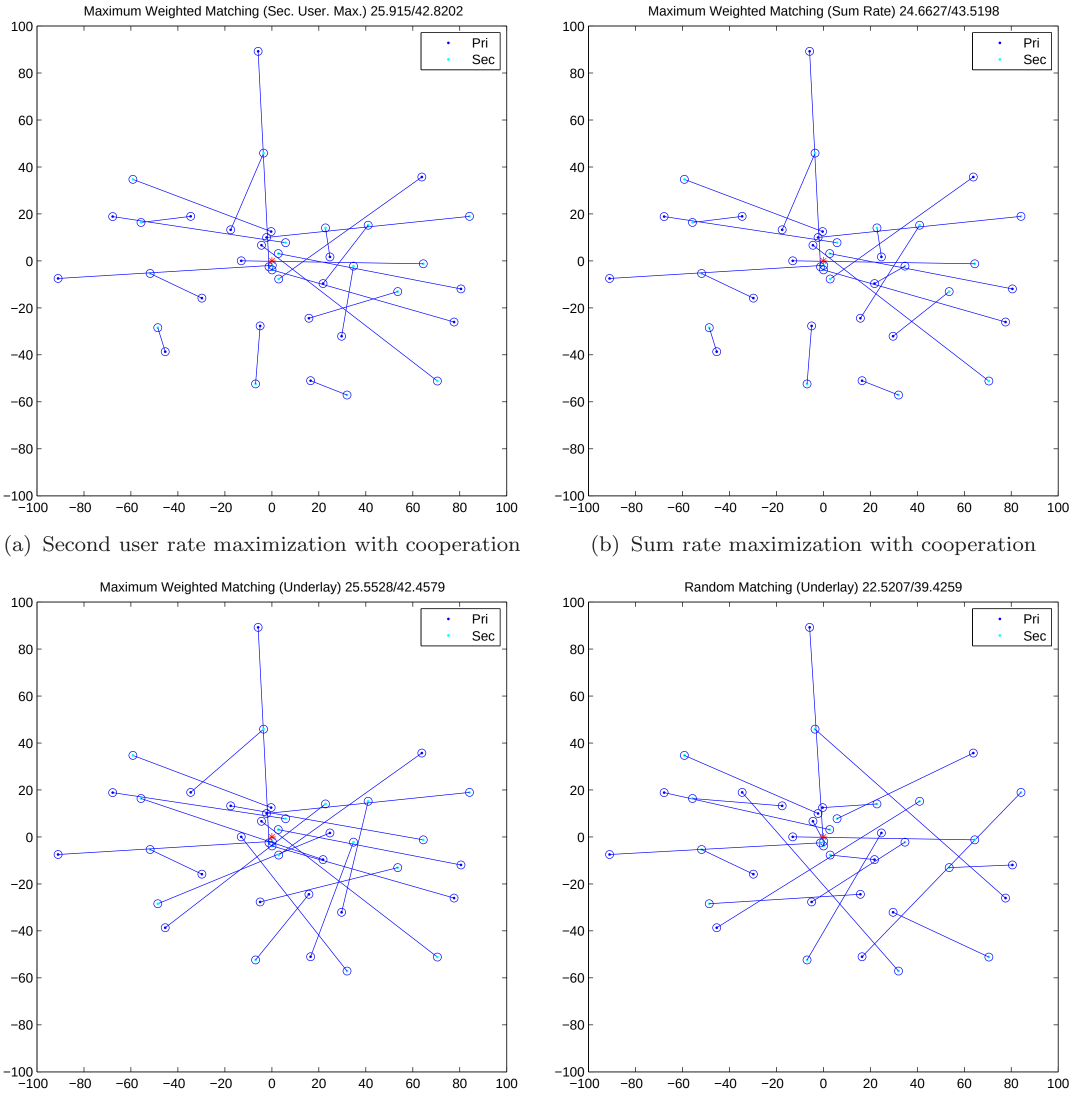
For optimal partner selection, we have to select one primary and one secondary user as cooperating partners, repeating until there is no other users left. After that step, we have to try all possible partnering combinations, which will result in a combinatorial complexity algorithm. Instead of this approach, we may morph this problem into a graph theory weighted matching problem, which can be solved optimally with $O(n^3)$ complexity by *Edmond's Maximum Weighted Matching* (MWM) algorithm in [20]. To apply the algorithm, we have to define all users as edges, and sum rates (or secondary user rates if desired) as weights. As it can be seen, cognitive setup differs from the mutual cooperation setup in Chapter 1 from graph theory perspective, and resulting graph will be bipartite. However, MWM algorithm can be applied to bipartite graphs with same complexity. After computing every weight on the graph, MWM can be run over the resulting graph and optimum matching scheme can be obtained. MWM algorithm can also be applied to underlay scenario where the only change is the computation of weight due to change in power control policy.

