COMPRESS AND FORWARD BASED COOPERATION STRATEGIES IN WIRELESS NETWORKS

MEHMET GÜNEŞ
B.S., Electronics Engineering, Işık University, 2008

Submitted to the Graduate School of Science and Engineering
in partial fulfillment of the requirements for the degree of
Master of Science
in
Electronics Engineering

IŞIK UNIVERSITY
2011

IŞIK UNIVERSITY
GRADUATE SCHOOL OF SCIENCE AND ENGINEERING

COMPRESS AND FORWARD BASED COOPERATION STRATEGIES IN WIRELESS NETWORKS

MEHMET GÜNEŞ

APPROVED BY:

Asisst. Prof. Onur Kaya (Thesis Supervisor)	Işık University	____________
Assoc. Prof. Mengüç Öner	Işık University	____________
Assist. Prof. Ali İnan	Işık University	____________

APPROVAL DATE: ..../..../....

COMPRESS AND FORWARD BASED COOPERATION STRATEGIES IN WIRELESS NETWORKS

Abstract

Cooperative communications are emerging as an alternative technology to MIMO communications. In this thesis, we first summarize information theoretical analysis of known channel types and also consider three fundamental relaying techniques known as Amplify and Forward, Decode and Forward and Compress and Forward. We propose two compress-and forward based cooperation protocols for a two-user multiple access channel, and obtain their achievable rate regions. The performance of the proposed models are is compared with the performance of other two-user cooperative protocols proposed in literature.

SIKIŞTIR İLET YÖNTEMİNE DAYALI İSBİRLİKLİ HABERLEŞME STRATEJİLERİ

Özet

İşbirlikli haberleşme, çok girişli çok çıkışlı sistemlere alternatif gelişmekte olan bir teknolojidir. Öncelikle bu çalışmada bilgi kuramsal kanal tipleri özetlenecek olup bilinen röle protokollerine; Yükselt-İlet, Çöz-İlet ve Sıkıstır-İlet değinilecektir. Sıkıştır-İlet tekniğini kullanan iki kullanıcılı işbirlikli ağ modeli için erişilebilir hız bölgesi türetilecektir. Ayrıca önerilen ağ modelinin başarımı, literatürde önerilmiş diğer iki kullanıcılı sistemlerin başarımları ile kıyaslanmıştır.

Acknowledgements

There are many people who helped me to complete my dissertation at Işık University. First, I would like to express my deepest acknowledgements to my supervisor Assist. Prof. Onur Kaya for his support, guidance and patience in this long journey. I am also thankful to Işık University for providing me all the facilities and for giving me the chance for being a part of such a wonderful environment. I would like to thank my friends and colleagues, Res. Assist. Murat İşleyen, Res Assist. Burak Çizmeçi, Res Assist. Engin Tamer, Res. Assist. Teoman Mert, and Res. Assist. Doğan Kırcalı and the other Engineering Faculty professors for their support. And lastly, I like to present all my respect and love to my mother Belkıs and my father Rıza because of their endless love, encouragement and motivation.

To my family...

Table of Contents

List of Tables

List of Figures

List of Symbols

C	Capacity
$E[\bullet]$	Expected Value
I	Mutual Information
$\max(\bullet)$	Maximum Value
$\min(\bullet)$	Minimum Value

List of Abbreviations

AF	Amplify-and-Forward
AWGN	Additive White Gaussian Noise
CF	Compress-and-Forward
DF	Decode-and-Forward
FCC	Federal Communications Commission
MAC	Multiple Access Channel
MAC-GF	Multiple Access Channel with Generalized Feedback
RF	Radio Frequency
SNR	Signal-to-Noise Ratio
Joint CF/DF	Joint Compress and Decode Forward

Chapter 1

Introduction

While wireless communication was not more than a dream before the last quarter of the 20th century, there are more than three billion mobile phone subscribers worldwide, according to 2007 ICT statistics. The demand for a variety of applications are growing everyday including, smart phone applications, high quality video streaming, smart home applications etc., which also forces hardware industry to design new backbone technologies which supports higher data rates and compatible end-user devices. However, wireless communication is the fastest growing branch of telecommunication industry; and technical problems remain in designing wireless network infrastructure, which is necessary to support emerging applications. In this introductory chapter, a brief history of wireless communication, motivation of this thesis, a detailed literature survey and on outline of this thesis are provided.

Wireless communication systems and devices were introduced in the early 1980s. First generation wireless devices were based on analog FM modulation technique and used narrow band circuit switched voice services. In the beginning of 1990s, with the introduction of digital modulation schemes second generation mobile devices were introduced and offered higher spectral efficiency with higher voice quality. Today, second generation mobile phones are still in use in least developed and developing countries for voice communications. The third generation wireless systems, which support higher data rates for voice, video-telephony and data communication were launched in early 2000s. Fourth generation wireless systems, currently under development, aim to offer high mobility with higher data rates for mobile users and also higher data rates for stationary users.

In the presence of interference, transmission path loss, shadowing, delay spread and doppler spread, signal transmission in wireless channel become a challenging task. Achieving higher data rates in such channel is possible by increasing transmission bandwidth or transmission power. However, in most of the communication systems consuming these resources is not a clever idea because of the need of battery power and large frequency band requirement. One appealing idea is to use multiple input multiple output technology (MIMO) in order to combat detrimental effects of the wireless channel. MIMO technologies improve received signal quality by combining signals received from multi-path wireless channels created by using multiple antennas in a device. Nevertheless, assembling multiple antennas should not be a practical idea, because of mobile device size limitations. Development of ad-hoc and sensor network applications bring the idea of cooperation in wireless communication systems. In such applications, information sent to the destination passes through other nodes, or information is shared among all nodes. The simplest and oldest form of cooperation is perhaps multi-hopping, which consist of point-to-point links between source and destination. Apart from any adverse effect of the environment, signal transmitted from source attenuates over long ranges which makes point-to-point links impractical. The problem is solved by one strong link with intermediate nodes including repeaters in each node. Wireless channel has a broadcast nature itself; other nodes (users) receive the end-user's signals, for a long time this was thought as interference and waste of energy, because nodes were not allowed to process end-user's information. However, in cooperative communication, nodes are able to process and forward end-user's information to an ultimate receiver which makes wireless channel more robust to rapid changes of the channel states and also increases achievable data rates. By using cooperative communication, transmitter can be considered as multiple virtual antenna transmitter which also allows single antenna mobiles to reap some benefits of MIMO systems.

Cooperative communication became more popular in the last decade. In the past years, in wireless network and communication standards like IEEE802.11, IEEE802.16 Coded OFDM (COFDM) using Adaptive modulation and Coding Schemes and multiple antennas are used, and also relaying protocols are discussed for IEEE802.16 both for Physical Layer and MAC Layer. Since limited number of antennas on mobile communication hardware can not satisfy growing service demands and make use of communication networks more efficiently network coding techniques, particularly relaying become an emerging solution.

1.0.1 Related Works

First idea about relaying was introduced by Van Der Meulen [1]. Van der Meulen derived upper and lower bounds on the capacity of relay channel. The capacity of relay channel is still unknown, but Cover and El-Gamal improved the bounds significiantly by developing two fundamental coding techniques called , Decode and Forward (DF) and Compress and Forward (CF) and also combination of these two in [2, Theorem 7]. Capacity theorems for both degraded and reversely degraded relay channels and relay channel with feedback are also included in [2]. In [3], Kramer et al. generalized DF and CF protocols suggested in [2]. Although the capacity of relay channel is unknown, it can be shown that, the DF and CF protocols can be capacity achieving under certain relay position(s). In addition to these coding schemes, other relaying strategies such as, Amplify and Forward (AF), Partial Decoding (PD), Linear Relaying (LR) is proposed in literature. There are numerous works on these coding strategies under different channel topologies in literature. No one can say one of the strategies always outperforms, but each of the strategies has own potential strengths and weaknesses under different network topologies and various channel conditions.

In [4], a fundamental three node-network consisting of a source, a relay and a destination, is analyzed. The relay is assumed to operate in half duplex mode and time sharing is employed for simplicity in calculations. Under a practical scenario of Gaussian Vector Channels [4] (similar to uplink channel of IEEE802.16), CF strategy always outperforms direct-link and AF strategy, since time sharing parameter can be optimized and particularly, while AF protocol amplifies the message it also amplifies noise component. On the other hand, CF strategy has approximately same performance with DF strategy. Three node fundamental wireless relay network in Rayleigh fading environment is considered in [5]. Under this model, an upper bound, achievable rate region for DF and CF are evaluated. Furthermore, bounds on outage and ergodic capacity are studied in certain conditions. It is also found, relaying outperforms with compared direct transmission (without relay), in terms of ergodic and outage capacity. In [6], bi-directional relay channel model where, two nodes exchange independent messages with help of a relay is studied. Relay uses one of four transmission strategies, such as, AF, DF, CF and a mixed strategy where relay does CF in one way and DF in another way. Achievable rate regions for bi-directional relay channel under time division broadcast protocol (TDBC) and multiple access broadcast protocol (MABC) are calculated and results extended to the Gaussian Case. With numerical results it can be shown that, while MABC outperforms in Low SNR regime, at high SNR regime TDBC gave better performance. Gaussian Relay Channels in Half-Duplex assumption is studied in [7]. In [7], noises at relay and destination are assumed to be arbitrarily correlated, which is a reasonable model for sensor networks. Achievable rates are evaluated for DF, AF and CF strategy for various channel settings and comprehensive numerical results are presented to highlight while DF disregards noise correlation CF and AF could exploit extra information. In [8], separated two way relay channel in which users do not receive each other's signal is studied. Achievable rate regions are characterized for DF, a combination of partial DF and CF strategy with single layer and two layer quantization (one of the user receives a better description of relay's received signal). Extension of these achievable schemes to Gaussian Case is also presented. Main result of [8] shows two layered quantization could enhance one of the user's rate region with using combination of partial DF and CF transmission strategy. Several protocols are analyzed for Gaussian Half Duplex Multiple Relay networks in [9]. The authors calculated an upper bound and present achievable rate regions forDF, CF, a multilevel partial DF strategy and multihopping. There also a CF strategy and a mixed strategy for two relays in which relay nodes either do DF or CF with regular encoding, are proposed. In addition, numerical calculations are done for both random and fixed transmission strategies. Among all, mixed strategy seems to have rate improvement under certain parameters and conditions. Delay limited capacity of half duplex relay channel is investigated for various cooperative protocols in [10]. Non-orthogonal Amplify and Forward (NAF), CF, a simpler version Estimate and Forward (EF), and Hybrid Relaying schemes was chosen and performance analysis has been done under fundamental three-node network model. Under long term average power constraint, Hybrid Relaying which uses either CF or DF gave best results in terms of delay limited capacity. In other comparison without optimal time allocation, CF strategy was found to always outperform than EF and NAF. In [11], a novel encoding strategy is proposed for fundamental three node network and corresponding achievable rate region for user cooperation channel is calculated. The results are also extended to the Gaussian Case. The fundamental ideas of Khojastepour et al. are decoding each codeword completely and discarding residuals after decoding limits relay channel capacity to the capacity between source and the relay. Results show that, an improvement is obtained with respect to the results of Cover and El-Gamal. Compress and Forward and other three coding strategies for fundamental three node network are investigated in [12]. Achievable rate regions are presented for symmetric and asymmetric three node network topologies. Mainly, effect of node position and data correlation is studied. It is shown that, while Compress and Forward Strategy gave best performance in asymmetric geometrical topologies, in symmetric topologies other coding strategies gave better performance under certain conditions. Xie [13] suggest an improvement on [2, Theorem 7], by stating decoding of compressed version of message is not necessary at decoder, although joint decoding is easier than successive decoding under fundamental three node network. Decoding strategies are investigated in [14]. The authors proposes two novel decoding strategies under fundamental three node network, called "Sequential Backward Decoding" (SeqBack) and "Simultaneous Backward Decoding" (SimBack).It is found that, achievable rate region for SimBack Decoding Strategy includes Cover and El Gamal rate region and generalizes a lower bound, and also under the assumption of zero-mean, jointly Gaussian distribution these strategies outperform the Cover and El Gamal's generalized strategy. In [15], authors found the equivalence of rates achieved by SeqBack decoding and SimBack decoding strategies. It is shown that, jointly decoding of all parameters simultaneously does not improve the rate. In [16], an achievable rate region for a two-user cooperative multiple access channel, where one user performs DF and the other user performs Wyner-Ziv-type CF is derived. It is shown that, hybrid cooperation scheme extends the rate region with respect to rate achieved by the pure DF cooperation where the channel link between a user and the destination is poor. Although there are extensive works in literature, there remain open problems related to cooperative relay networks in general. Particularly, there is no practical channel code design for CF strategy, which precludes the implementation of a communication protocol. On the other hand, CF strategy requires channel state information at relay, which makes relay position more significant and in some cases makes using CF strategy unfeasible. To understand cooperative networks in deep and elaboration of suggested solutions will help us to understand the performance limits of future communication networks.

In this thesis, we will consider a mutual cooperation scheme for a two user cooperative network, for which we will characterize an achievable rate region using CF relaying protocol in Chapter 3. We will also consider a mutual cooperation scheme based on [2, Theorem 7] for the same system model. The results obtained in Chapter 3 and Chapter 4 compared with the benchmark system provided [17] which uses pure DF system for same network structure and also a hybrid cooperation scheme provided by [16]. In addition, we will analyze and discuss the performance of CF relaying protocol considering different channel conditions. In the results section we will highlight the strengths and weaknesses of the protocol supported by the results in chapter 3 and Chapter 4.

Chapter 2

Background Theory

In telecommunications, an information bearing signal is transferred between nodes via a communication channel. A communication channel is a physical transmission medium such as, copper wire, an underwater channel, an optical wire, free space etc...However, in information theory, a channel refers to a theoretical model which has certain parameters and error characteristics. In a wireless channel, a transmitted signal encounters some detrimental effects such as, noise, attenuation, distortion and interference.

Path loss is one of the adverse effects in wireless environment. It models the loss in transmitted power at receiver. Path loss depends on many factors related whole communication setup between transmitter and receiver. Generally, path loss measured in dB scale and corresponds to a nonnegative number. Since the channel does not contain any active nodes, path gain is defined as the negative of path loss which corresponds a negative number. Shadow fading is another detrimental effect presented by channel, which is caused by obstacles in physical environment. Considering one transmitter and two receivers at same distance, but in different locations received signal power at both transmitters are not the same. Since we cannot know the exact locations of the obstacles, shadow fading modeled as a random variable and added to path loss component. It has been found experimentally, when shadow fading measured in dB it follows zero-mean Gaussian distribution with standard deviation σ also measured in dB. The shadow fading measured in dB follows log-normal distribution and for this reason, shadow fading is also called "log-normal fading".

Since shadow fading and path loss models the long distance effects of wireless environment, we also need to define small distance effects of the channel. In wireless medium, when a single transmitted signal arrived to a receiver, probably have multiple copies caused by reflective, refractive and absorbing properties of physical channel. This type of channel is called multipath channel as shown in figure 2.1, since transmitted signal have multiple copies come from different paths.

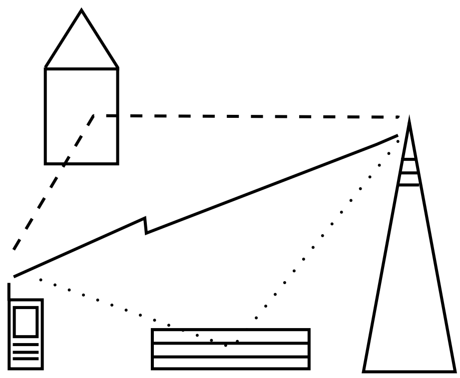

Figure 2.1: Multipath Channel

In addition, the presence of motion at transmitter,receiver or obstacle objects, the transmitted signal copies may received in different time instants each having a different amplitude and phase.

The channel delay spread refers to the time difference between first arrived signal and last arrived signal to the receiver. It is important to note that, symbol duration should be chosen smaller than delay spread to mitigate inter symbol interference in multipath channels.

The channel coherence bandwidth defined as the frequency spectrum in which channel response have same amplitude and linear phase change for all frequencies.If the transmitted signal's bandwidth exceeds the coherence bandwidth, the signal sees different attenuation levels. This type of channel considered as a frequency selective channel, and also called a broadband channel. On the other hand, if the transmitted signal's bandwidth less than the coherence bandwidth, channel can be considered as flat fading channel and also called narrowband channel.

The motion of the transmitter, the receiver, or the obstacles changes channel transfer characteristics with introducing frequency shifts called Doppler shift. The channel transfer characteristics generally modeled as a random variable if the channel is time invariant, and as a random process with each realization a different random variable for time varying channels. Fourier transform of the correlation function between the channel coefficient realizations is known as doppler spectrum. In addition, doppler spread is the measure that doppler spectrum is nonzero over frequency. Since the channel coherence time refers to time period that makes correlation between two realizations of channel's response zero, we can think doppler spread can be considered as inverse of channel coherence time. It is important to note that, doppler spread gives us the rate of change of fading. If the doppler spread is smaller than the transmitted signal bandwidth, the channel introduces slow fading, and if the doppler spread is larger than the transmitted signal's bandwidth channel is said to be fast fading.

Likewise in an information theoretical channel model, these effects are considered in statistical sense.Information theoretical channel models are used to find the bounds on channel capacity; the coding mechanisms and the entire transmission schemes are different than the practical scenarios. In this chapter, we will give a detailed information on channel models including, Single user Gaussian Channel, Gaussian Broadcast Channel, Gaussian Multiple Access Channel, Relay Channel and Multiple Access Channel with Generalized Feedback. We will also mention coding strategies, Amplify and Forward, Decode and Forward and Compress and Forward for relay channel.

2.1 Channel Models

In order to understand cooperative communications, we need to clarify information theoretical channel models and its properties.

2.1.1 Single User Gaussian Channel

Gaussian channel is a very useful model to characterize many practical channels like, satellite and radio links. By the central limit theorem, summation of random effects converges to normal distribution, which proves its practicality. Gaussian channel is a continuous alphabet channel as shown in figure 2.2

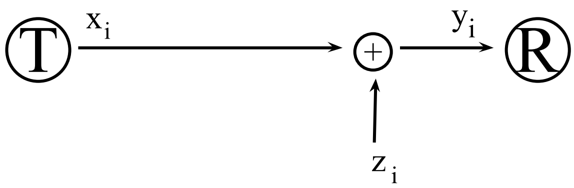

Figure 2.2: Single user Gaussian Channel

and can be modelled by 2.1,

$$Y_i = X_i + Z_i \qquad Z_i \sim \mathcal{N}(0, N) \tag{2.1}$$

where X_i is the channel input, Z_i is i.i.d Gaussian distributed noise component with variance N. Although Gaussian channel is a continuous alphabet channel, in practical cases Gaussian channel converted to a discrete channel which is easier to process with compared to a continuous channel. Information capacity of Gaussian channel with power constraint is given by,

$$C = \max_{E[X^2] \leqslant P} I(X;Y) \tag{2.2}$$

We can expand 2.2 as follows,

$$\begin{aligned} I(X;Y) &= h(Y) - h(Y \mid X) \\ &= h(Y) - h(X + Z \mid X) \\ &= h(Y) - h(Z \mid X) \\ &= h(Y) - h(Z) \end{aligned} \tag{2.3}$$

where,

$$\begin{aligned} E[Y^2] &= E[X+Z]^2 \\ &= E[X^2] + \underbrace{2E[X]E[Z]}_{from\, independence} + E[Z^2] \\ &= E[Z^2] \\ &= P + N \end{aligned} \tag{2.4}$$

$$h(Y) = \frac{1}{2}\log_2(2\pi e(P+N)) \tag{2.5}$$

and,

$$\begin{aligned} h(Z) &= E[Z^2] \\ &= N \end{aligned} \tag{2.6}$$

$$h(Z) = \frac{1}{2}\log_2(2\pi e N) \tag{2.7}$$

From the last line of 2.3 the capacity of single user gaussian channel can be found,

$$\begin{aligned} I(X;Y) &= h(Y) - h(Z) \\ &= \frac{1}{2}\log_2(1 + \frac{P}{N}) \end{aligned} \tag{2.8}$$

2.1.2 Gaussian Broadcast Channel

A broadcast channel consist of one transmitter and two or more receivers as shown in figure 2.3. To characterize Gaussian broadcast channel's capacity region we

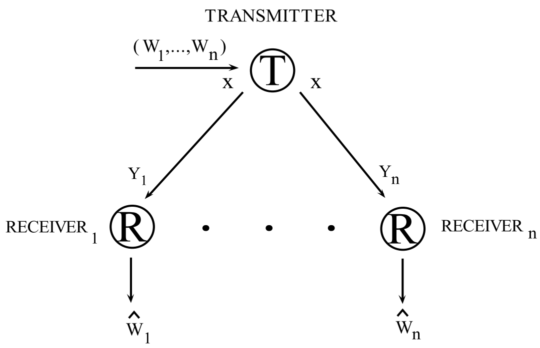

Figure 2.3: Gaussian Broadcast Channel

need to define degraded broadcast channels. A two user broadcast channel is physically degraded if,

$$p(y_1, y_2 \mid x) = p(y_1 \mid x)p(y_2 \mid y_1) \tag{2.9}$$

and the capacity region for sending independent information over the degraded broadcast channel $X \to Y_1 \to Y_2$ is the convex hull of the closure of all (R_1, R_2) satisfying,

$$\begin{aligned} R_1 &\leq I(U; Y_2) \\ R_2 &\leq I(X; Y_1 \mid U) \end{aligned} \tag{2.10}$$

for some joint distribution $p(u)p(x|u)p(y_1, y_2|x)$ where the auxiliary random variable U has cardinality bounded by $\mid U \mid \leq min\{\mid X \mid, \mid Y_1 \mid, \mid Y_2 \mid\}$ [Theorem 15.6.2,1]. The capacity region of a broadcast channel depends only on conditional marginal distributions $p(y_1 \mid x)\, and\, p(y_2 \mid x)$.Assuming a transmitter with power P and two receivers with noise variances N_1 and N_2 where $N_1 < N_2$ capacity region for Gaussian broadcast channel is given by,

$$\begin{aligned} R_1 &\leq \frac{1}{2}\log_2(1 + \frac{\alpha P}{N_1}) \\ R_2 &\leq \frac{1}{2}\log_2(1 + \frac{(1-\alpha)P}{\alpha P + N_2}) \end{aligned} \tag{2.11}$$

where α parameter chosen arbitrarily to trade off between the rates $R_1 and R_2$.Two codebooks are generated one with power αP at a rate R_1 and another codebook with $\alpha' P$ at a rate R_2. Transmitter chooses $X(w_1)$ from $w_1 \epsilon \{1, 2, \ldots, 2^{nR_1}\}$ and $X(w_2)$ from $w_2 \epsilon \{1, 2, \ldots, 2^{nR_2}\}$and sends the sum of two codewords X. Receiver 1 with high signal to noise ratio (SNR) starts to decode second receiver's codeword $\widehat{X_2}$ after decoding subtracts it from its received vector Y_1. Second receiver starts to decode its own message from second codebook with rate R_2. It is interesting to note that; receiver with high SNR always knows the other receiver's message. Corresponding proofs of the theorems are stated in Chapter 15.6 in [18].

2.1.3 Gaussian Multiple Access Channel

A multiple access channel consists of m transmitters communicating with a common receiver as shown in 2.4. Considering two transmitters and one receiver for

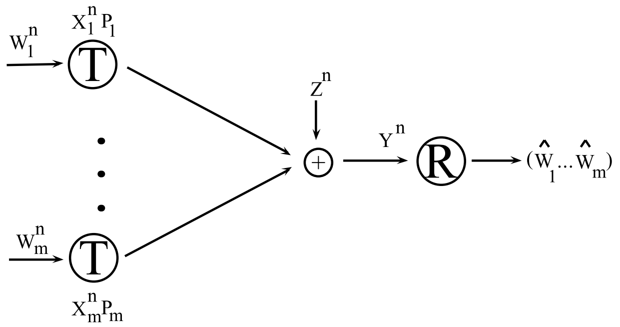

Figure 2.4: Gaussian Multiple Access Channel

Gaussian multiple access channel, the received signal at a time instant i can be given as,

$$Y_i = X_{1i} + X_{2i} + Z_i \tag{2.12}$$

where Z_i is i.i.d. zero mean Gaussian random variable with variance N and both transmitters subject to the power constraint,

$$\begin{aligned} E[X_1^2] &\leq P_1 \\ E[X_2^2] &\leq P_2 \end{aligned} \tag{2.13}$$

the capacity region of two user Gaussian multiple access channel to be the convex hull of rate pairs satisfying,

$$
\begin{aligned}
R_1 &\leq I(X_1; Y \mid X_2) \\
R_2 &\leq I(X_2; Y \mid X_1) \\
R_1 + R_2 &\leq I(X_1, X_2; Y)
\end{aligned}
\tag{2.14}
$$

for some input distribution $f(x_1)f(x_2)$.By expanding 2.14 we can find,

$$
\begin{aligned}
I(X_1; Y|X_2) &= h(Y|X_1, X_2) \\
&= h(X_1 + X_2 + Z \mid X_2) - h(X_1 + X_2 + Z \mid X_1, X_2) \\
&= h(X_1 + Z \mid X_2) - h(Z \mid X_1, X_2) \\
&= h(X_1 + Z \mid X_2) - h(Z) \\
&= h(X_1 + Z) - h(Z) \\
&= h(X_1 + Z) - \frac{1}{2}\log_2(2\pi e N) \\
&\leq \frac{1}{2}\log_2(2\pi e P_1 + N) \\
&= \frac{1}{2}\log_2(1 + \frac{P_1}{N})
\end{aligned}
\tag{2.15}
$$

and similarly,

$$
R_2 = \frac{1}{2}\log_2(1 + \frac{P_2}{N}) \tag{2.16}
$$

then from 2.14 sum rate can be calculated as,

$$\begin{aligned}
R_1 + R_2 &\leq I(X_1, X_2; Y) \\
&= h(Y) - h(Y|X_1, X_2) \\
&= h(X_1 + X_2 + Z) - h(X_1 + X_2 + Z|X_1, X_2) \\
&= h(X_1 + X_2 + Z) - h(Z|X_1, X_2) \\
&= h(X_1 + X_2 + Z) - h(Z) \\
&\leq \frac{1}{2} log_2((2\pi e)(P_1 + P_2 + N)) - \frac{1}{2} log_2(2\pi e N) \\
R_1 + R_2 &= \frac{1}{2} log_2(1 + \frac{P_1 + P_2}{N})
\end{aligned} \tag{2.17}$$

Two codebooks are generated one having 2^{nR_1} codewords of power P_1, another having codewords 2^{nR_2} of power P_2. Each transmitter chooses an arbitrary codeword from its own codebook and transmits simultaneously. Receiver starts to decode from second user's information considering first user's information as noise component so,

$$R_2 \leq \frac{1}{2} \log_2(1 + \frac{P_2}{P_1 + N}) \tag{2.18}$$

After decoding second user's codeword successfully, it can be subtracted out and first user's codeword can be decoded at a rate of,

$$R_1 \leq \frac{1}{2} \log_2(1 + \frac{P_1}{N}) \tag{2.19}$$

It is interesting to note that, if we generalize for m users with equal power P, the sum rate will be,

$$R_{SUM} \leq \frac{1}{2} \log_2(1 + \frac{mP}{N}) \tag{2.20}$$

as $m \to \infty$ the sum rate also goes infinity. But individual average rate will be,

$$R_{IND} \leq \frac{1}{m} \log_2(1 + \frac{mP}{N}) \tag{2.21}$$

2.1.4 Relay Channel

A relay channel consists of either one or more intermediate nodes, helping to transmit source's information to an intended receiver. The capacity of relay channel is still unknown ,but it is derived for particular cases e.g., for physically degraded channels [2], and asymptotic capacity for Gaussian relay channel in [19]. In this section, we will consider the simplest relay case consisting a transmitter,a relay and a receiver as shown in figure 2.5.

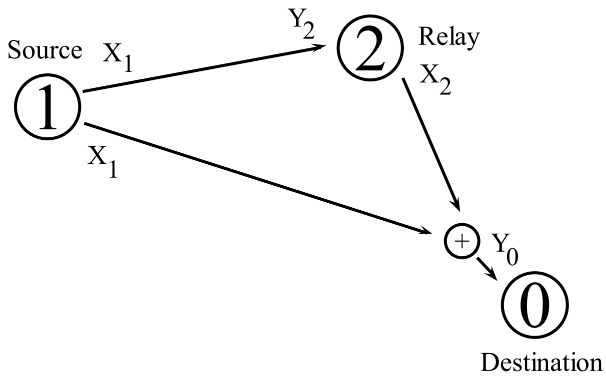

Figure 2.5: Relay Channel

The relay channel in figure 2.5 consist of four finite sets $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{Y}_0$, $\mathcal{Y}_2$ and a probability distribution $p(y_0, y_2|x_1, x_2)$.

An (M, n) code for the relay channel consist of a set of integers $\mathcal{M} = \{1, 2, \ldots, M\}$ and encoding function $X : \mathcal{M} \to \mathcal{X}^n$ is allowed to depend only on past observations such that,

$$X_{2i} = f_i(Y_{21}, Y_{22}, \ldots, Y_{2i-1}) \tag{2.22}$$

and a decoding function,

$$G : \mathcal{Y}^n \to \mathcal{M} \tag{2.23}$$

Hence, for any choice of $p(w)$,$w \epsilon \mathcal{M}$, any code choice $x : \mathcal{M} \to \mathcal{X}^n$ and relay functions $\{f_i\}_{i=1}^n$, the joint probability distribution on $\mathcal{M} \times \mathcal{X}_1^n \times \mathcal{X}_2^n \times \mathcal{Y}_2^n \times \mathcal{Y}_0^n$ is given by,

$$p(w, x_1^n, x_2^n, y_2^n, y_0^n) = p(w) \prod_{i=1}^{n} p(x_{1i}|w) p(x_{2i}|y_{21}, y_{22}, \ldots, y_{2i-1}).p(y_{2i}, y_{0i}|x_{1i}, x_{2i}) \tag{2.24}$$

By applying max flow min cut theorem, capacity upper bound can be found as [2],

$$C = \sup_{p(x_1,x_2)} \min\{I(X_1, X_2; Y_0), I(X_1; Y_0, Y_2|X_2)\} \tag{2.25}$$

The first term in 2.25 bounds the rates of source and relay to destination node which corresponds to multiple access channel, if the relay knows the complete message $I(X_1, X_2; Y_0)$ can be achievable. The second term bounds the rate from source to destination and source to relay channel which corresponds to a broadcast channel. And the rate $I(X_1; Y_0, Y_2|X_2)$ can be achieved if source node knows X_2 and the receiver node knows Y_2.

The degraded relay channel particularly is in interest, because in degraded relay channel relay received signal y_2 is better than ultimate receiver input y_0 which relay can contribute new information to the receiver. The degradedness is similar to broadcast channel as in 2.9 which one receiver is degraded version of other receiver.

The capacity of degraded relay channel can be characterized as,

$$C \leq \sup_{p(x_1,x_2)} \min\{I(X_1, X_2; Y_0), I(X_1; Y_2|X_2)\} \tag{2.26}$$

The corresponding proofs can be found in [2], with using superposition coding, Slepian-Wolf partitioning and coding for cooperative MAC channel.

Reversely degraded channel, is another form of degraded channel which relay received signal y_2 is worse than ultimate receiver's signal y_0 in such scenario, the relay node can not cooperate to send x_1, and just facilitates the transmission of the best x_2.

The capacity of reversely degraded channel can be characterized as,

$$C = \max_{x_2 \epsilon \mathcal{X}_2} \max_{p(x_2)} \{I(X_2; Y_0|x_2)\} \tag{2.27}$$

In next section, we will consider the Gaussian relay channel and discuss encoding and decoding mechanisms used in Gaussian relay channel.