5.4 Simulation Results

In our simulations, we have considered $K = 20$ primary and secondary users, located randomly with uniform distribution on a cell with radius $r = 100m$, where the receiver is located in the center. Partner selection is done once, and partners do not change throughout the transmission. Each primary and secondary user has the same power constraint.

We have considered both sum rate optimization and second user rate optimization problems in both proposed cooperative and underlay scenarios. We applied MWM to both scenarios and in underlay scenario we also considered a system where primary and secondary users are randomly matched.

As expected, the maximum sum rate of the system can be achieved by sum rate maximization power control with MWM where throughput of the system is 43.5198, total rate of the secondary users is 24.6627 and matching can be observed from Figure 5.3(b). Sum rate maximization may increase rate of primary and secondary users, therefore, primary users may benefit from this scheme. However,



(a) Second user rate maximization with cooperation

(b) Sum rate maximization with cooperation

(c) Underlay system matched by MWM

(d) Underlay system with random matching

Figure 5.3: Example Simulation Results

cognitive setup may be used to enable transmission of secondary users without affecting the primary users' communication, thus second user maximization may be more important from this point of view. In secondary user maximization power control and matching in Figure 5.3(a), the total rate of secondary users is 25.9150, which is the maximum of all benchmark systems. The system throughput is lower in comparison to sum rate maximization, yet this is not the main consideration in this scheme. In underlay scenarios, both the system throughput and sum of secondary user rates is are lower than proposed schemes while it gets worse when MWM is not applied.

5.5 Conclusion

In this chapter, a cooperative cognitive overlay communication setup was introduced and optimum power allocation schemes for sum rate and secondary user rate maximization were found. The matchings of the primary and secondary users is done optimally by MWM. The system was simulated and results show that, proposed system can be used to maximize system throughput or maximize the secondary users' rates. Therefore, it can be seen that, cooperation in cognitive setup can be a promising technique and deserves further research.

Conclusion

There are several techniques to increase the system throughput in wireless communication and one of the promising techniques is cooperation. In this dissertation, we have introduced cooperation with joint power control to maximize partners total rate. Then we have divided a system into two user partitions, maximized the rate and selected partners optimally by MWM from graph theory so that we could maximize the system throughput. Then we have expanded these ideas and applied them to multi-cell scenario with a novel FFR scheme which encourages cooperation. At last, we have enabled cooperation in an overlay cognitive setup with joint power control and found optimal partnering strategy to maximize system throughput or second users' rates. The system was compared to underlay cognitive setups and shown that proposed system performs better in terms of throughput. We have shown that cooperation is a promising technique, with novel ideas and multidisciplinary study, wireless communication systems can be more optimized.

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Curriculum Vitae

Saygın Bakşi was born on 28 April 1987, in Adana. He received his B.S. degree in Electronics Engineering from Işık University in 2010. He worked as a research assistant at the Department of Electrical and Electronics Engineering at Işık University between 2010 and 2013. The research projects supported by TUBITAK which he has taken part are "Encoding and Decoding Strategies, Achievable Rates and Resource Allocation for Cooperative Cognitive Radio Systems" between 2012–2013 and "Orthogonal Frequency Division Multiplexing (OFDM)-Based Resource Allocation in Cooperative Wireless Communications" between 2010–2011. The courses which he assisted include Introduction to Communication Systems, Electronics, Circuit Theory Laboratory and Electronics Laboratory. His research interests are cooperative communications, resource allocation in wireless networks, fractional frequency reuse and cognitive radio.

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