2.1.4.1 Gaussian Relay Channel

Gaussian relay channel consists of three terminals a transmitter, a relay and a receiver illustrated in figure 2.6. Transmitter wishes to send information to receiver aided by the relay, which doesn't have its own information. Gaussian

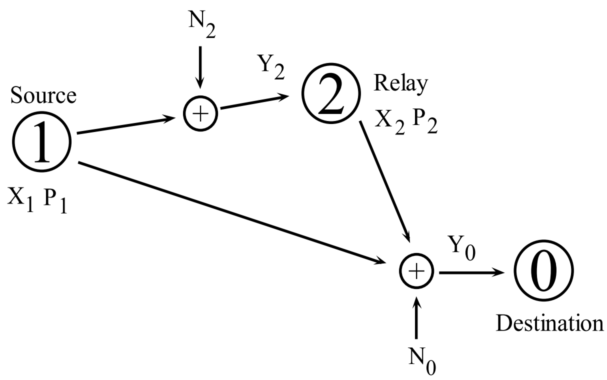

Figure 2.6: Gaussian Relay Channel

relay channel is modelled by,

$$\begin{aligned} Y_2 &= X_1 + Z_2 \\ Y_0 &= X_1 + Z_2 + X_2 + Z_0 \end{aligned} \tag{2.28}$$

where Z_2 and Z_0 are Gaussian noise with noise variances N_2 and N_0. The capacity of Gaussian Relay Channel [3] is given by,

$$C = \max_{0 \leq \alpha \leq 1} \min \left\{ C\left(\frac{P_1 + P_2 + 2\sqrt{\alpha' P_1 P_2}}{N_2 + N_0} \right), C\left(\frac{\alpha P_1}{N_2} \right) \right\} \tag{2.29}$$

where $\alpha' = 1 - \alpha$.In order to understand the improvement gained by relay channel, let's consider the case where, $\alpha = 1$ and for a large N_0 and if we set,

$$\left(\frac{P_2}{N_0}\right) \geq \left(\frac{P_1}{N_2}\right) \tag{2.30}$$

where relay's SNR at destination is chosen greater than transmitter's SNR at relay node.Since the relay to destination link's SNR is better than transmitter to relay's SNR in this scenario, the bottleneck in this system becomes source to relay channel, and the capacity,

$$C = \frac{1}{2} log_2 \left(\alpha \frac{P_1}{N_2}\right) \tag{2.31}$$

can be achieved from the transmitter to relay. In addition, the rate increased from $1/2\log_2(1 + (P_1/N_1 + N_2))$ to $1/2\log_2(1 + P_1/N_2)$by the presence of relay.We will now describe the coding mechanism to achieve the capacity expression in 2.29. We will generate two codebooks, the first one with 2^{nR_1} words of power αP_1 and the second with 2^{nR_0} codewords of power $\alpha' P_1$ where $R_1 < 1/2\log_2(1 + \alpha P_1/N_2)$. We will also assume 2.30 holds. In the first block, transmitter sends a codeword from the first codebook. Relay can decode reliably, because $R_1 < 1/2\log_2(1+\alpha P_1/N_2)$. The intended receiver cannot decode reliably since the capacity between transmitter and relay is greater than the capacity between source and destination, but receiver constructs a list of all possible codewords of size $2^{n(R_1 - 1/2log_2(\alpha P_1/N_2 + N_0))}$ instead of decoding. In the next block, first codebook is partitioned randomly into 2^{nR_0} cells to with an equal number of codewords in each cell. The relay and the transmitter find the cell of the partition in which the codeword from the first codebook lies, and cooperatively send the codeword from second codebook with that index. On the other hand, relay scales this codeword to satisfy its power constraint P_2. At receiver side, since the transmission of cooperative information takes place coherently,

$$\begin{aligned} TotalPoweratReceiver &= (\sqrt{\alpha' P_1} + \sqrt{P_2})^2 \\ CombiningGain &= 2\sqrt{\alpha' P_1 P_2} \end{aligned} \tag{2.32}$$

The transmitter finally transmits fresh information from first codebook by adding it on cooperative signal from second codebook in the second block. Decoding proceeds as follows. First, receiver finds the cooperative index from second codebook by looking the closest codeword in the second codebook. After the codeword subtracted from received signal, receiver calculates a list of indices corresponding the codewords from first codebook that may have been sent in the second block with size 2^{nR_0}. At last, the receiver compares the list of all possible codewords that may have been sent in first block with the cell of partition observed from cooperative signal in the second block. In the intersection there would be only one codeword with high probability.

However, the relay node processing capability is an important constraint in relay channel. The relay node can operate either in half-duplex or full-duplex mode. While transmission and reception takes place simultaneously in same frequency band in full duplex mode, in half duplex mode relay transmits and receives in orthogonal channels. Although full-duplex mode seems unfeasible in practical cases, it contributes to understand characteristics of relay channel. In order to study practical cases, half-duplex assumption is required.

2.1.5 Multiple Access Channel With Generalized Feedback

The fundamental three node network consisting one relay is the simplest example of cooperative network. In such network, relay node doesn't have its own information to send the intended receiver.The idea presented in [20] , the users in network aims to send information to an ultimate receiver with utilizing feedbacks received from channel. The general capacity of MAC-GF is still unknown, but in literature achievable rate regions are derived for specific cases.

Carleial in [21] and Willems in [22] derived achievable rate regions using superposition and block Markov coding. Using the results obtained in [21] and [22] Zeng and Kuhlmann compared the achivable rate regions and showed Willem's achievable rate region is larger than Carleial's for some particular cases in [23]. Another achievable rate region developed by Gastpar in [24] which is the extension of linear feedback strategy proposed by Ozarow [25]. Furthermore, Gastpar and Kramer [26] derived the outer bounds for a special case of MAC-GF which noisy feedback is the degraded version of channel output.

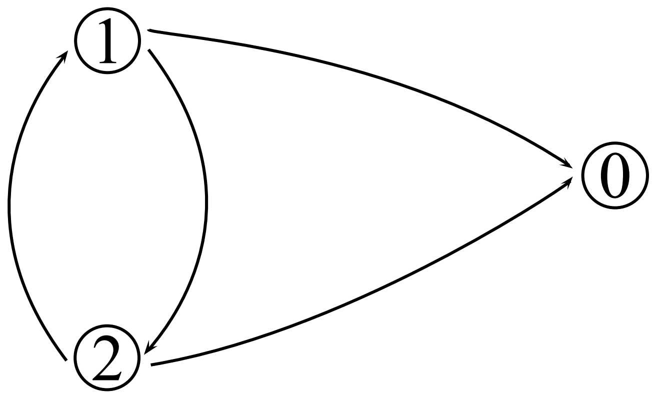

Figure 2.7: User Cooperation Channel

An achievable rate region for a two-user cooperative multiple access channel, where one user performs DF and the other user performs Wyner-Ziv-type CF is derived in [16]. It is shown that, hybrid cooperation scheme extends the rate region with respect to rate achieved by the pure DF cooperation where the channel link between a user and the destination is poor.

2.2 Cooperation Protocols

In cooperative wireless communication, a source transmits a message to a destination with the assistance of a relay. The relay receives to the signal from source and may retransmit the signal using one of the fundamental cooperation protocols; amplify and forward(AF), decode and forward(DF), and compress and forward(CF) to the destination.There is also another idea called *facilitation*, is mostly of theoretical interest.By combining the source and relay transmissions, and depending on the relaying protocol used, the destination can achieve diversity against fading without the use of an antenna array at any terminal. Increasing diversity makes the communication channel more robust against the fading[17][27]. Diversity gain is defined as the rate of decrease in probability of error with increasing SNR. It is shown that, using proposed cooperative protocols in [28] higher diversity gains can be achieved. However, the another term multiplexing gain describes, how the actual communication rate increases with increasing SNR. Of course, there is a trade-off between diversity and multiplexing ,because with multiplexing gain throughput can be maximized and similarly with diversity gain channel can be considered more robust against the fading.In MIMO systems this is n important performance criterion ,and the user cooperation can be considered as substitution to MIMO, diversity multiplexing trade-off becomes an important problem in cooperative networks.

There is no single cooperation protocol works well for the general relay channel. Source and relay nodes share their resources in order to achieve highest throughput via using these well-known protocols. In this section we will explain amplify-and-forward, decode-and-forward and compress-and-forward and we exclude the most theoretical interest *facilitation*, and also we will consider three node fundamental network topology as shown in 2.8 in order to explain cooperative protocols above.

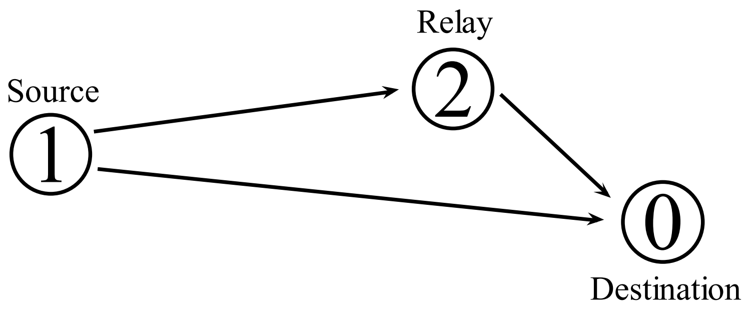

Figure 2.8: Fundamental Relay Channel

2.2.1 Amplify And Forward

Amplify and forward one of the fundamental transmission techniques used in cooperative communications. In this scheme, relay node simply amplifies the received signal and send it to the destination node. However, amplifying the received signal at relay may also cause the amplification of noise component which is the main problem of this scheme. Although, the noise component is also amplified the destination receives two independently faded version of transmitted signal which increases diversity.

2.2.2 Decode And Forward

The decode and forward coding technique first introduced by Cover and El-Gamal considering single relay case as illustrated in figure 2.8.

where source node intends to transmit information to the destination node by using the direct link between source and destination as well as aided by relay node. In DF protocol, relay node decodes and re-encodes its received signal and forwards it to the destination node. However, by decoding the received signal at relay, a hard decision is made by relay node, if the relay channel is not physically degraded, decoding at relay may limits the channel capacity. This strategy can achieve rates up to,

$$R_1 \leq \sup\{\min_{p(x_1,x_2)}(I(X_1X_2;Y_0), I(X_1;Y_2|X_2))\} \tag{2.33}$$

There have been several methods proposed to achieve R_1 in literature. Cover and El-Gamal used irregular block Markov superposition encoding and successive decoding in [2]. In [21], Carleial considered block Markov superposition encoding and backward decoding. Backward decoding strategies are investigated in detail in [14]. The authors proposes two novel decoding strategies under fundamental three node network, called "Sequential Backward Decoding" (SeqBack) and "Simultaneous Backward Decoding" (SimBack).It is found that, achievable rate region for SimBack Decoding Strategy includes Cover and El Gamal rate region and generalizes a lower bound, and also under the assumption of zero-mean, jointly Gaussian distribution these strategies outperform the Cover and El Gamal's generalized strategy. In [15], authors found the equivalence of rates achieved by SeqBack decoding and SimBack decoding strategies. It is shown that, jointly decoding of all parameters simultaneously does not improve the rate. If the relay channel is not physically degraded, partial decode and forward strategy may outperform than DF strategy. By using partial DF, relay decodes a part of the source message and remaining part is directly sent to destination without help of the relay.

2.2.3 Compress and Forward

In the DF protocol, since relay is forced to make a hard decision, it may limit the achievable rate region in cases where, erroneous blocks received at relay. In such cases, instead of decoding the received signal, compression based schemes are preferred. The relay compresses its received signal, maps into a channel codeword and forwards it to the destination. Depending on operating mode, compressed signal forward to destination either in same block or in next block. If relay operates in half duplex mode, message forwarded to the destination in next block, or operates in full-duplex mode; relay can listen and forward the compressed signal to the destination simultaneously.

After all B blocks transmitted, receiver starts to decode from relay's signal X_2 considering source signal as noise component. After successfully recovering $\hat{Y}_2$, receiver uses both $\hat{Y}_2$ and Y_0 to determine X_1 if,

$$R_{CF} < I(X_1; \hat{Y}_2 Y_0 | X_2) \tag{2.34}$$

subject to the constraint,

$$I(\hat{Y}_2; Y_2|X_2) \leq I(X_2; Y_0) \tag{2.35}$$

From 2.35, compression rate is chosen lower than the relay to receiver achievable rate, which guarantees the reliable decoding of $\hat{Y}_2$ at receiver. The performance of CF scheme can be improved by using Wyner-Ziv type compression presented in [1].An example shown in table 2.1 for two blocks of transmission using Wyner-Ziv type CF. Transmission of B blocks performed in $B+2$ blocks, in the last two blocks cooperation information sent to the intended receiver. X_1 is generated using Block Markov encoding. Receiver first decodes $x_2(\hat{y}_{2,3})$, and $x_2(\hat{y}_{2,2})$ and recovers $\hat{y}_{2,3}$.This can be done reliably if,

$$\hat{R} \leq I(X_2; Y_0) + I(\hat{Y}_2; Y_0|X_2) \tag{2.36}$$

Table 2.1: Example transmission scheme for two blocks Wyner-Ziv Type CF Cooperation

Block 1	**Block 2**	**Block 3**	**Block 4**
$x_1(w_{1,1}\|1)$	$x_1(w_{1,2}\|w_{1,1})$	$x_1(1\|w_{1,2})$	$x_1(1\|1)$
$x_2(1)$	$x_2(\hat{y}_{2,1})$	$x_2(\hat{y}_{2,2})$	$x_2(\hat{y}_{2,3})$

Employing received signal from previous block at intended receiver as the side information instead of noise component allow us to reduce the compression rate from $I(\hat{Y}_2; Y_2|X_2)$ to $I(\hat{Y}_2; Y_2|X_2Y_0)$and compression rate can be chosen,

$$\hat{R} = I(\hat{Y}_2; Y_2|X_2Y_0) \tag{2.37}$$

and achievable rate remains same in equation2.34.

However, CF strategy requires channel state information at relay, which makes relay position more significant and in some cases makes using CF strategy unfeasible.

Chapter 3

Achievable Rate with Mutual Compress and Forward

In this chapter, we propose a bidirectional cooperation model based on compress and forward strategy, for a Gaussian multiple access channel. In section 3.1, we introduce the system model. In section 3.2, we give the details of the codebook generation, encoding and decoding steps of our compress and forward scheme for an arbitrary MAC. In section 3.3, we apply our encoding strategy for the Gaussian MAC, introduced in section 3.1, and derive the resulting achievable rate regions.

3.1 System Model:

We consider a two user MAC where the users can hear each other as shown in figure 3.1. The received signals are given by,

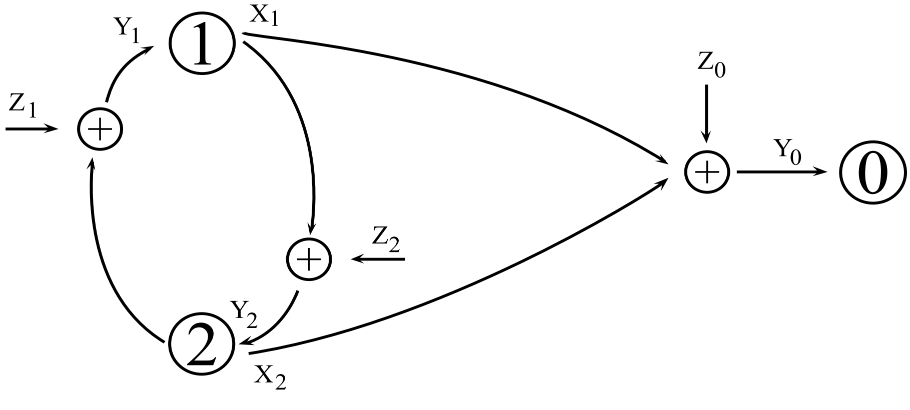

Figure 3.1: Two User Cooperation Channel

$$Y_1 = X_1 + X_2 + Z_1 \tag{3.1}$$

$$Y_2 = X_1 + X_2 + Z_2 \tag{3.2}$$

$$Y_0 = X_1 + X_2 + Z_0 \tag{3.3}$$

Noise components are chosen i.i.d Gaussian with variances N_0, N_1, N_2,

$$Z_0 \sim \mathcal{N}(0, N_0) \tag{3.4}$$

$$Z_1 \sim \mathcal{N}(0, N_1) \tag{3.5}$$

$$Z_2 \sim \mathcal{N}(0, N_2) \tag{3.6}$$

$$\tag{3.7}$$

3.2 Codebook Generation:

A block Markov encoding similar to the scheme in [2] is used. Outline of the encoding and decoding procedure is given below:

Random Coding:

1. Choose 2^{nR_1} and 2^{nR_2} i.i.d x_{10} and x_{20} with probability $p(x_{10}) = \prod_{i=1}^{n} p(x_{10i})$ and $p(x_{20}) = \prod_{i=1}^{n} p(x_{20i})$. Label these $x_{10}(w_1)$ and $x_{20}(w_2)$ respectively.

2. Choose $2^{nR_{12}}$ and $2^{nR_{21}}$ i.i.d x_{12} and x_{21} with probability $p(x_{12}) = \prod_{i=1}^{n} p(x_{12i})$ and $p(x_{21}) = \prod_{i=1}^{n} p(x_{21i})$. Label these $x_{12}(s_1)$ and $x_{21}(s_2)$,$s_1 \in [1, 2^{nR_{12}}]$ and $s_2 \epsilon [1, 2^{nR_{21}}]$ respectively.

3. Choose for each $x_{12}(s_1)$, $2^{n\hat{R}_1}$ i.i.d. $\hat{Y}_1$ each with probability $p(\hat{y}_1|x_{12}(s_1)) = \prod_{i=1}^{n} p(\hat{y_{1i}}|x_{12}(s_1))$ and, $x_{21}(s_2)$, $2^{n\hat{R}_2}$ i.i.d. $\hat{Y}_2$ each with probability $p(\hat{y}_2|x_{21}(s_2)) = \prod_{i=1}^{n} p(\hat{y_{2i}}|x_{21}(s_2))$ where for $x_{12} \epsilon \mathcal{X}_{12}$ and $\hat{y}_1 \in \hat{\mathcal{Y}}_1$, $x_{21} \in \mathcal{X}_{21}$ and $\hat{y}_2 \in \hat{\mathcal{Y}}_2$,
   we define
   $p(\hat{y}_1|x_{12}) = \sum_{x_{20},y_0,y_1} p(x_{20}) p(y_0, y_1|x_{20}, x_{12}) p(\hat{y}_1|y_1, x_{12})$
   $p(\hat{y}_2|x_{21}) = \sum_{x_{10},y_0,y_2} p(x_{10}) p(y_0, y_2|x_{10}, x_{21}) p(\hat{y}_2|y_2, x_{21})$

Label these $\hat{y}_1(z_1|s_1)$, $s_1 \in [1, 2^{nR_{12}}]$,$z_1 \in [1, 2^{n\hat{R}_1}]$ and $\hat{y}_2(z_2|s_2)$, $s_2 \in [1, 2^{nR_{21}}]$,$z_2 \in [1, 2^{n\hat{R}_2}]$.

4. Randomly partition the set $\{1, 2, \ldots, 2^{n\hat{R}_1}\}$ into $2^{nR_{12}}$ cells, $S_{s_1} \in [1, 2^{nR_{12}}]$ and $\{1, 2, \ldots, 2^{n\hat{R}_2}\}$ into $2^{nR_{21}}$ cells, $S_{s_2} \in [1, 2^{nR_{21}}]$.

Encoding:

Let w_{1i} and w_{2i} be the messages to be sent in block i and assume that the $(\hat{y}_1(z_{1i-1}|s_{1i-1}), y_1(i-1), x_{12}(s_{1i-1}))$ and $(\hat{y}_2(z_{2i-1}|s_{2i-1}), y_2(i-1), x_{21}(s_{2i-1}))$are jointly $\epsilon-$ typical and that, $z_{1i-1} \in S_{s_{1i}}$,$z_{2i-1} \in S_{s_{2i}}$. Then the codeword pairs $(x10, x_{21})$ and (x_{20}, x_{12}) will be transmitted in block i.

Remark:

X_{12} and X_{21} codewords corresponds to compressed observation of each user and X_{10} and X_{20} corresponds to each user's own information.

Decoding:

At the end of the block i, The receiver estimates both s_{1i} and s_{2i} by looking for the unique typical $x_{12}(s_1)$ and $x_{21}(s_2)$ with $y_0(i)$. If $R_{12} < I(X_{12}, \hat{Y}_1; Y_0|X_{21})$ and $R_{21} < I(X_{21}, \hat{Y}_2; Y_0|X_{12})$ and n is sufficiently large, then this decoding operation will incur with small probability of error. The receiver calculates a set $L_1(y(i-1))$ of z_1 and $L_2(y(i-1))$ of z_2 such that $z_1 \in L_1(y(i-1))$ and $z_2 \in L_2(y(i-1))$ if

$$(\hat{y}_1(z_1|\hat{s}_{1i-1}), x_{12}(\hat{s}_{1i-1}), y_0(i-1)) \tag{3.8}$$

and

$$(\hat{y}_2(z_2|\hat{s}_{2i-1}), x_{21}(\hat{s}_{2i-1}), y_0(i-1)) \tag{3.9}$$

are jointly $\epsilon-$ typical. The receiver then declares that z_{1i-1} and z_{2i-1} were sent if

$$z_{1i-1} \in S_{\hat{s}_{1i}} \cap L_1(y_0(i-1)) \tag{3.10}$$

$$z_{2i-1} \in S_{\hat{s}_{2i}} \cap L_2(y_0(i-1)) \tag{3.11}$$

$$\hat{R}_1 < I(\hat{Y}_1; Y_0|X_{12}) + R_{12} \tag{3.12}$$

$$\hat{R}_2 < I(\hat{Y}_2; Y_0|X_{21}) + R_{21} \tag{3.13}$$

Using both $(\hat{y}_2(\hat{z}_{2i-1}|\hat{s}_{2i-1}))$ and $y(i-1)$, the receiver finally declares that $\hat{w}_{1i-1}$ was sent in block $i-1$ if $(x_{10}(\hat{w}_{1i-1}), (\hat{y}_2(\hat{z}_{2i-1}|\hat{s}_{2i-1}), y(i-1), x_{21}(\hat{s}_{2i-1}))$ are jointly $\epsilon-$ typical. And for the second user, using both $(\hat{y}_1(\hat{z}_{1i-1}|\hat{s}_{1i-1})$ and $y(i-1)$, the receiver finally declares that $\hat{w}_{2i-1}$ was sent in block $i-1$ if $(x_{20}(\hat{w}_{2i-1}), (\hat{y}_1(\hat{z}_{1i-1}|\hat{s}_{1i-1}),$ $y(i-1), x_{12}(\hat{s}_{1i-1}))$ are jointly $\epsilon-$ typical. Thus $\hat{w}_{1i-1} = w_{1i-1}$ and $\hat{w}_{2i-1} = w_{2i-1}$ if

$$R_1 \leq I(X_1; Y_0, \hat{Y}_1, \hat{Y}_2|X_{12}, X_{21}, X_2) \tag{3.14}$$

$$R_2 \leq I(X_2; Y_0, \hat{Y}_2, \hat{Y}_1|X_{21}, X_{12}, X_1) \tag{3.15}$$

After receiving $y_1(i)$ first user decides that z_1 is received if $(\hat{y}_1(z_1|s_{1i}), y_1(i), x_{12}(s_{1i}))$ are jointly $\epsilon-$typical.And for the second user,after receiving $y_2(i)$ second user decides that z_2 is received if $(\hat{y}_2(z_2|s_{2i}), y_2(i), x_{21}(s_{2i}))$ are jointly $\epsilon-$typical. There will exist such a z_1 and z_2 with high probability if,

$$\hat{R}_1 > I(\hat{Y}_1; Y_1|X_{12}) \tag{3.16}$$

$$\hat{R}_2 > I(\hat{Y}_2; Y_2|X_{21}) \tag{3.17}$$

And let,

$$R_{12} = I(X_{12}; Y_0) - \epsilon \tag{3.18}$$

$$\hat{R}_1 = I(\hat{Y}_1; Y_1|X_{12}) + \epsilon \tag{3.19}$$

$$R_{21} = I(X_{21}; Y_0) - \epsilon \tag{3.20}$$

$$\hat{R}_2 = I(\hat{Y}_2; Y_2|X_{21}) + \epsilon \tag{3.21}$$

And these with (3.12) and (3.13) will yield,

$$I(\hat{Y}_1; Y_1|X_{12}, X_1) \leq I(X_{12}, \hat{Y}_1; Y_0|X_{21}) \tag{3.22}$$

$$I(\hat{Y}_2; Y_2|X_{21}, X_2) \leq I(X_{21}, \hat{Y}_2; Y_0|X_{12}) \tag{3.23}$$

$$I(\hat{Y}_1; Y_1|X_{12}, X_1) + I(\hat{Y}_2; Y_2|X_{21}, X_2) \leq I(X_{12}, X_{21}; Y_0) + I(\hat{Y}_1; Y_0|X_{12}, X_{21}) + I(\hat{Y}_2; Y_0|X_{12}, X_{21}) \tag{3.24}$$

the rates satisfying,

$$R_1 \leq I(X_1; Y_0, \hat{Y}_1, \hat{Y}_2|X_{12}, X_{21}, X_2) \tag{3.25}$$

$$R_2 \leq I(X_2; Y_0, \hat{Y}_1, \hat{Y}_2|X_{12}, X_{21}, X_1) \tag{3.26}$$

$$R_1 + R_2 \leq I(X_1, X_2; Y_0, \hat{Y}_1, \hat{Y}_2|X_{12}, X_{21}) \tag{3.27}$$

are achievable for any distribution of the from

$$p(x_{12})p(x_1|x_{12})p(\hat{y}_1|x_{12}, x_1, y_1)p(x_{21})p(x_2|x_{21})p(\hat{y}_2|x_{21}, x_2, y_2)p(y_0, y_1, y_2|x_1, x_2) \tag{3.28}$$

subject to the constraints (3.22) - (3.24).

3.3 Compress and Forward Cooperation for Gaussian MAC:

In [29] the effects of user cooperation on secrecy is investigated. The rate expressions in [29, Theorem 2], can be used in order to characterize an achievable rate region for proposed two-user cooperative network as shown in figure 3.1.

Our proposed achievable rate region depends on compress and forward relaying protocol mentioned in Chapter 2. We used a block Markov encoding in order to characterize achievable rate region expressions.The channel input for each users can be expressed as,

$$X_1 = X_{10} + X_{12} \tag{3.29}$$

$$X_2 = X_{20} + X_{21} \tag{3.30}$$

where, X_{10}, X_{20}, X_{12}, X_{21} are i.i.d. Gaussian distributed random variables with powers P_{10}, P_{20}, P_{12} and P_{21} respectively as,

$$X_{10} \sim \mathcal{N}(0, P_{10}) \tag{3.31}$$

$$X_{20} \sim \mathcal{N}(0, P_{20}) \tag{3.32}$$

$$X_{12} \sim \mathcal{N}(0, P_{12}) \tag{3.33}$$

$$X_{21} \sim \mathcal{N}(0, P_{21}) \tag{3.34}$$

where $P_1 = P_{10} + P_{12}$ and $P_2 = P_{20} + P_{21}$ and power is splitted among X_{10} and X_{12} and X_{20} and X_{21} respectively.

Each user divided its power in order to cooperate and also send its own information. The power is not allocated adaptively. The user 1 transmits its own information with power P_{10} and sends its compressed observation about user 2 with power P_{12}. Same power allocation applies for the second user.

Compression noise components are chosen zero-mean Gaussian with noise variances N_{c_1} , N_{c_2} respectively.

$$Z_{c_1} \sim \mathcal{N}(0, N_{c_1}) \tag{3.35}$$

$$Z_{c_2} \sim \mathcal{N}(0, N_{c_2}) \tag{3.36}$$

With the conditioning of X_1 to $\hat{Y}_1$ and X_2 to $\hat{Y}_2$ in (3.22) it is assumed that each user can cancel out its own information from the channel feedback. This is also discussed both in [29] *Remark* 3 and *Remark* 2 in [30].

and $\hat{Y}_1$ and $\hat{Y}_2$ can be expressed as,

$$\hat{Y}_1 = Y_1 - X_1 + Z_{c_1} \tag{3.37}$$

$$\hat{Y}_1 = X_2 + Z_1 + Z_{c_1} \tag{3.38}$$

$$\hat{Y}_2 = Y_2 - X_2 + Z_{c_2} \tag{3.39}$$

$$\hat{Y}_2 = X_1 + Z_2 + Z_{c_2} \tag{3.40}$$

At the end of any block i, X_{12} and X_{21} codewords are used in order to resolve the uncertainity of the receiver about the previously sent source messages X_1 and X_2.

The rate expressions can be calculated as follows,

$$\begin{aligned}
R_1 &\leq I(X_1; Y_0, \hat{Y}_1, \hat{Y}_2 | X_{12}, X_{21}, X_2) \\
&= h(Y_0, \hat{Y}_1, \hat{Y}_2 | X_{12}, X_{21}, X_2) - h(Y_0, \hat{Y}_1, \hat{Y}_2 | X_{12}, X_{21}, X_2, X_1) \\
&= h(X_{10} + Z_0, Z_1 + Z_{c_1}, X_{10} + Z_2 + Z_{c_2}) - h(Z_0, Z_1 + Z_{c_1}, Z_2 + Z_{c_2}) \\
&= \frac{1}{2} \log_2 \left\{ \frac{(2\pi e) \begin{vmatrix} P_{10}+N_0 & 0 & P_{10} \\ 0 & N_1+N_{c_1} & 0 \\ P_{10} & 0 & P_{10}+N_2+N_{c_2} \end{vmatrix}}{(2\pi e) N_0 (N_2 + N_{c_2})(N_1 + N_{c_1})} \right\} \\
&= \frac{1}{2} \log_2 \left(1 + P_{10} \frac{N_0 + N_2 + N_{c_2}}{N_0 (N_2 + N_{c_2})} \right)
\end{aligned} \tag{3.41}$$

$$
\begin{aligned}
R_2 &\leq I(X_2; Y_0, \hat{Y}_1, \hat{Y}_2 | X_{12}, X_{21}, X_1) \\
&= h(Y_0, \hat{Y}_1, \hat{Y}_2 | X_{12}, X_{21}, X_1) - h(Y_0, \hat{Y}_1, \hat{Y}_2 | X_{12}, X_{21}, X_1, X_2) \\
&= h(X_{20} + Z_0, X_{20} + Z_1 + Z_{c_1}, Z_2 + Z_{c_2}) - h(Z_0, Z_1 + Z_{c_1}, Z_2 + Z_{c_2}) \\
&= \frac{1}{2} \log_2 \left\{ \frac{(2\pi e) \begin{vmatrix} P_{20}+N_0 & P_{20} & 0 \\ P_{20} & P_{20}+N_1+N_{c_1} & 0 \\ 0 & 0 & N_2+N_{c_2} \end{vmatrix}}{(2\pi e) N_0 (N_2 + N_{c_2})(N_1 + N_{c_1})} \right\} \\
&= \frac{1}{2} \log_2 \left(1 + P_{20} \frac{N_0 + N_1 + N_{c_1}}{N_0 (N_1 + N_{c_1})} \right) \qquad (3.42)
\end{aligned}
$$

$$
\begin{aligned}
R_1 + R_2 &\leq I(X_1, X_2; Y_0, \hat{Y}_1, \hat{Y}_2 | X_{12}, X_{21}) \\
&= h(Y_0, \hat{Y}_1, \hat{Y}_2 | X_{12}, X_{21}) - h(Y_0, \hat{Y}_1, \hat{Y}_2 | X_{12}, X_{21}, X_1, X_2) \\
&= h(X_{10} + X_{20} + Z_0, X_{20} + Z_1 + Z_{c_1}, X_{10} + Z_2 + Z_{c_2}) \qquad (3.43) \\
&\quad - h(Z_0, Z_1 + Z_{c_1}, Z_2 + Z_{c_2}) \\
&= \frac{1}{2} \log_2 \left\{ \frac{(2\pi e) \begin{vmatrix} P_{10}+P_{20}+N_0 & P_{20} & P_{10} \\ P_{20} & P_{20}+N_1+N_{c_1} & 0 \\ P_{10} & 0 & P_{10}+N_2+N_{c_2} \end{vmatrix}}{(2\pi e) N_0 (N_2 + N_{c_2})(N_1 + N_{c_1})} \right\} \\
&= \frac{1}{2} \log_2 \Bigg(1 + P_{10} \frac{N_0 + N_2 + N_{c_2}}{N_0 (N_2 + N_{c_2})} + P_{20} \frac{N_0 + N_1 + N_{c_1}}{N_0 (N_1 + N_{c_1})} \\
&\qquad + P_{10} P_{20} \frac{N_0 + N_1 + N_{c_1} + N_2 + N_{c_2}}{N_0 (N_1 + N_{c_1})(N_2 + N_{c_2})} \Bigg) \qquad (3.44)
\end{aligned}
$$

subject to the constraints,

$$
\begin{aligned}
I(\hat{Y}_1; Y_1 | X_{12}, X_1) &\leq I(X_{12}, \hat{Y}_1; Y_0 | X_{21}) \qquad (3.45) \\
I(\hat{Y}_2; Y_2 | X_{21}, X_2) &\leq I(X_{21}, \hat{Y}_2; Y_0 | X_{12}) \qquad (3.46) \\
I(\hat{Y}_1; Y_1 | X_{12}, X_1) + I(\hat{Y}_2; Y_2 | X_{21}, X_2) &\leq I(X_{12}, X_{21}; Y_0) + \qquad (3.47) \\
I(\hat{Y}_1; Y_0 | X_{12}, X_{21}) + I(\hat{Y}_2; Y_0 | X_{12}, X_{21}) &
\end{aligned}
$$

Chapter 4

Achievable Rates with Mutual Compress/Decode and Forward

In this chapter, we propose a bidirectional cooperation model based on joint decode/compress forward strategy for a Gaussian multiple access channel. In section 4.1, we introduce the system model. In section 4.2, we give the details of the codebook generation, encoding and decoding steps of our partial decode compress forward scheme for an arbitrary MAC. In section 4.3, we apply our encoding strategy for the Gaussian MAC, introduced in section 4.1, and derive the resulting achievable rates. In section 4.4, we provide simulation results, comparing our achievable rate region to those of known cooperation strategies.

4.1 System Model:

We consider a two user MAC where the users can hear each other as shown in figure 4.1. The received signals are given by,

$$Y_1 = X_1 + X_2 + Z_1 \tag{4.1}$$

$$Y_2 = X_1 + X_2 + Z_2 \tag{4.2}$$

$$Y_0 = X_1 + X_2 + Z_0 \tag{4.3}$$

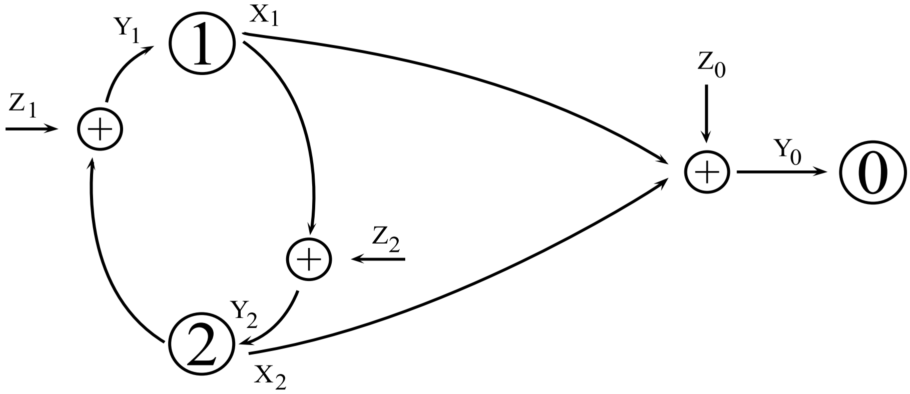

Figure 4.1: Two User Cooperation Channel

Noise components are chosen i.i.d Gaussian with variances σ_0^2, σ_1^2, σ_2^2,

$$Z_0 \sim \mathcal{N}(0, \sigma_0^2) \tag{4.4}$$

$$Z_1 \sim \mathcal{N}(0, \sigma_1^2) \tag{4.5}$$

$$Z_2 \sim \mathcal{N}(0, \sigma_2^2) \tag{4.6}$$

$$\tag{4.7}$$

4.2 Codebook Generation:

A regular block Markov superposition encoding and backward decoding employed. Outline of the encoding and decoding procedure is given below:

- Generate $2^{n(R_{1d}+R_{2d})}$ sequences $\underline{u}^n$. Label these $\underline{u}^n(w'_{1d}, w'_{2d})$, for each $w'_{1d} \in \{1, \ldots, 2^{nR_{1d}}\}$, $w'_{2d} \in \{1, \ldots, 2^{nR_{2d}}\}$
- For each $\underline{u}^n$, generate $2^{nR_{1d}}$ sequences $\underline{x}_{1d}^n$. Label these $\underline{x}_{1d}^n \{w_{1d}, \underline{u}^n(w'_{1d}, w'_{2d})\}$ for each $w_{1d} \in \{1, \ldots, 2^{nR_{1d}}\}$.
- For each $\underline{u}^n$, generate $2^{nR_{2d}}$ sequences $\underline{x}_{2d}^n$. Label these $\underline{x}_{2d}^n \{w_{2d}, \underline{u}^n(w'_{1d}, w'_{2d})\}$ for each $w_{2d} \in \{1, \ldots, 2^{nR_{2d}}\}$.
- For each $\underline{u}^n$, generate $2^{nR_{10}}$ sequences $\underline{x}_{10}^n$. Label these $\underline{x}_{10}^n \{w_{10}, \underline{u}^n(w'_{1d}, w'_{2d})\}$ for each $w_{10} \in \{1, \ldots, 2^{nR_{10}}\}$.

- For each $\underline{u}^n$, generate $2^{nR_{20}}$ sequences $\underline{x}_{20}^n$. Label these $\underline{x}_{20}^n\left\{w_{20}, \underline{u}^n(w'_{1d}, w'_{2d})\right\}$ for each $w_{20} \in \left\{1, \ldots, 2^{nR_{20}}\right\}$.

- For each $\underline{u}^n$, generate $2^{nR_{12}}$ sequences $\underline{x}_{12}^n$. Label these $\underline{x}_{12}^n\left\{z'_{12}, \underline{u}^n(w'_{1d}, w'_{2d})\right\}$ for each $z'_{12} \in \left\{1, \ldots, 2^{nR_{12}}\right\}$.

- For each $\underline{u}^n$, generate $2^{nR_{21}}$ sequences $\underline{x}_{21}^n$. Label these $\underline{x}_{21}^n\left\{z'_{21}, \underline{u}^n(w'_{1d}, w'_{2d})\right\}$ for each $z'_{21} \in \left\{1, \ldots, 2^{nR_{21}}\right\}$.

- For each $\underline{u}^n$and $\underline{x}_{12}^n$, generate $2^{nR_{1c}}$ sequences $\underline{X}_{1c}^n$. Label these $\underline{X}_{1c}^n\left\{w_{1c}, \underline{u}^n(w'_{1d}, w'_{2d})\right\}$ for each, $w_{1c} \in \left\{1, \ldots, 2^{nR_{1c}}\right\}$

- For each $\underline{u}^n$ and $\underline{x}_{21}^n$, generate $2^{nR_{2c}}$ sequences $\underline{X}_{2c}^n$. Label these $\underline{X}_{2c}^n\left\{w_{2c}, \underline{u}^n(w'_{1d}, w'_{2d})\right\}$ for each, $w_{2c} \in \left\{1, \ldots, 2^{nR_{2c}}\right\}$

- For each $\underline{u}^n$, $\underline{x}_{1d}^n$, $\underline{x}_{2d}^n$, $\underline{x}_{21}^n$, $\underline{x}_{2c}^n$ $\underline{x}_{20}^n$ generate $2^{nR_{21}}$ sequences $\underline{\hat{y}}_2^n$. Label these $\underline{\hat{y}}_2^n\,(z_{21},\underline{x}_{21}^n\{z'_{21}, \underline{u}^n(w'_{1d}, w'_{2d})\}, \underline{x}_{1d}^n\{w_{1d}, \underline{u}^n(w'_{1d}, w'_{2d})\}, \underline{x}_{20}^n\{w_{20}, \underline{u}^n(w'_{1d}, w'_{2d})\},$ $\underline{x}_{2d}^n\{w_{2d}, \underline{u}^n(w'_{1d}, w'_{2d})\})$ for each $z_{21} \in \left\{1, \ldots, 2^{nR_{21}}\right\}$.

- For each $\underline{u}^n$, $\underline{x}_{1d}^n$, $\underline{x}_{2d}^n$, $\underline{x}_{12}^n$, $\underline{x}_{1c}^n$ $\underline{x}_{10}^n$ generate $2^{nR_{12}}$ sequences $\underline{\hat{y}}_1^n$. Label these $\underline{\hat{y}}_1^n\,(z_{12},\underline{x}_{12}^n\{z'_{12}, \underline{u}^n(w'_{1d}, w'_{2d})\}, \underline{x}_{2d}^n\{w_{2d}, \underline{u}^n(w'_{1d}, w'_{2d})\}, \underline{x}_{10}^n\{w_{10}, \underline{u}^n(w'_{1d}, w'_{2d})\},$ $\underline{x}_{1d}^n\{w_{1d}, \underline{u}^n(w'_{1d}, w'_{2d})\})$ for each $z_{12} \in \left\{1, \ldots, 2^{nR_{12}}\right\}$.

Encoding and Decoding at each User

Supposse user 1 knows z'_{12}, w'_{2d} from previous block. User 1 decodes w_{2d} by finding $\hat{w}_{2d}$ such that, $(u(w'_{1d}, w'_{2d}), x_{2d}(\hat{w}_{2d}), x_1, y_1)$ are jointly $\epsilon-$ typical. For decoding with arbitrarily low $Pr(e)$, we need

$$R_{2d} < I(X_{2d}; Y_1|U, X_{1d}, X_{12}, X_{1c}) \tag{4.8}$$

then user 1 estimates $\hat{z}_{12}$ such that,

$$(u^n(w'_{1d}, w'_{2d}), x_{2d}(w_{2d}), x_{12}(z'_{12}, u^n(w'_{1d}, w'_{2d})), \tag{4.9}$$

$$x_{1c}(w_{1c}), x_{1d}(w_{1d}), \hat{y}_1(\hat{z}_{12}, x_{12}(z'_{12}, u^n(w'_{1d}, w'_{2d})), x_{2d}(w_{2d}), u^n(w'_{1d}, w'_{2d})), y_1) \tag{4.10}$$

are jointly typical. For this we need,

$$R_{12} > I(Y_1; \hat{Y}_1 | U, X_{1d}, X_{2d}X_{12}, X_{1c}) \tag{4.11}$$

Similarly, for second user we need,

$$R_{21} > I(Y_2; \hat{Y}_2 | U, X_{1d}, X_{2d}X_{21}, X_{2c}) \tag{4.12}$$

and,

$$R_{1d} < I(X_{1d}; Y_2 | U, X_{2d}, X_{21}, X_{2c}) \tag{4.13}$$

Decoding at Receiver

The receiver starts decoding only after receiving the last block $y_0^n(B+2)$. Assume that $w_{1d}, z_{21}, w_{2d}, z_{12}$ have been decoded accurately from block $i+1$. The receiver determines the unique $\hat{w}'_{1d}$and $\hat{w}'_{2d}$ such that $(u^n(\hat{w}'_{1d}, \hat{w}'_{2d}), x_{2d}(\hat{w}'_{2d}, w_{2d}), x_{1d}(\hat{w}'_{1d}, w_{1d}), y_0)$ are jointly $\epsilon-$ typical. For sufficiently large n, $\hat{w}'_{1d} = w'_{1d}$, $\hat{w}'_{2d} = w'_{2d}$ with high probability if

$$R_{1d} + R_{2d} < I(Y_0; U, X_{1d}, X_{2d}) \tag{4.14}$$

Then, the receiver searches for the unique w_{10} and w_{20} such that,$(u^n(w'_{1d}, w'_{2d}), x^n_{1d}(w'_{1d}, w_{1d}), x^n_{2d}(w'_{2d}, w_{2d}), x^n_{10}(w_{1d}, u^n(w'_{1d}, w'_{2d})), x^n_{20}(w_{2d}, u^n(w'_{1d}, w'_{2d})\hat{w}_{1c}), y_0)$ are jointly $\epsilon-$ typical.It is well known that $(u^n(w'_{1d}, w'_{2d}), x_{2d}(w'_{2d}, w_{2d}), x_{1d}(w'_{1d}, w_{1d}), x^n_{10}(w_{1d}, u^n(w'_{1d}, w'_{2d})), y_0)$ and$(u^n(w'_{1d}, w'_{2d}), x_{2d}(w'_{2d}, w_{2d}), x_{1d}(w'_{1d}, w_{1d}), x^n_{20}(w_{2d}, u^n(w'_{1d}, w'_{2d})), y_0)$ also has to be jointly $\epsilon-$ typical. For sufficiently large n $\hat{w}_{10} = w_{10}$ and $\hat{w}_{20} = w_{20}$ if

$$R_{10} < I(X_{10}; Y_0|U, X_{20}, X_{1d}, X_{2d}) \quad (4.15)$$

$$R_{20} < I(X_{20}; Y_0|U, X_{10}, X_{1d}, X_{2d}) \quad (4.16)$$

$$R_{10} + R_{20} < I(X_{10}, X_{20}; Y_0|U, X_{1d}, X_{2d}) \quad (4.17)$$

Next, it searches for the unique $\hat{z}'_{21}$ such that $(u^n(\hat{w}'_{1d}, \hat{w}'_{2d}), x_{2d}(\hat{w}'_{2d}, w_{2d}), x_{1d}(\hat{w}'_{1d}, w_{1d}), \hat{y}_2^n(w'_{1d}, w_{1d}, \hat{z}'_{21}, z_{21}), x_{21}^n(w'_{1d}, \hat{z}'_{21}), y_0)$ are jointly $\epsilon-$ typical. For sufficiently large n $\hat{z}'_{21} = z'_{21}$ with high probability if

$$R_{21} < I(X_{21}, \hat{Y}_2; Y_0|U, X_{1d}, X_{2d}, X_{12}) \quad (4.18)$$

And similarly, for the second user receiver searches for the unique $\hat{z}'_{12}$ such that $(u^n(\hat{w}'_{1d}, \hat{w}'_{2d}), x_{2d}(\hat{w}'_{2d}, w_{2d}), x_{1d}(\hat{w}'_{1d}, w_{1d}), \hat{y}_1^n(w'_{2d}, w_{2d}, \hat{z}'_{12}, z_{12}), x_{12}^n(w'_{2d}, \hat{z}'_{12}), y_0)$ are jointly $\epsilon-$ typical. For sufficiently large n $\hat{z}'_{12} = z'_{12}$ with high probability if

$$R_{12} < I(X_{12}, \hat{Y}_1; Y_0|U, X_{1d}, X_{2d}, X_{21}) \quad (4.19)$$

Finally, the receiver searches the list for the unique $\hat{w}_{1c}$ and $\hat{w}_{2c}$ such that, $(u^n(w'_{1d}, w'_{2d}), x_{1d}^n(w'_{1d}, w_{1d}), x_{2d}^n(w'_{2d}, w_{2d}), x_{12}^n(w'_{2d}, z'_{12}), x_{21}^n(w'_{1d}, z'_{21}), x_{1c}^n(w'_{1d}, w_{1d}, \hat{w}_{1c}), x_{2c}^n(w'_{2d}, w_{2d}, \hat{w}_{2c}), \hat{y}_1^n(w'_{2d}, w_{2d}, z'_{12}, z_{12}), \hat{y}_2^n(w'_{1d}, w_{1d}, z'_{21}, z_{21}), y_0)$ are jointly $\epsilon-$ typical. It is well known that $(u^n(w'_{1d}, w'_{2d}), x_{2d}(w'_{2d}, w_{2d}), x_{1d}(w'_{1d}, w_{1d}), x_{1c}^n(w'_{1d}, w_{1d}, \hat{w}_{1c}), x_{21}^n(w'_{1d}, z'_{21}), \hat{y}_2^n(w'_{1d}, w_{1d}, z'_{21}, z_{21}), y_0)$ and $(u^n(w'_{1d}, w'_{2d}), x_{2d}(w'_{2d}, w_{2d}), x_{1d}(w'_{1d}, w_{1d}), x_{2c}^n(w'_{2d}, w_{2d}, \hat{w}_{2c}), x_{12}^n(w'_{2d}, z'_{12}), \hat{y}_1^n(w'_{2d}, w_{2d}, z'_{12}, z_{12}), y_0)$ also has to be jointly $\epsilon-$ typical. For sufficiently large n, $\hat{w}_{1c} = w_{1c}$ and $\hat{w}_{2c} = w_{2c}$ with high probability if

$$R_{1c} < I(X_{1c}; \hat{Y}_2 Y_0|U, X_{1d}, X_{2d}, X_2, X_{12}, X_{21}) \quad (4.20)$$

$$R_{2c} < I(X_{2c}; \hat{Y}_1 Y_0|U, X_{1d}, X_{2d}, X_1, X_{12}, X_{21}) \quad (4.21)$$

$$R_{1c} + R_{2c} < I(X_{1c}X_{2c}; \hat{Y}_1\hat{Y}_2Y_0|U, X_{1d}, X_{2d}, X_{12}, X_{21}) \tag{4.22}$$

The rates achievable for any two user cooperative multiple access channel are given by,

$$R_1 < \min\{(4.13) + (4.20) + (4.15), (4.14) + (4.20) + (4.15)\} \tag{4.23}$$

$$R_2 < \min\{(4.8) + (4.21) + (4.15), (4.14) + (4.21) + (4.15)\} \tag{4.24}$$

$$R_1 + R_2 < \min\{(4.13) + (4.8) + (4.22) + (4.15), (4.14) + (4.22)\} \tag{4.25}$$

subject to the constraints,

$$I(X_{12}, \hat{Y}_1; Y_0|U, X_{1d}, X_{2d}, X_{21}) > I(Y_1; \hat{Y}_1|U, X_{2d}, X_{12}, X_{1d}, X_{1c}) \tag{4.26}$$

$$I(X_{21}, \hat{Y}_2; Y_0|U, X_{1d}, X_{2d}, X_{12}) > I(Y_2; \hat{Y}_2|U, X_{2d}, X_{21}, X_{1d}, X_{2c}) \tag{4.27}$$

$$I(X_{12}, X_{21}; Y_0|U, X_{1d}, X_{2d}) + I(\hat{Y}_2; Y_0|U, X_{1d}, X_{2d}, X_{12}, X_{21}) \tag{4.28}$$

$$+ I(\hat{Y}_1; Y_0|U, X_{1d}, X_{2d}, X_{12}, X_{21}) >$$

$$I(Y_1; \hat{Y}_1|U, X_{2d}, X_{12}, X_{1d}, X_{1c}) + I(Y_2; \hat{Y}_2|U, X_{2d}, X_{21}, X_{1d}, X_{2c}) \tag{4.29}$$

4.3 Partial Decode Compress Forward for Gaussian MAC:

In this section, we will characterize the achievable rate region using partial decode compress forward for proposed two-user cooperative network as shown in 4.1. Our proposed achievable rate region depends on [2, Theorem 7]which is a combination of the Decode and Forward strategy and Compress and Forward strategy. We used regular block Markov superposition encoding and backward decoding as

Willems suggests in [22], the channel input for each user can be expressed as,

$$X_1 = \sqrt{P_{1c}}X_{1c} + \sqrt{P_{12}}X_{12} + \sqrt{P_{1d}}X_{1d} + \sqrt{P_{10}}X_{10} + \sqrt{P_{1u}}U \tag{4.30}$$

$$X_2 = \sqrt{P_{2c}}X_{2c} + \sqrt{P_{21}}X_{21} + \sqrt{P_{2d}}X_{2d} + \sqrt{P_{20}}X_{20} + \sqrt{P_{2u}}U \tag{4.31}$$

where $X_{1c}, X_{2c}, X_{12}, X_{21}, X_{1d}, X_{2d}, X_{1u}, X_{2u}$ are i.i.d. Gaussian distributed random variables with powers $P_{1c}, P_{12}, P_{2c}, P_{21}, P_{1d}, P_{2d}, P_{u1}, P_{u2}$

$$\begin{aligned} X_{1c} &\sim \mathcal{N}(0, P_{1c}) & X_{2c} &\sim \mathcal{N}(0, P_{2c}) \\ X_{12} &\sim \mathcal{N}(0, P_{12}) & X_{21} &\sim \mathcal{N}(0, P_{21}) \\ X_{1d} &\sim \mathcal{N}(0, P_{1d}) & X_{2d} &\sim \mathcal{N}(0, P_{2d}) \\ X_{1u} &\sim \mathcal{N}(0, P_{u1}) & X_{2u} &\sim \mathcal{N}(0, P_{u2}) \end{aligned}$$

where $P_1 = P_{1c} + P_{12} + P_{1d} + P_{u1}$ and $P_2 = P_{2c} + P_{21} + P_{2d} + P_{u2}$. Compression noise components are chosen zero-mean Gaussian with noise variances $\sigma^2_{w_1}$,$\sigma^2_{w_2}$.

$$N_{w_1} \sim \mathcal{N}(0, \sigma^2_{w_1}) \tag{4.32}$$

$$N_{w_2} \sim \mathcal{N}(0, \sigma^2_{w_2}) \tag{4.33}$$

It is assumed that each user can cancel out its own information from the channel feedback same in Chapter 3.

The rate expressions can be evaluated as follows,

$$\begin{aligned} R_{1d} &< I(X_{1d}; Y_2 | U, X_{2d}, X_{21}, X_{2c}) \\ &= h(Y_2|U, X_{2d}, X_{21}, X_{2c}) - h(Y_2|U, X_{2d}, X_{21}, X_{2c}, X_{1d}) \\ &< \frac{1}{2}\log_2\left(1 + \frac{P_{1d}}{P_{1c} + P_{12} + P_{10} + \sigma_2^2}\right) \end{aligned} \tag{4.34}$$

We can show that,

$$\begin{aligned} R_{10} &< I(X_{10}; Y_0 | U, X_{20}, X_{1d}, X_{2d}) \\ &= h(Y_0|U, X_{20}, X_{1d}, X_{2d}) - h(Y_0|U, X_{10}, X_{1d}, X_{2d}, X_{20}) \\ &< \frac{1}{2}\log_2\left(1 + \frac{P_{10}}{P_{1c} + P_{12} + P_{2c} + P_{21} + \sigma_0^2}\right) \end{aligned} \tag{4.35}$$

Similar to evaluation of (4.34),

$$\begin{aligned} R_{2d} &< I(X_{2d}; Y_1|U, X_{1d}, X_{12}, X_{1c}) \\ &= h(Y_1|U, X_{1d}, X_{12}, X_{1c}) - h(Y_1|U, X_{1d}, X_{12}, X_{1c}, X_{2d}) \\ &< \frac{1}{2}\log_2\left(1 + \frac{P_{2d}}{P_{2c} + P_{21} + P_{20} + \sigma_1^2}\right) \end{aligned} \tag{4.36}$$

We can also show that,

$$\begin{aligned} R_{20} &< I(X_{20}; Y_0|U, X_{10}, X_{1d}, X_{2d}) \\ &= h(Y_0|U, X_{10}, X_{1d}, X_{2d}) - h(Y_0|U, X_{10}, X_{1d}, X_{2d}, X_{20}) \\ &< \frac{1}{2}\log_2\left(1 + \frac{P_{20}}{P_{1c} + P_{12} + P_{2c} + P_{21} + \sigma_0^2}\right) \end{aligned} \tag{4.37}$$

For joint decoding of the cooperative messages w_{1d} and w_{2d} at the receiver, we need

$$\begin{aligned} R_{1d} + R_{2d} &< I(Y_0; U, X_{1d}, X_{2d}) \\ &= h(Y_0) - h(Y_0|U, X_{1d}, X_{2d}) \\ &< \frac{1}{2}\log_2\left(1 + \frac{P_{1d} + P_{u1} + P_{u2} + P_{2d} + P_{10} + P_{20} + 2\sqrt{P_{u1}P_{u2}}}{P_{1c} + P_{12} + P_{2c} + P_{21} + \sigma_0^2}\right) \end{aligned} \tag{4.38}$$

We can also consider,

$$\begin{aligned} R_{10} + R_{20} &< I(X_{10}, X_{20}; Y_0|U, X_{1d}, X_{2d}) \\ &= h(Y_0|U, X_{1d}, X_{2d}) - h(Y_0|U, X_{1d}, X_{2d}, X_{10}, X_{20}) \\ &< \frac{1}{2}\log_2\left(1 + \frac{P_{10} + P_{20}}{P_{1c} + P_{12} + P_{2c} + P_{21} + \sigma_0^2}\right) \end{aligned} \tag{4.39}$$

Finally we can consider,

$$
\begin{aligned}
R_{1c} &< I(X_{1c}; \hat{Y}_2 Y_0 | U, X_{1d}, X_{2d}, X_2, X_{12}, X_{21}) \\
&= h(\hat{Y}_2 Y_0 | U, X_{1d}, X_{2d}, X_2, X_{12}, X_{21}) \\
&\quad - h(\hat{Y}_2 Y_0 | U, X_{1d}, X_{2d}, X_2, X_{12}, X_{21}, X_{1c}) \\
&< \frac{1}{2}\log_2\left(1 + \frac{P_{1c}(\sigma_2^2 + \sigma_{w_2}^2 + \sigma_0^2)}{(\sigma_2^2 + \sigma_{w_2}^2)\sigma_0^2}\right)
\end{aligned} \tag{4.40}
$$

Similar to evaluation of (4.40)

$$
\begin{aligned}
R_{2c} &< I(X_{2c}; \hat{Y}_1 Y_0 | U, X_{2d}, X_{1d}, X_1, X_{21}, X_{12}) \\
&= h(\hat{Y}_1 Y_0 | U, X_{2d}, X_{1d}, X_1, X_{21}, X_{12}) \\
&\quad - h(\hat{Y}_1 Y_0 | U, X_{2d}, X_{1d}, X_1, X_{21}, X_{12}, X_{2c}) \\
&< \frac{1}{2}\log_2\left(1 + \frac{P_{2c}(\sigma_1^2 + \sigma_{w_1}^2 + \sigma_0^2)}{(\sigma_1^2 + \sigma_{w_1}^2)\sigma_0^2}\right)
\end{aligned} \tag{4.41}
$$

We can consider,

$$
\begin{aligned}
R_{1c} + R_{2c} &< I(X_{1c}X_{2c}; \hat{Y}_1 \hat{Y}_2 Y_0 | U, X_{1d}, X_{2d}, X_{12}, X_{21}) \\
&= h(\hat{Y}_1 \hat{Y}_2 Y_0 | U, X_{1d}, X_{2d}, X_{12}, X_{21}) \\
&\quad - h(\hat{Y}_1 \hat{Y}_2 Y_0 | U, X_{1d}, X_{2d}, X_{12}, X_{21}, X_{1c}, X_{2c})
\end{aligned}
$$

Hence, we have

$$
R_{1c} + R_{2c} < \frac{1}{2}\log_2\Big(1 + \frac{P_{1c} + P_{2c}}{\sigma_0^2} + \frac{P_{1c}}{\sigma_2^2 + \sigma_{w_2}^2} + \frac{P_{2c}}{\sigma_1^2 + \sigma_{w_1}^2} \tag{4.42}
$$

$$
+ \frac{P_{1c}P_{2c}(\sigma_1^2 + \sigma_{w_1}^2 + \sigma_2^2 + \sigma_{w_2}^2 + \sigma_0^2)}{\sigma_0^2(\sigma_1^2 + \sigma_{w_1}^2)(\sigma_2^2 + \sigma_{w_1}^2)}\Big) \tag{4.43}
$$

The detailed computations can be found in Appendix A.

4.4 Simulation Results:

In this section, we provided simulation results comparing all different possible channel conditions. We used two-user mutual cooperation based on DF and CF strategy in order to compare mutual cooperation based on partial DF and CF strategy. In figure 4.2, we considered an extra case which is based on a hybrid strategy mentioned in [16]where User 1 performs DF to User 2 and User 2 performs CF to User 1. In the table 4.1 we can see all different possible channel conditions.

Table 4.1: Scenarios

#	User1 to User2	User1 to Receiver	User2 to User1	User2 to Receiver
1	Weak	Weak	Weak	Weak
2	Weak	Strong	Weak	Weak
3	Weak	Weak	Strong	Weak
4	Weak	Weak	Strong	Strong
5	Weak	Strong	Weak	Strong
6	Weak	Strong	Strong	Weak
7	Weak	Strong	Strong	Strong
8	Strong	Weak	Strong	Weak
9	Strong	Weak	Strong	Strong

We used several channel conditions in order to compare the performances of each relaying protocol. In the scenario, where all channels have same quality , both users can achieve higher data rates individually using CF protocol, but in the sum rate region, DF outperforms CF as in figure 4.2. Since the joint CF/DF scheme can be achieved by the time sharing between the CF and DF protocols both users can achieve higher data rates with using joint CF/DF. In the scenario where, the link from user 1 to user 2 weaker than the link from user 2 to user 1, we can observe from figure 4.8 if second user tries to use DF it gives worse results with respect to CF and joint CF/DF strategy at individual part of rate regions. At sum rate part DF gives better results and also joint CF/DF always outperforms with respect to other strategies. In figure 4.9, it is obviously seen that, when the inter-user links are strong DF always outperforms CF and our combined strategy based on [2, Theorem 7] gave the same results. On the other hand, we observed that, hybrid strategy is one of the specific case of [2, Theorem 7] and does not extend the achievable rate region for mutual two-user cooperative MAC.

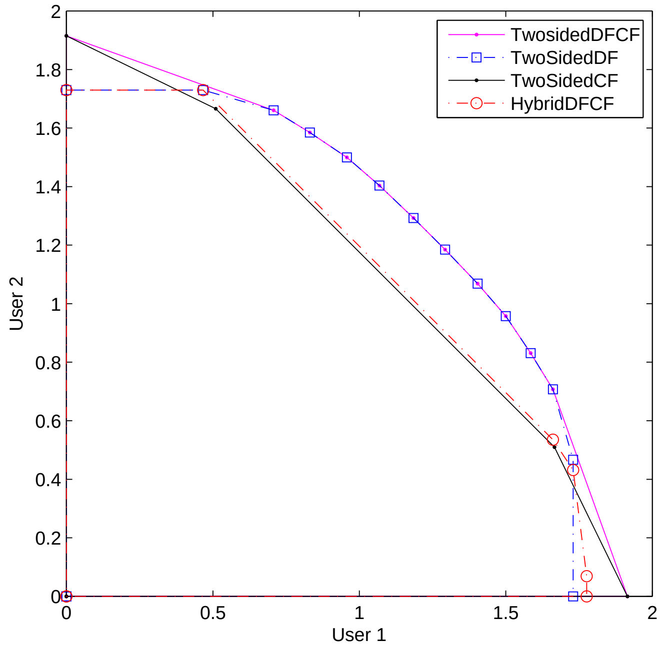

Figure 4.2: Comparison of achievable rate regions for a two-user cooperative network when $P_1 = 10 \quad P_2 = 10 \quad \sigma_0^2 = 1 \quad \sigma_1^2 = 1 \quad \sigma_2^2 = 1$.

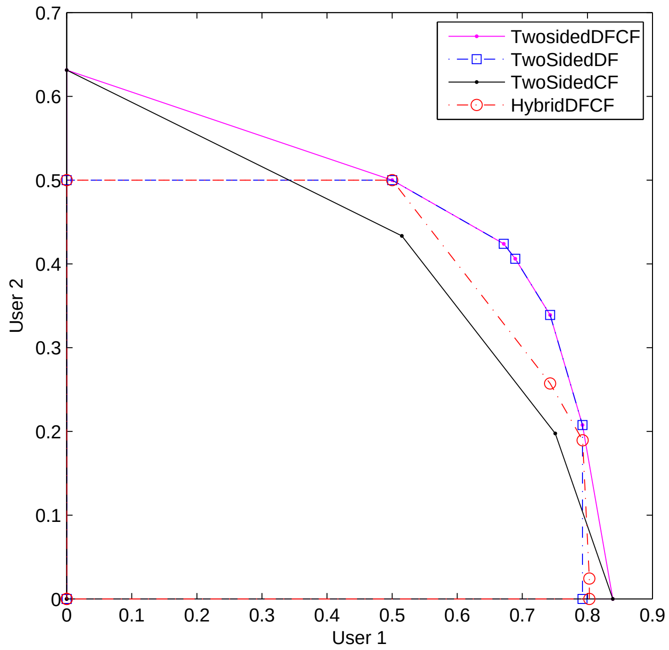

Figure 4.3: Comparison of achievable rate regions for a two-user cooperative network when $P_1 = 10 \quad P_2 = 5 \quad \sigma_0^2 = 5 \quad \sigma_1^2 = 5 \quad \sigma_2^2 = 10$.

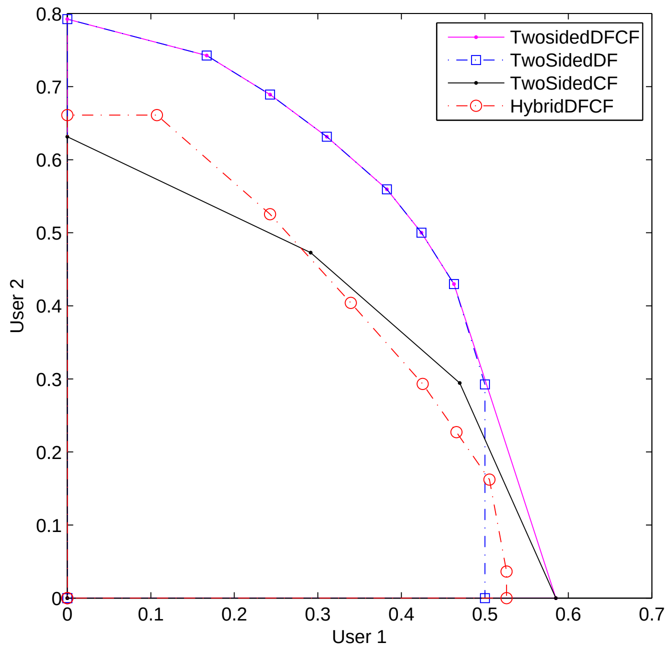

Figure 4.4: Comparison of achievable rate regions for a two-user cooperative network when $P_1 = 10 \quad P_2 = 10 \quad \sigma_0^2 = 10 \quad \sigma_1^2 = 5 \quad \sigma_2^2 = 10$

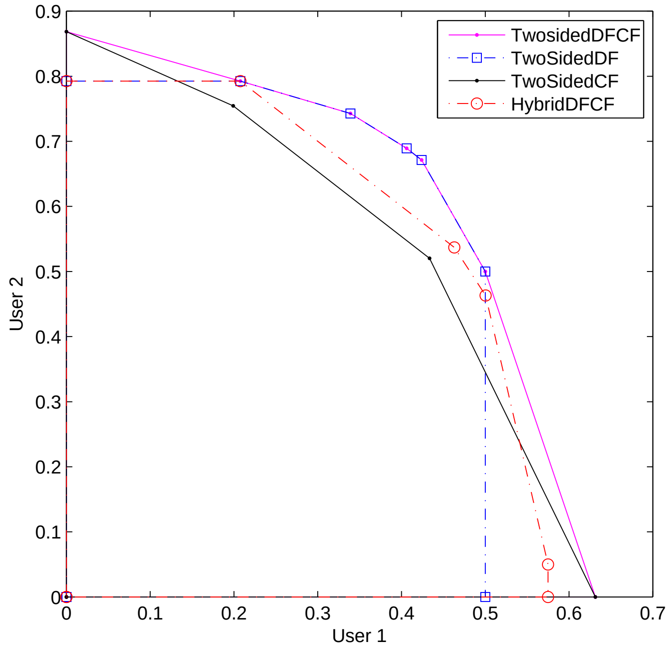

Figure 4.5: Comparison of achievable rate regions for a two-user cooperative network when$P_1 = 5 \quad P_2 = 10 \quad \sigma_0^2 = 5 \quad \sigma_1^2 = 5 \quad \sigma_2^2 = 5$.

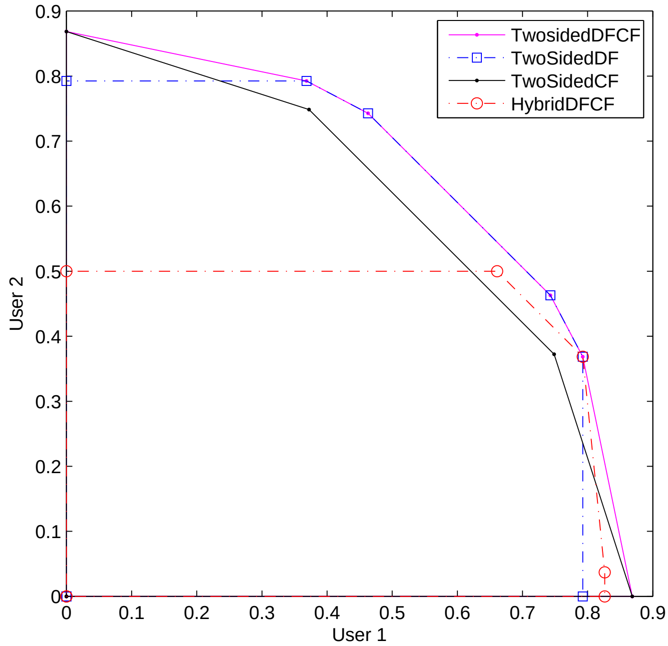

Figure 4.6: Comparison of achievable rate regions for a two-user cooperative network when$P_1 = 10 \quad P_2 = 10 \quad \sigma_0^2 = 5 \quad \sigma_1^2 = 10 \quad \sigma_2^2 = 10$.

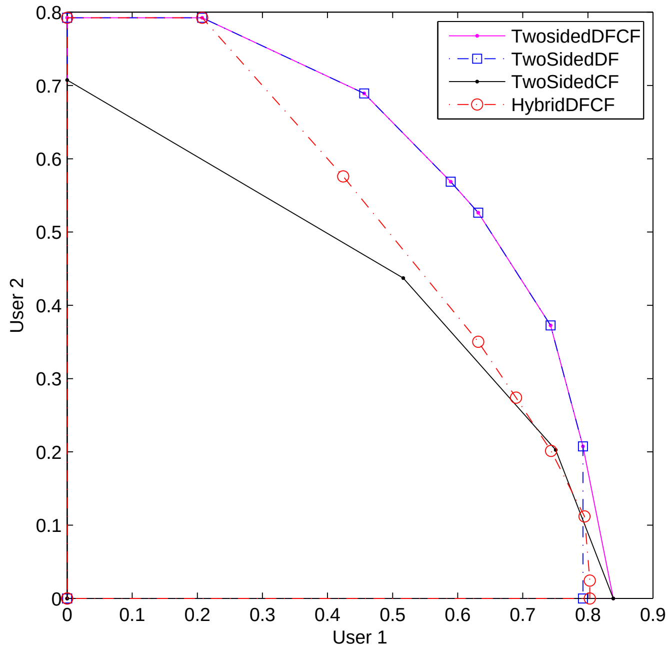

Figure 4.7: Comparison of achievable rate regions for a two-user cooperative network when$P_1 = 10 \quad P_2 = 5 \quad \sigma_0^2 = 5 \quad \sigma_1^2 = 2.5 \quad \sigma_2^2 = 10$.

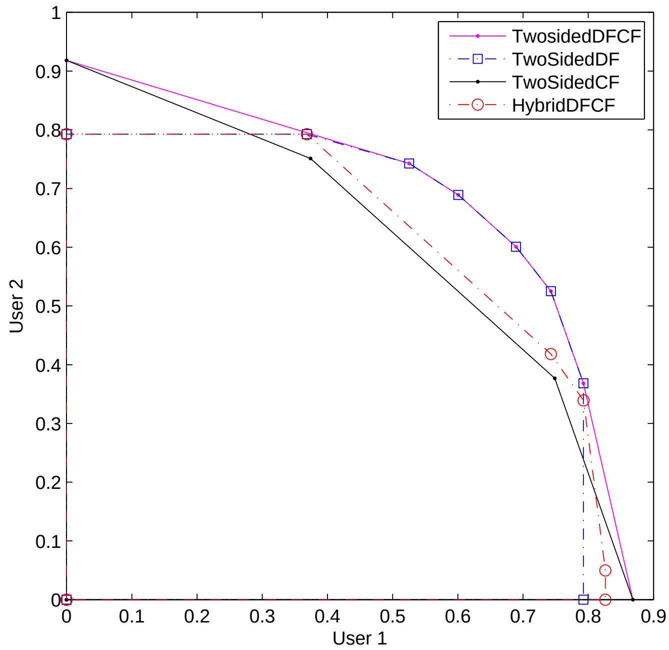

Figure 4.8: Comparison of achievable rate regions for a two-user cooperative network when$P_1 = 10 \quad P_2 = 10 \quad \sigma_0^2 = 5 \quad \sigma_1^2 = 5 \quad \sigma_2^2 = 10$.

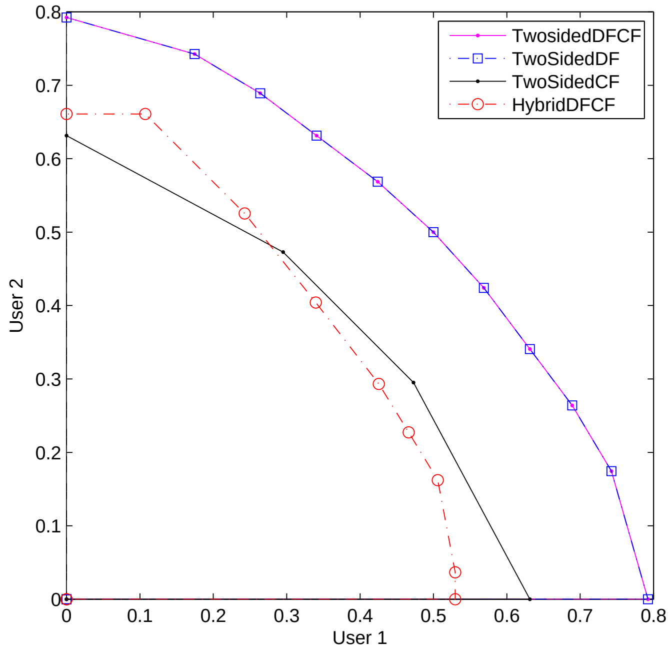

Figure 4.9: Comparison of achievable rate regions for a two-user cooperative network when$P_1 = 10$ $P_2 = 10$ $\sigma_0^2 = 10$ $\sigma_1^2 = 5$ $\sigma_2^2 = 5$.

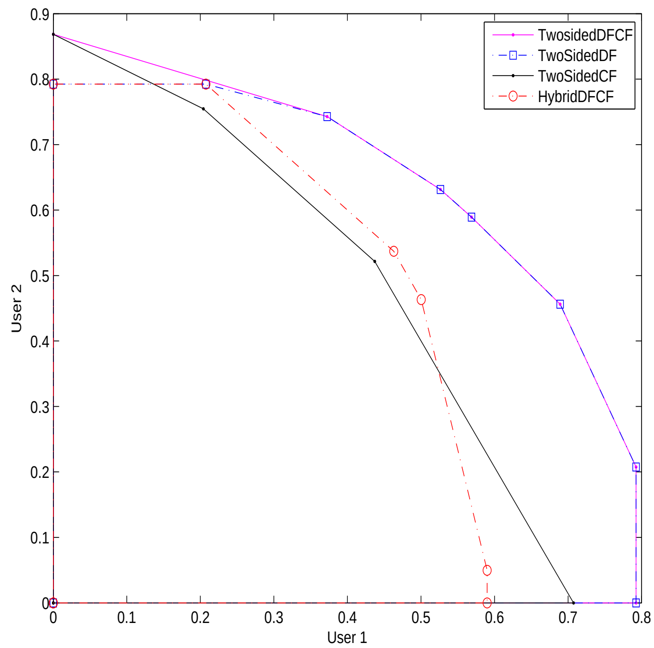

Figure 4.10: Comparison of achievable rate regions for a two-user cooperative network when$P_1 = 5$ $P_2 = 10$ $\sigma_0^2 = 5$ $\sigma_1^2 = 5$ $\sigma_2^2 = 2.5$.

Conclusion

In this thesis, we presented two new achievable rate regions for a two-user cooperative multiple access channel where both users (i) for the case employ compress and forward,(ii) joint CF/DF. We compared the performance of our proposed schemes with other mutual cooperation strategies where both users perform DF; and one user perform DF and other user performs CF scheme. In Chapter 3, we derived and characterized an achievable rate region for compress and forward for a two-user mutually cooperative MAC. Our achievable rate scheme is based on [2, Theorem 6]. In chapter 4, we derived and characterized an achievable rate region for joint CF/DF for same system setup in Chapter 3. Our achievable rate scheme is based on [2, Theorem 7]. The strategies make use of regular block Markov superposition encoding and Willems' backward decoding. From the results we can see that, DF always outperforms CF and the rate regions achievable using joint CF/DF strategy can also be achieved by time sharing between the achievable rate regions of individual DF and CF policies.

References

[1] E. C. van der Meulen. "Three-terminal communication channels". *Advances in Applied Probability*, 3:121, 1971.

[2] T. Cover and A.E. Gamal. "Capacity theorems for the relay channel". *IEEE Transactions on Information Theory*, 25(5):572 – 584, sep 1979.

[3] G. Kramer, M. Gastpar, and P. Gupta. "Cooperative Strategies and Capacity Theorems for Relay Networks". *IEEE Transactions on Information Theory*, 51(9):3037 – 3063, sept. 2005.

[4] S. Simoens, O. Munoz, and J. Vidal. "Achievable Rates of Compress-and-Forward Cooperative Relaying on Gaussian Vector Channels". In *IEEE International Conference on Communications ICC '07.*, pages 4225 –4231, june 2007.

[5] A. Host-Madsen and J. Zhang. "Capacity bounds and power allocation for wireless relay channels". *IEEE Transactions on Information Theory* , 51(6): 2020 –2040, june 2005.

[6] Sang Joon Kim, N. Devroye, P. Mitran, and V. Tarokh. "Comparison of bi-directional relaying protocols". In *IEEE Sarnoff Symposium, 2008*, pages 1 –5, april 2008.

[7] Lili Zhang, Jinhua Jiang, and Shuguang Cui. "Study of Half-Duplex Gaussian Relay Channels with Correlated Noises". In *IEEE Wireless Communications and Networking Conference (WCNC), 2010* , pages 1 –5, april 2010.

[8] D. Gunduz, E. Tuncel, and J. Nayak. “Rate regions for the separated two-way relay channel”. In *46th Annual Allerton Conference on Communication, Control, and Computing, 2008* , pages 1333 –1340, sept. 2008.

[9] P. Rost and G. Fettweis. “Achievable Rates in Gaussian Half-Duplex Multiple Relay Networks”. In *IEEE Global Telecommunications Conference, 2008. IEEE GLOBECOM 2008.* , pages 1 –6, 30 2008-dec. 4 2008.

[10] Jie Yang, D. Gunduz, D.R. Brown, and E. Erkip. “Resource allocation for cooperative relaying”. In *42nd Annual Conference on Information Sciences and Systems, 2008. CISS 2008.*, pages 848 –853, march 2008.

[11] M.A. Khojastepour, A. Sabharwal, and B. Aazhang. “Improved achievable rates for user cooperation and relay channels”. In *International Symposium on Information Theory, 2004. ISIT 2004. Proceedings.* , page 4, june-2 july 2004.

[12] L. Ong and M. Motani. “Coding Strategies for Multiple-Access Channels With Feedback and Correlated Sources”. *IEEE Transactions on Information Theory*, 53(10):3476 –3497, oct. 2007.

[13] Liang-Liang Xie. *arXiv:0908.0163v2.*

[14] H.-F. Chong, M. Motani, and H. K. Garg. “Generalized Backward Decoding Strategies for the Relay Channel”. *IEEE Transactions on Information Theory*, 53(1):394 –401, jan. 2007.

[15] Hon-Fah Chong and M. Motani. “On Achievable Rates for the General Relay Channel”. *IEEE Transactions on Information Theory*, 57(3):1249 –1266, march 2011.

[16] Qian Li, K.H. Li, and K.C. Teh. “An achievable rate region of cooperative multiple-access channels with hybrid CF and DF cooperation”. *IEEE Communications Letters*, 13(8):591 –593, august 2009.

[17] E. Erkip A. Sendonaris and B. Aazhang. "User cooperation diversity-Part i: System description". *IEEE Transactions on Communications*, 51(11):1927 –1938, november 2003.

[18] Thomas M. Cover and Joy A. Thomas. *Elements of Information Theory.* Wiley-Interscience, New Jersey,USA, 1991.

[19] M. Gastpar and M. Vetterli. "On the capacity of wireless networks: the relay case". In *IEEE Twenty-First Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. INFOCOM 2002.*, volume 3, pages 1577 – 1586 vol.3, 2002.

[20] E. C. van der Meulen F. M. J. Willems and J. P. M. Schalkwijk. An achievable rate region for the multiple access channel with generalized feedback. In *Proc. Allerton Conf., Monticello, IL.* IEEE Press.

[21] A. Carleial. "Multiple-access channels with different generalized feedback signals". *IEEE Transactions on Information Theory*, 28(6):841 – 850, nov 1982.

[22] F. M. J. Willems. *"*I*nformation theoretical results for the discrete memoryless multiple access channel".* PhD thesis, Katholieke Universiteit, November 1982.

[23] C.-M. Zeng, F. Kuhlmann, and A. Buzo. "Achievability proof of some multiuser channel coding theorems using backward decoding". *IEEE Transactions on Information Theory*, 35(6):1160 –1165, nov 1989.

[24] M. Gastpar. "On noisy feedback in Gaussian networks". *Proc. Annu. Allerton Conf. Communication, Control, Computing*, Sept 2005.

[25] L. Ozarow. "The capacity of the white Gaussian multiple access channel with feedback". *IEEE Transactions on Information Theory*, 30(4):623 – 629, jul 1984.

[26] M. Gastpar and G. Kramer. "On Cooperation Via Noisy Feedback". In *2006 International Zurich Seminar on Communications*, pages 146 –149, 0-0 2006.

[27] E. Erkip A. Sendonaris and B. Aazhang. "User cooperation diversity-Part ii: Implementation aspects and performance analysis". *IEEE Transactions on Communications*, 51(11):1939 –1948, november 2003.

[28] J.N. Laneman, D.N.C. Tse, and G.W. Wornell. "Cooperative diversity in wireless networks: Information Theory, Efficient protocols and outage behavior". *IEEE Transactions on Information Theory*, 50(12):3062 – 3080, dec. 2004.

[29] E. Ekrem and S. Ulukus. "Effects of cooperation on the secrecy of multiple access channels with generalized feedback". In *42nd Annual Conference on Information Sciences and Systems, 2008. CISS 2008.*, pages 791 –796, march 2008.

[30] R. Tannious and A. Nosratinia. "Relay Channel With Private Messages". *IEEE Transactions on Information Theory*, 53(10):3777 –3785, oct. 2007.

Appendix A:Derivations

In this section we will give the details of the computations in Chapter 4. From (4.34) we can compute,

$$\begin{aligned}
R_{1d} &< I(X_{1d};Y_2|U,X_{2d},X_{21},X_{2c}) \\
&= h(Y_2|U,X_{2d},X_{21},X_{2c}) - h(Y_2|U,X_{2d},X_{21},X_{2c},X_{1d}) \\
&= h(X_{1d}+X_{1c}+X_{12}+X_{10}+N_2) - h(X_{1c}+X_{12}+X_{10}+N_2) \\
&= \frac{1}{2}\log_2\left(\frac{(2\pi e)(P_{1d}+P_{1c}+P_{12}+P_{10}+\sigma_2^2)}{(2\pi e)(P_{1c}+P_{12}+P_{10}+\sigma_2^2)}\right) \\
&< \frac{1}{2}\log_2\left(1+\frac{P_{1d}}{P_{1c}+P_{12}+P_{10}+\sigma_2^2}\right) \qquad (44)
\end{aligned}$$

From (4.35), let us consider,

$$\begin{aligned}
R_{10} &< I(X_{10};Y_0|U,X_{20},X_{1d},X_{2d}) \\
&= h(Y_0|U,X_{20},X_{1d},X_{2d}) - h(Y_0|U,X_{10},X_{1d},X_{2d},X_{20}) \\
&= h(X_{1c}+X_{12}+X_{10}+X_{2c}+X_{21}+N_0) - h(X_{1c}+X_{12}+X_{2c}+X_{21}+N_0) \\
&= \frac{1}{2}\log_2\left(\frac{(2\pi e)(P_{12}+P_{10}+P_{1c}+P_{21}+P_{2c}+\sigma_0^2)}{(2\pi e)(P_{12}+P_{1c}+P_{21}+P_{2c}+\sigma_0^2)}\right) \\
&< \frac{1}{2}\log_2\left(1+\frac{P_{10}}{P_{1c}+P_{12}+P_{2c}+P_{21}+\sigma_0^2}\right) \qquad (45)
\end{aligned}$$

We can compute $I(X_{2d}; Y_1|U, X_{1d}, X_{12}, X_{1c})$ as follows,

$$
\begin{aligned}
R_{2d} &< I(X_{2d}; Y_1|U, X_{1d}, X_{12}, X_{1c}) \\
&= h(Y_1|U, X_{1d}, X_{12}, X_{1c}) - h(Y_1|U, X_{1d}, X_{12}, X_{1c}, X_{2d}) \\
&= h(X_{2d} + X_{2c} + X_{21} + X_{20} + N_1) - h(X_{2c} + X_{21} + X_{20} + N_1) \\
&= \frac{1}{2}\log_2\left(\frac{(2\pi e)(P_{2d} + P_{2c} + P_{21} + P_{20} + \sigma_1^2)}{(2\pi e)(P_{2c} + P_{21} + P_{20} + \sigma_1^2)}\right) \\
&< \frac{1}{2}\log_2\left(1 + \frac{P_{2d}}{P_{2c} + P_{21} + P_{20} + \sigma_1^2}\right) \qquad (46)
\end{aligned}
$$

Next, let us consider,

$$
\begin{aligned}
R_{20} &< I(X_{20}; Y_0|U, X_{10}, X_{1d}, X_{2d}) \\
&= h(Y_0|U, X_{10}, X_{1d}, X_{2d}) - h(Y_0|U, X_{20}, X_{1d}, X_{2d}, X_{10}) \\
&= h(X_{1c} + X_{12} + X_{20} + X_{2c} + X_{21} + N_0) - h(X_{1c} + X_{12} + X_{2c} + X_{21} + N_0) \\
&= \frac{1}{2}\log_2\left(\frac{(2\pi e)(P_{21} + P_{20} + P_{2c} + P_{12} + P_{1c} + \sigma_0^2)}{(2\pi e)(P_{21} + P_{2c} + P_{12} + P_{1c} + \sigma_0^2)}\right) \\
&< \frac{1}{2}\log_2\left(1 + \frac{P_{20}}{P_{2c} + P_{21} + P_{1c} + P_{12} + \sigma_0^2}\right) \qquad (47)
\end{aligned}
$$

From (4.39) we can also consider,

$$
\begin{aligned}
R_{10} + R_{20} &< I(X_{10}, X_{20}; Y_0|U, X_{1d}, X_{2d}) \\
&= h(Y_0|U, X_{1d}, X_{2d}) - h(Y_0|U, X_{1d}, X_{2d}, X_{10}, X_{20}) \\
&= h(X_{1c} + X_{12} + X_{10} + X_{2c} + X_{21} + X_{20} + N_0) \\
&\quad - h(X_{1c} + X_{12} + X_{2c} + X_{21} + N_0) \\
&= \frac{1}{2}\log_2\left(\frac{(2\pi e)(P_{10} + P_{12} + P_{20} + P_{1c} + P_{21} + P_{2c} + \sigma_0^2)}{(2\pi e)(P_{12} + P_{1c} + P_{21} + P_{2c} + \sigma_0^2)}\right) \\
&< \frac{1}{2}\log_2\left(1 + \frac{P_{10} + P_{20}}{P_{1c} + P_{12} + P_{2c} + P_{21} + \sigma_0^2}\right) \qquad (48)
\end{aligned}
$$

We can compute $I(Y_0; U, X_{1d}, X_{2d})$ as,

$$
\begin{aligned}
R_{1d} + R_{2d} &< I(Y_0; U, X_{1d}, X_{2d}) \\
&= h(Y_0) - h(Y_0|U, X_{1d}, X_{2d}) \\
&= h(X_1 + X_2 + N_0) - h(X_{1c} + X_{12} + X_{2c} + X_{21} + N_0) \\
&= h(U + X_{1c} + X_{12} + X_{10} + X_{1d} + X_{2c} + X_{21} + X_{20} + X_{2d} + N_0) \\
&\quad - h(X_{1c} + X_{12} + X_{10} + X_{2c} + X_{21} + X_{20} + N_0) \\
&< \frac{1}{2}\log_2\left(1 + \frac{P_{10} + P_{20} + P_{1d} + P_{1u} + P_{2u} + P_{2d} + 2\sqrt{P_{1u}P_{2u}}}{P_{1c} + P_{12} + P_{2c} + P_{21} + \sigma_0^2}\right) \quad (49)
\end{aligned}
$$

We can also compute (4.40),(4.41)and (4.42) as follows,

$$
\begin{aligned}
R_{1c} &< I(X_{1c}; \hat{Y}_2 Y_0 | U, X_{1d}, X_{2d}, X_2, X_{12}, X_{21}) \\
&= h(\hat{Y}_2 Y_0 | U, X_{1d}, X_{2d}, X_2, X_{12}, X_{21}) \\
&\quad - h(\hat{Y}_2 Y_0 | U, X_{1d}, X_{2d}, X_2, X_{12}, X_{21}, X_{1c}) \\
&= h(X_{1c} + \sigma_2^2 + \sigma_{w_2}^2, X_{1c} + \sigma_0^2) - h(\sigma_1^2 + \sigma_{w_1}^2, \sigma_0^2) \\
&= \frac{1}{2}\log_2\left\{\frac{(2\pi e)\begin{vmatrix} P_{1c}+\sigma_2^2+\sigma_{w_2} & P_{1c} \\ P_{1c} & P_{1c}+\sigma_0^2 \end{vmatrix}}{(2\pi e)\sigma_0^2(\sigma_1^2 + \sigma_{w_1^2})}\right\} \\
&< \frac{1}{2}\log_2\left(1 + \frac{P_{1c}(\sigma_2^2 + \sigma_{w_2}^2 + \sigma_0^2)}{(\sigma_2^2 + \sigma_{w_2}^2)\sigma_0^2}\right) \quad (50)
\end{aligned}
$$

And,

$$
\begin{aligned}
R_{2c} &< I(X_{2c}; \hat{Y}_1 Y_0 | U, X_{2d}, X_{1d}, X_1, X_{21}, X_{12}) \\
&= h(\hat{Y}_1 Y_0 | U, X_{2d}, X_{1d}, X_1, X_{21}, X_{12}) \\
&\quad - h(\hat{Y}_1 Y_0 | U, X_{2d}, X_{1d}, X_1, X_{21}, X_{12}, X_{2c}) \\
&= h(X_{2c} + \sigma_1^2 + \sigma_{w_1}^2, X_{2c} + \sigma_0^2) - h(\sigma_2^2 + \sigma_{w_2}^2, \sigma_0^2) \\
&= \frac{1}{2}\log_2\left\{\frac{(2\pi e)\begin{vmatrix} P_{2c}+\sigma_1^2+\sigma_{w_1}^2 & P_{2c} \\ P_{2c} & P_{2c}+\sigma_0^2 \end{vmatrix}}{(2\pi e)\sigma_0^2(\sigma_2^2 + \sigma_{w_2}^2)}\right\} \\
&< \frac{1}{2}\log_2\left(1 + \frac{P_{2c}(\sigma_1^2 + \sigma_{w_1}^2 + \sigma_0^2)}{(\sigma_1^2 + \sigma_{w_1}^2)\sigma_0^2}\right) \quad (51)
\end{aligned}
$$

Hence,

$$
\begin{aligned}
R_{1c} + R_{2c} &< I(X_{1c}X_{2c}; \hat{Y}_1\hat{Y}_2Y_0|U, X_{1d}, X_{2d}, X_{12}, X_{21}) \\
&= h(\hat{Y}_1\hat{Y}_2Y_0|U, X_{1d}, X_{2d}, X_{12}, X_{21}) \\
&\quad - h(\hat{Y}_1\hat{Y}_2Y_0|U, X_{1d}, X_{2d}, X_{12}, X_{21}, X_{1c}, X_{2c}) \\
&= h(X_{2c} + \sigma_1^2 + \sigma_{w_1}^2, X_{1c} + \sigma_2^2 + \sigma_{w_2}^2 + X_{1c} + X_{2c} + \sigma_0^2) \\
&\quad - h(\sigma_1^2 + \sigma_{w_1}^2, \sigma_2^2 + \sigma_{w_2}^2, \sigma_0^2) \\
&= \frac{1}{2}\log_2 \left\{ \frac{(2\pi e)\begin{vmatrix} P_{2c}+\sigma_1^2+\sigma_{w_1}^2 & 0 & P_{2c} \\ 0 & P_{1c}+\sigma_2^2+\sigma_{w_2}^2 & P_{1c} \\ P_{2c} & P_{1c} & P_{1c}+P_{2c}+\sigma_0^2 \end{vmatrix}}{(2\pi e)\sigma_0^2(\sigma_2^2 + \sigma_{w_2}^2)(\sigma_1^2 + \sigma_{w_1}^2)} \right\} \\
&< \frac{1}{2}\log_2 \Bigg(1 + \frac{P_{1c} + P_{2c}}{\sigma_0^2} + \frac{P_{1c}}{\sigma_2^2 + \sigma_{w_2}^2} + \frac{P_{2c}}{\sigma_1^2 + \sigma_{w_1}^2} \\
&\qquad + \frac{P_{1c}P_{2c}(\sigma_1^2 + \sigma_{w_1}^2 + \sigma_2^2 + \sigma_{w_2}^2 + \sigma_0^2)}{\sigma_0^2(\sigma_1^2 + \sigma_{w_1}^2)(\sigma_2^2 + \sigma_{w_1}^2)} \Bigg) \qquad (52)
\end{aligned}
$$

Curriculum Vitae

Mehmet GÜNEŞ was born in 13 August 1985, in İstanbul. He received his B.S. degree in Electronics Engineering in 2008 from Işık University. He worked as a teaching assistant at the Department of Electronics Engineering of Işık University from 2008 to 2011. His research interests include wireless communication, information theory, and cooperative communication strategies.